Automatica 78 (2017) 123–134

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Composability and controllability of structural linear time-invariant systems: Distributed verification[☆]

J. Frederico Carvalho[e,1], Sérgio Pequito[a,b,1], A. Pedro Aguiar[c], Soummya Kar[a], Karl H. Johansson[d]

[a] Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, United States
[b] Institute for Systems and Robotics, Instituto Superior Técnico, Technical University of Lisbon, Lisbon, Portugal
[c] Department of Electrical and Computer Engineering, Faculty of Engineering, University of Porto, Porto, Portugal
[d] ACCESS Linnaeus Center, School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden
[e] Center for Autonomous Systems CAS/RPL, KTH Royal Institute of Technology, Stockholm, Sweden

ARTICLE INFO

Article history:
Received 18 June 2015
Received in revised form
16 April 2016
Accepted 21 November 2016

Keywords:
Control system analysis
Controllability
Structural properties
Graph theory
Combinatorial mathematics

ABSTRACT

Motivated by the development and deployment of large-scale dynamical systems, often comprised of geographically distributed smaller subsystems, we address the problem of verifying their controllability in a distributed manner. Specifically, we study controllability in the structural system theoretic sense, *structural* controllability, in which rather than focusing on a specific numerical system realization, we provide guarantees for equivalence classes of linear time-invariant systems on the basis of their structural sparsity patterns, i.e., the location of zero/nonzero entries in the plant matrices. Towards this goal, we first provide several necessary and/or sufficient conditions that ensure that the overall system is structurally controllable on the basis of the subsystems' structural pattern and their interconnections. The proposed verification criteria are shown to be efficiently implementable (i.e., with polynomial time-complexity in the number of the state variables and inputs) in two important subclasses of interconnected dynamical systems: *similar* (where every subsystem has the same structure) and *serial* (where every subsystem outputs to at most one other subsystem). Secondly, we provide an iterative distributed algorithm to verify structural controllability for general interconnected dynamical system, i.e., it is based on communication among (physically) interconnected subsystems, and requires only local model and interconnection knowledge at each subsystem.

1. Introduction

In recent years we have witnessed an explosion in the use of large-scale dynamical systems, notably, those with a modular structure (Davison, 1977; Davison & Özgüner, 1983; Özgüner & Hemani, 1985), such as content delivery networks, social networks, robot swarms, and smart grids. Such systems, often geographically distributed, are comprised of smaller subsystems (which we may refer to as *agents*), and a typical concern is ensuring that the system, as a whole, performs as intended. More than often, when analyzing these interconnected dynamical systems, which in this paper we consider to consist of continuous linear-time invariant (LTI) subsystems, we do not know the exact parameters of the plant matrices. Therefore, we focus on the zero/nonzero pattern of the system's plant, which we refer to as *sparsity pattern*, and we focus on structural counterpart of controllability, i.e., *structural controllability* (Dion, Commault, & der Woude, 2003).

It is worthwhile noting that these agents may be *homogeneous* or *heterogeneous*, from its structure point of view. When the agents are homogeneous, their plants and connections (when used) have the same sparsity pattern and the system is referred to as a *similar system*. Otherwise, the agents are heterogeneous and two possible scenarios are conceivable: (i) an agent may receive information from (possibly several) other agents but it only transmits to one other agent, the overall system is referred to as *serial*, and

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Delin Chu under the direction of Editor Ian R. Petersen.

E-mail addresses: jfpbdc@kth.se (J.F. Carvalho), sergio.pequito@gmail.com (S. Pequito), pedro.aguiar@fe.up.pt (A.P. Aguiar), soummyak@andrew.cmu.edu (S. Kar), kallej@kth.se (K.H. Johansson).

[1] Both authors contributed equally to this work.

http://dx.doi.org/10.1016/j.automatica.2016.12.016

commonly arises in peer-to-peer communication schemes; and (ii) the communications between agents can be arbitrary, which commonly arise in broadcast communication setups. All the above subclasses of interconnected dynamical systems are of interest and explored in detail in this paper. More precisely, we provide several necessary and/or sufficient conditions to ensure key properties of the system, which can be verified resorting to efficient (i.e., with polynomial time complexity in the number of state variables) algorithms.

In some applications, the problem of composability is particularly relevant. Consider, for example, a swarm of robots possessing similar structure where the communication topology may change over time, or where robots may join or leave the swarm over time. Then, the existence of necessary and/or sufficient conditions on the structure and interconnection between these agents contribute to *controllability-by-design* schemes, i.e., we ensure that by inserting an agent into the interconnected dynamical system, we obtain a controllable dynamical system. Consequently, we can specify with which agents an agent should interact with such that those conditions hold.

A swarm of robots can also be composed by a variety of heterogeneous agents in which case controllability-by-design is also important, yet due to constraints on the communication range, the interaction between agents is merely local, even if some additional information is known. Therefore, in the context of serial systems we can equip each subsystem with the capability of inferring if the entire system is structurally controllable, i.e., we provide distributed algorithms that rely only on the interaction between a subsystem and its neighbors, where information about their structure may be shared. In particular, if we equip the robots in the swarm with actuation capabilities that can be activated when the interconnected dynamical system is not structurally controllable, we can render this interconnected dynamical system structurally controllable.

