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Brief paper

Distributed optimal coordination for multiple heterogeneous Euler–Lagrangian systems ☆

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ABSTRACT

In this paper, we consider the distributed optimal coordination (DOC) problem for multi-agent systems with the agents in the form of Euler–Lagrangian (EL) dynamics. We propose two different distributed protocols for the heterogeneous continuous-time EL agents to achieve the optimization task. By constructing suitable Lyapunov functions, we prove the global convergence to the optimal coordination of the EL systems in the case with parametric uncertainties, and the global exponential convergence in the case without parametric uncertainties. Furthermore, we estimate the regret bound for an uncertain DOC problem with time-varying cost functions and inexact gradients. Finally, we provide a numerical example to validate the theoretical results.

1. Introduction

The distributed optimization problem (DOP) has attracted much attention due to various practical applications in convex computation, resource allocation, and localization (Nedic & Ozdaglar, 2009; Yi, Hong, & Liu, 2016; Zhang, Lou, Hong & Xie, 2015), where a group of agents cooperatively solve a global optimization problem with the objective function as the sum of privately-known local cost functions. Recently, more and more efforts have been made for the continuous-time distributed optimization algorithm design (Kia, Cortés, & Martínez, 2015; Wang, Yi, & Hong, 2014), mainly because practical continuous-time systems are required to achieve the optimization and some well-known control techniques may facilitate the analysis and algorithm design of DOP. For example, control perspective was employed to the design of distributed optimization algorithms based on the standard Lyapunov theory in Kia et al. (2015) and Wang and Elia (2011), while an internal-model-based design was proposed to solve the DOP with disturbance rejection in Wang et al. (2014).

The last decade has witnessed the flourishing research activities on cyber–physical systems, with an integration of computation and communication with physical processes. In fact, distributed optimization can be completed based on effective combination of the (cyber) computation/communication and the (physical) dynamics/control in many applications such as the target aggregation in robotic networks (Meng et al., 2014), and the optimal power flow in smart grids (Zhang, Papachristodoulou, & Li, 2015). In these situations, a group of continuous-time physical systems, such as robots and power generators, are considered to cooperatively achieve the corresponding optimal performance, in addition to their stabilization or tracking concerns. To avoid the confusion with the conventional DOP, we term this optimization problem with physical dynamics as DOC problems.

One of the important physical systems is the EL system, which can be used to describe many mechanical systems, such as mobile robots, rigid bodies, and autonomous vehicles (Spong, Hutchinson, & Vidyasagar, 2006). Motivated by various applications, including spacecraft formation, attitude control of multiple rigid bodies, cooperative search of multiple mobile robots, the study on the distributed control of EL systems has drawn much attention in recent years (referring to Cai & Huang, 2014; Foderaro, Ferrari, & Wettergren, 2014; Meng et al., 2014). Also, few results have been obtained for the DOC of the EL systems such as the semi-global optimization results in Deng and Hong (2016).

The main objective of this paper is to study the general DOC designs with global convergence for multiple heterogeneous EL systems, even in the presence of uncertainties. The technical

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contributions of the paper include: (i) We formulate a DOC problem for EL systems to investigate how a group of EL agents cooperatively achieve optimization. This formulation can be viewed as extensions of some well-known problems, including DOP discussed in Kia et al. (2015) and Wang and Elia (2011) by adding EL dynamics, multi-agent consensus of EL agents (see Cai & Huang, 2014 and reference therein) by adding optimization concerns, and the set aggregation of EL agents in Meng et al. (2014) by considering general optimization functions. (ii) With convex optimization and nonlinear control techniques, we provide two distributed gradient-based algorithms for the multiple EL agents. Then we prove the global convergence to the optimal coordination of the system with parametric uncertainty, and the global exponential convergence in the case without parametric uncertainty. Note that the results are different from that for the double-integrator agents without any convergence rate analysis by LaSalle's invariance principle in Zhang and Hong (2014) and that with only semi-global results in Deng and Hong (2016). (iii) We investigate the uncertain DOC problem with time-varying cost functions and inexact gradients for EL agents and estimate the online regret bound in this case.

This paper is organized as follows. Section 2 formulates the DOC problem with related preliminaries. Section 3 presents two different distributed algorithms for the multiple EL agents to achieve DOC along with their convergence analysis. Section 4 discusses an uncertain optimization problem with the regret bound analysis. Section 5 gives an example to illustrate the proposed algorithms. Finally, Section 6 gathers concluding remarks.

Notations: $\|\cdot\|$ denotes the Euclidean norm of a vector or matrix. For $x_i \in \mathbb{R}^m$, $i = 1, \ldots, n$, $col(x_1, \ldots, x_n) = (x_1', \ldots, x_n')'$, where x' is the transpose of x. Denote the increasingly ordered eigenvalues of matrix $X \in \mathbb{R}^{n\times n}$ by $\lambda_1(X), \ldots, \lambda_n(X)$. Let $\mathbf{1}_n$ and $\mathbf{0}_n$ be the n-dimensional vectors of all entries as 1 and 0, respectively.

2. Preliminaries and formulation

In this section, we first review related preliminaries from graph theory (Godsil & Royle, 2001) and convex analysis (Rockafellar, 1972), and then formulate our problem.

2.1. Preliminaries

An undirected graph is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. An edge $(i, j) \in \mathcal{E}$ shows that vertices i, j can communicate with each other. If there is a path between any two vertices of $\mathcal{G}$, then $\mathcal{G}$ is connected. The weighted adjacency matrix of $\mathcal{G}$ is denoted by $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The degree matrix $D = \mathrm{diag}(d_1, \ldots, d_n) \in \mathbb{R}^{n\times n}$ is a diagonal matrix with elements $d_i = \sum_{j=1}^n a_{ij}$, $i \in \mathcal{V}$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = D - A$. If $\mathcal{G}$ is connected, then the null space of L is spanned by $\mathbf{1}_n$, and all the other $n - 1$ eigenvalues of L are strictly positive.

