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Brief paper

Bipartite containment tracking of signed networks ☆

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ABSTRACT

This paper considers a class of bipartite containment tracking problems for leader-following networks associated with signed digraphs. It admits multiple leaders which can be not only stationary but also dynamically changing via interactions between them and their neighbored leaders. It is shown that the followers can converge to the convex hull containing each leader's trajectory as well as its symmetric trajectory which is the same as it in modulus but different in sign. In particular, if the adjacency weight matrix of the digraph is not signed but nonnegative, then the leader-following network achieves the traditional containment tracking. Simulation tests are performed to illustrate the observed bipartite containment tracking performances of signed networks.

1. Introduction

Networks as well as coordination of nodes included in them have attracted considerable attention over the past decade (see, e.g., the surveys of Cao, Yu, Ren, & Chen, 2013; Olfati-Saber, Fax, & Murray, 2007 and references therein). Consensus has been considered as one of the fundamental coordination problems on networks, which means the agreement of all nodes on a common quantity (Olfati-Saber et al., 2007). In particular, when there is only one leader (i.e., the node without neighbors) in the network, each follower (i.e., the node with at least one neighbor) achieves consensus in the way that it converges to the leader's state (see, e.g., Hu & Hong, 2007; Ni & Cheng, 2010; Su, Chen, Lam, & Lin, 2013). This actually is the so-called leader-following consensus of networks, which however fails to work when there exist multiple leaders. Since each leader is not influenced by any of the other nodes (leaders or followers) in the network, its dynamic behaviors are usually known as predetermined information. By noting this property, containment control has been proposed instead of consensus in networks with multiple leaders such that all followers converge to the convex hull spanned by the leaders (Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008).

As a more general concept than leader-following consensus, containment control has been studied in many applications and areas. In Klotz, Cheng, and Dixon (2016), Mei, Ren, Chen, and Ma (2013), Mei, Ren, and Ma (2012) and Meng, Ren, and You (2010), distributed containment control protocols are proposed for Lagrangian networks which are known as suitable descriptions in many mechanical systems, such as autonomous vehicles and robot manipulators. There are also various studies on containment control for networks in the presence of single-integrator (Cao, Ren, & Egerstedt, 2012; Liu, Xie, & Wang, 2012), double-integrator (Li, Ren, & Xu, 2012; Lou & Hong, 2012), and general linear (Li, Ren, Liu, & Fu, 2013; Liu, Cheng, Tan, & Hou, 2015) dynamics, respectively. Recently, containment control problems have been successfully solved for social networks (Kan, Klotz, Pasiliao, & Dixon, 2015) and random networks (Kan, Shea, & Dixon, 2016), respectively. Moreover, a common property of containment control results (see, e.g., Cao et al., 2012; Li et al., 2013; Li et al., 2012; Liu et al., 2015; Liu et al., 2012; Lou & Hong, 2012; Mei et al., 2012; Meng et al., 2010) is that they allow not only stationary but also dynamic leaders. It is worth noting, however, that the dynamic changing of leaders differs from that of followers. Generally, the leaders' dynamics are caused by some external driving signals, whereas the followers are enabled to dynamically evolve due to interactions between them and their neighbored nodes.

Another fact worth noting is that the aforementioned studies are devoted to the *traditional networks* including only cooperative interactions. This class of traditional networks is conveniently described by graphs associated with adjacency weight matrices which are nonnegative. By contrast, there exists a more general class of networks, which will be called *signed networks*, since they are associated with signed graphs that can admit not only

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positive but also negative adjacency weights. Signed networks may have cooperative and antagonistic interactions represented by positive and negative adjacency weights, respectively, due to which they have wide applications in, e.g., social networks and economic systems. Take, for example, the social networks, where a cooperative/antagonistic interaction can be considered to reflect a friend/foe, trust/distrust, or like/dislike relationship between nodes (Altafini, 2013). The existence of antagonistic interactions produces more complex dynamic behaviors of signed networks than those of traditional networks, see, e.g., bipartite consensus of Altafini (2013) and Valcher and Misra (2014), interval bipartite consensus of Hu and Zheng (2014) and Meng, Du, and Jia (2016), and bipartite flocking of Fan, Zhang, and Wang (2014). As pointed out in Altafini (2013) and Valcher and Misra (2014), bipartite consensus means that the nodes in signed networks reach agreement on a quantity which is the same for all only in modulus but different in sign, and in particular includes as a special case the standard consensus of traditional networks (since they can be viewed as a special case of structurally balanced signed networks). But, unlike standard consensus which holds under the spanning tree condition (Ren & Beard, 2005), bipartite consensus may not hold though signed networks fulfill the spanning tree condition and instead interval bipartite consensus that possesses more convergence behaviors may occur (Hu & Zheng, 2014; Meng et al., 2016). With this observation, a further question is what the convergence behaviors will be for leader-following signed networks including multiple separate leaders since they violate the spanning tree condition of, e.g., Hu and Zheng (2014) and Meng et al. (2016) requiring globally reachable (root) nodes. To our knowledge, no studies have been reported on answering this question.