Nonetheless, imposing *a priori* knowledge of the structure of the interconnections in the system (for instance, whether it is a serial system) can be restrictive, so distributed algorithms to verify structural controllability of general interconnected dynamical systems are in need. Hereafter, we provide such an algorithm: It requires the interaction between a subsystem and its neighbors, but it does not require to share the structure of the subsystems involved. Instead, it requires only partial information about its structure, which leads to a certain level of privacy of the intervenients in the communication. The proposed scheme is also particularly suitable to other applications such as the smart grid of the future, that consists of entities described by subsystems deployed over large distances; in particular, notice that in these cases, the different entities may not be willing to share information about their structure due to security or privacy reasons.

Related Work: Structural controllability was introduced by Lin (1974) in the context of single-input single-output (SISO) systems, and extended to multi-input multi-output (MIMO) systems by Shields and Pearson (1976). A recent survey of the results in structural systems theory, where several necessary and sufficient conditions are presented, can be found in Dion et al. (2003).

In this paper, we focus on the composability aspects that ensure structural controllability. In other words, we are interested in understanding how the connection between different dynamical subsystems enables or jeopardizes the structural controllability of the overall system. The presented problem statement fits the general framework presented in Anderson and Hong (1982). Nevertheless, the verification procedures proposed in Anderson and Hong (1982) based on matrix nets lead to a computational burden which increases exponentially with the dimension of the problem. Alternatively, in Davison (1977) an efficient method is proposed that takes into account the whole system instead of local properties (i.e., the components of the system and their interconnections), however this method does not apply to an arbitrary system. More precisely, it is assumed that when connected, the state space digraph (to be defined later) is spanned by a disjoint union of cycles, which is called a *rank constraint*. In contrast, in Li, Xi, and Zhang (1996) and Rech and Perret (1991), the authors have presented results on the structural controllability of interconnected dynamical systems, by focusing on the cascade interconnection of system structures that ensure the structural controllability of the interconnected dynamical system. Nevertheless, these structures are not unique, and the interconnection of these is established assuming such connectible structures are given, therefore, no practical criteria to compute the structures and verify the results are given. More recently, in Blackhall and Hill (2010) similar results were obtained by exploring which variables may belong to a structure and referred to as controllable state variable. Thus, similarly to Li et al. (1996) and Rech and Perret (1991), the results depend on the identified structures, but no method to systematically identify these structures is provided. In Yang and Zhang (1995) the study is conducted assuming that all the subsystems except a *central* subsystem, which is allowed to communicate with every other subsystem, have the same dynamic structure, and the interconnection between the several subsystems also has the same structure (even though they may not be used). This, study considers the impact of local interactions into the system structural controllability, which results can be obtained with the solution proposed hereafter.

In Pequito, Kar, and Aguiar (2016a), we studied the problem of determining the sparsest input matrix to ensure structural controllability in a centralized fashion. Furthermore, polynomial algorithms with computational complexity $\mathcal{O}(n^3)$ were provided to both problems, where n is the number of state variables. In Pequito, Kar, and Aguiar (2015), we studied the setting where the selection of inputs is constrained to a given collection, and shown to be NP-hard. Finally, in Pequito, Kar, and Aguiar (2016b), the problem in Pequito et al. (2016a) was further extended to determining the input matrix incurring in the minimum cost when the state variables actuated incur in different costs while ensuring structural controllability. Furthermore, procedures with $\mathcal{O}(n^{\omega})$ computational complexity were provided, where $\omega < 2.373$ is the lowest known exponent associated with the complexity of multiplying two $n \times n$ matrices. All these contrasts with the problem addressed in the current paper in the sense that we aim to verify structural controllability properties in a *distributed fashion*. In particular, it requires identifying specific network conditions on the network structure under which we can use efficient algorithms, i.e., polynomial in the dimension of the state space, or provide distributed algorithms suitable to address the proposed problem.

On the other hand, composability aspects regarding controllability have been heavily studied by several authors, see for instance, Chen and Desoer (1967), Davison and Wang (1975), Wang and Davison (1973), Wolovich and Hwang (1974), Yonemura and Ito (1972) and Zhou (2015). Briefly, all these studies resort to the well known Popov–Belevitch–Hautus (PBH) eigenvalue controllability criterion for LTI systems (Hespanha, 2009). We notice that this criterion requires the knowledge of the overall system to infer its controllability. The reason is closely related with the loss of degrees of freedom imposed by interconnected dynamical systems, as well as conservation laws in general, that reflects in the decrease of the rank of the system's dynamics matrix when compared with the sum of the rank of the dynamics matrices of each subsystem. Consequently, even if all subsystems are controllable, their interconnection may not be. Notwithstanding, the same does not happen when dealing with structural systems, where if all subsystems are structurally controllable, then the overall system is structurally controllable. So, while not guaranteeing that a system