A differentiable function $f(\cdot) : \mathbb{R}^m \to \mathbb{R}$ is convex if $f(x) - f(y) \geq \nabla f(y)'(x-y)$, $\forall x, y \in \mathbb{R}^m$, where ∇f stands for the gradient of f. f is ω-strongly convex ($\omega > 0$) on $\mathbb{R}^m$ if $(\nabla f(x)-\nabla f(y))'(x-y) \geq \omega\|x - y\|^2$, $\forall x, y \in \mathbb{R}^m$. Moreover, f is θ-Lipschitz on $\mathbb{R}^m$ if $\|f(x) - f(y)\| \leq \theta\|x - y\|$, $\forall x, y \in \mathbb{R}^m$.

2.2. Problem formulation

Consider a network system composed of n heterogeneous EL agents with an associated undirected communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The dynamics of each agent $i \in \mathcal{V}$ is described as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \quad (1)$$

where $q_i, \dot{q}_i \in \mathbb{R}^m$ denote the generalized position and velocity vectors, respectively; $M_i(q_i) \in \mathbb{R}^{m\times m}$ is the positive definite inertia matrix; $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^m$ is the vector for Coriolis and centripetal forces; $G_i(q_i) \in \mathbb{R}^m$ is the gravity vector; and $\tau_i \in \mathbb{R}^m$ is the control force. The dynamics of system (1) satisfies the following properties (Spong et al., 2006):

Property 1. *$\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.*

Property 2. *For any $x, y \in \mathbb{R}^m$, $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + G_i(q_i) = \Omega_i(q_i, \dot{q}_i, x, y)\varphi_i$, where $\varphi_i \in \mathbb{R}^p$ is a constant vector consisting of the uncertain parameters of EL system (1), and $\Omega_i(q_i, \dot{q}_i, x, y) \in \mathbb{R}^{m\times p}$ is a known regression matrix that is only dependent on state variables.*

In this EL multi-agent system, agent $i \in \mathcal{V}$ is endowed with a local cost function $f_i : \mathbb{R}^m \to \mathbb{R}$, which is only known by this agent. The global cost function $\tilde{f} : \mathbb{R}^{nm} \to \mathbb{R}$ of the whole system is defined as $\tilde{f}(q) := \sum_{i=1}^n f_i(q_i)$, where $q = col(q_1, \ldots, q_n)$. The objective of EL system (1) is to cooperatively achieve consensus with the minimum global cost, namely, to solve the following optimization problem:

$$\min_{p\in\mathbb{R}^m} f(p), \quad f(p) = \sum_{i=1}^n f_i(p). \quad (2)$$

Definition 1. The DOC is achieved for system (1), if, for any initial condition $(q_i(0), \dot{q}_i(0))$ with $i \in \mathcal{V}$, all the EL agents converge to the global optimal solution of problem (2), *i.e.*, $\lim_{t\to\infty} q_i(t) = p^*$, $\lim_{t\to\infty} \dot{q}_i(t) = \mathbf{0}_m$, $i \in \mathcal{V}$, where $p^* \in \arg\min_{p\in\mathbb{R}^m} f(p)$.

In practice, the local cost functions in (2) may not be accurately obtained, due to computational uncertainties or external disturbances. The related problems have been considered in many applications such as online optimization, target localization and trajectory optimization (Foderaro et al., 2014; Hosseini, Chapman, & Mesbahi, 2013; Yan, Sundaram, Vishwanathan, & Qi, 2013; Zhang et al., 2015). Therefore, we are also interested in the DOC problem in the presence of uncertainties. Consider the following uncertain DOC problem with time-varying cost functions for system (1), that is,

$$\min_{p\in\mathbb{R}^m} \bar{f}(p, t), \quad \bar{f}(p, t) = \sum_{i=1}^n \bar{f}_i(p, t), \quad (3)$$

where $\bar{f}_i(p, t) = f_i(p) + g_i(t)$ is the observed cost function of agent i at time t with a time-varying uncertain function $g_i(t)$, and $f_i(p)$ is the real cost function of agent i. Note that the uncertainty of $\bar{f}_i(p, t)$ results from its evolution in an uncertain way, and $f_i(p)$ in (2) can be regarded as its nominal one without the time-varying uncertainty. Different from the DOC problem, the uncertain DOC problem (3) addresses how well the EL agents can cooperatively complete the optimization task in the case of uncertain time-varying cost functions.

The regret function has been widely used in the online optimization literature (Hosseini et al., 2013; Yan et al., 2013) as a performance measurement for a given algorithm to measure the average difference between the actual total cost and the minimal cost resulting from the best fixed decision p^* over a given time interval $[0, T]$, $T \geq 0$, which is defined as

$$R(T) = \frac{1}{T}\int_0^T \sum_{i=1}^n \big(\bar{f}_i(q_i(t), t) - \bar{f}_i(p^*, t)\big)dt. \quad (4)$$

To proceed further, we introduce the following two assumptions, which were also used in Kia et al. (2015) and Wang et al. (2014).

A1: The graph $\mathcal{G}$ is connected.

A2: For any $i \in \mathcal{V}$, f_i is differentiable and ω-strongly convex, and ∇f_i is θ-Lipschitz on $\mathbb{R}^m$, where $\omega, \theta > 0$.

Under **A2**, f is strongly convex, which implies that problem (2) has a unique optimal solution p^*. Here we consider the case of $0 \leq \|p^*\| < +\infty$.