In this paper, we extend the notion of containment control to directed signed networks including multiple dynamic leaders. It is shown that all the followers can be guaranteed to converge to the convex hull which is spanned by not only all the leaders' trajectories but also their symmetric trajectories with the same moduli but different signs. That is, the "bipartite containment tracking" is ensured for signed networks. The dynamic leaders evolve owing to interactions between them and their neighbors, which is achieved via an extension of the traditional notion of leaders (see, e.g., Cao et al., 2012, Definition 2.1) by allowing them to have neighbors. Further, as a consequence of this extended notion of leaders, the spanning tree can be extended to a more general case, in which for each follower, there exists at least one leader having some paths to the follower. By this general connectivity condition, bipartite containment tracking is achieved for signed networks, regardless of whether they are structurally balanced or unbalanced. In particular, the bipartite containment tracking results collapse into the class of interval bipartite consensus results (see, e.g., Meng et al., 2016) when all leaders are within a unique strongly connected component of signed networks and become providing containment control results when signed networks collapse into traditional networks, respectively.

The remainder of our paper is organized as follows. We give the bipartite containment tracking problem for signed networks in Section 2 and propose main results in Section 3. In Section 4, simulations are provided. We conclude the paper in Section 5 and supply the proofs of claims and corollaries in Appendix.

Notations: Given integers $m > 0$ and $n > 0$, $\mathscr{I}_n = \{1, 2, \dots, n\}$, $1_n = [1, 1, \dots, 1]^{\mathrm{T}} \in \mathbb{R}^n$, $\mathrm{diag}\{d_1, d_2, \dots, d_n\}$ is a diagonal matrix with diagonal elements $d_1, d_2, \dots, d_n$, respectively, and $|A| = \left[|a_{ij}|\right]$ for any $A = \left[a_{ij}\right] \in \mathbb{R}^{m\times n}$, where $|a_{ij}| = \mathrm{sign}(a_{ij})a_{ij}$ and $\mathrm{sign}(a_{ij})$ is the sign function of any real scalar $a_{ij} \in \mathbb{R}$. We say that A is a nonnegative matrix, denoted by $A \geq 0$, if all its elements are nonnegative. By $A \geq B$, it means $A - B \geq 0$. A square matrix is said to be Hurwitz stable if all its eigenvalues have negative real parts.

2. Problem statement

We consider signed networks with n nodes. Let the ith node be given by (see Altafini, 2013; Meng et al., 2016)

$$\dot{x}_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}\left[x_j(t) - \mathrm{sign}(a_{ij})x_i(t)\right], \quad i \in \mathscr{I}_n \tag{1}$$

where $x_i(t) \in \mathbb{R}$ is the state, $a_{ij} \in \mathbb{R}$ is the adjacency weight to evaluate information interaction between the ith node and the jth node, and $\mathcal{N}_i = \{j : a_{ij} \neq 0\}$ is the label set of neighbors of the ith node. In fact, the system (1) provides a network description of nodes with single-integrator dynamics: $\dot{x}_i(t) = u_i(t)$, which operates under the action of the following distributed protocol:

$$u_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}\left[x_j(t) - \mathrm{sign}(a_{ij})x_i(t)\right], \quad i \in \mathscr{I}_n.$$

The interaction between nodes is conveniently described by a signed directed graph ("digraph" for short). A signed digraph $\mathscr{G}$ is represented by a triple $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathbf{A})$, where $\mathscr{V} = \{v_1, v_2, \dots, v_n\}$ is a node set, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V} = \{(v_i, v_j) : v_i, v_j \in \mathscr{V}\}$ is an edge set such that (v_j, v_i) is a directed edge from v_j to v_i just in the case when v_j is a neighbor of v_i, and $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is an adjacency weight matrix such that $(v_j, v_i) \in \mathscr{E} \Leftrightarrow a_{ij} \neq 0$ and otherwise, $a_{ij} = 0$. Clearly, the neighbors of each node v_i are defined by $\mathcal{N}_{v_i} = \{v_j : (v_j, v_i) \in \mathscr{E}\}$, where $\mathcal{N}_i$ is its label set equivalent to $\mathcal{N}_i = \{j : (v_j, v_i) \in \mathscr{E}\}$. Assume that there exist no self-loops in $\mathscr{G}$, i.e., $a_{ii} = 0$, $\forall i \in \mathscr{I}_n$ and the signed digraph $\mathscr{G}$ is digon sign-symmetric, i.e., $a_{ij}a_{ji} \geq 0$, $\forall i, j \in \mathscr{I}_n$ (Altafini, 2013). For $\mathscr{G}$, a (directed) path of length m from v_i to v_j is a finite sequence of edges in the form of $(v_{k_0}, v_{k_1}), (v_{k_1}, v_{k_2}), \dots, (v_{k_{m-1}}, v_{k_m})$, where $k_0 = i, k_m = j$ and $v_{k_0}, v_{k_1}, \dots, v_{k_m}$ are distinct nodes. If there exists some node such that $\mathscr{G}$ has paths from it to every other node, then $\mathscr{G}$ is said to have a spanning tree. Moreover, if there exist paths between every distinct pair of nodes in $\mathscr{G}$, then $\mathscr{G}$ is said to be strongly connected. In addition, a signed digraph $\mathscr{G}$ is structurally balanced if there exists a bipartition $\{\mathscr{V}^{(1)}, \mathscr{V}^{(2)}\}$ of nodes, where $\mathscr{V}^{(1)} \cup \mathscr{V}^{(2)} = \mathscr{V}$ and $\mathscr{V}^{(1)} \cap \mathscr{V}^{(2)} = \emptyset$, such that $a_{ij} \geq 0$, $\forall v_i, v_j \in \mathscr{V}^{(l)}$ for $l \in \{1, 2\}$ and $a_{ij} \leq 0$, $\forall v_i \in \mathscr{V}^{(l)}$, $\forall v_j \in \mathscr{V}^{(q)}$ for $l \neq q$ and $l, q \in \{1, 2\}$; and $\mathscr{G}$ is structurally unbalanced, otherwise. Note that the traditional graphs associated with the nonnegative adjacency weight matrices $\mathbf{A} \geq 0$ can be viewed as a special case of structurally balanced signed graphs, where one of the subcommunities $\mathscr{V}^{(1)}$ and $\mathscr{V}^{(2)}$ is empty (see also Altafini, 2013 for more discussions).

For two digraphs $\mathscr{G}_1 = (\mathscr{V}_1, \mathscr{E}_1, \mathbf{A}_1)$ and $\mathscr{G}_2 = (\mathscr{V}_2, \mathscr{E}_2, \mathbf{A}_2)$, $\mathscr{G}_1$ is called a subgraph of $\mathscr{G}_2$ if $\mathscr{V}_1 \subseteq \mathscr{V}_2$ and $\mathscr{E}_1 \subseteq \mathscr{E}_2$. Consider any subgraph $\mathscr{G}_s = (\mathscr{V}_s, \mathscr{E}_s, \mathbf{A}_s)$ of $\mathscr{G}$, and we extend the notion of neighbor set $\mathcal{N}_{v_i}$ for a node v_i of $\mathscr{G}$ to define the *neighbor set* $\mathcal{N}_{\mathscr{G}_s}$ *of the subgraph* $\mathscr{G}_s$ as

$$\mathcal{N}_{\mathscr{G}_s} = \left\{v_j : (v_j, v_i) \in \mathscr{E},\ \forall v_i \in \mathscr{V}_s,\ \forall v_j \in \mathscr{V} \setminus \mathscr{V}_s\right\}$$

where $\mathscr{V} \setminus \mathscr{V}_s = \left\{v_j : v_j \in \mathscr{V} \text{ but } v_j \notin \mathscr{V}_s\right\}$. It is not difficult to verify $\mathcal{N}_{\mathscr{G}_s} = \left(\bigcup_{v_i\in\mathscr{V}_s} \mathcal{N}_{v_i}\right) \setminus \mathscr{V}_s$. Based on this definition, we separate the nodes of networks into two groups according to the following extended notions of leaders and followers.

Definition 1. For networks associated with a signed digraph $\mathscr{G}$, a node v_i is called a *leader* if v_i is included in some strongly connected subgraph $\mathscr{G}_s$ of $\mathscr{G}$ that has no neighbors, i.e., $\mathcal{N}_{\mathscr{G}_s} = \emptyset$. The node v_i is called a *follower*, otherwise.

By Definition 1, we make an extension of the usual notion of leaders. A leader generally refers to an individual node without neighbors in the literature (see, e.g., Cao et al., 2012, Definition 2.1). Since any digraph consisting of only one node is strongly