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Brief paper

Set stabilization for switched Boolean control networks[☆]

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ABSTRACT

This paper studies the set stabilization of switched Boolean control networks, in which a feedback control design algorithm is presented by a constructive method. Necessary and sufficient conditions of the set stabilization of switched Boolean control networks under arbitrary switching signals are presented for the cases of the control input relying on switching signals or not, respectively. Furthermore, the corresponding switching-signal-independent and switching-signal-dependent controllers are provided for these two cases, respectively. It is shown that the condition of the switching-signal-dependent controller is less conservative than the one of the switching-signal-independent controller. Finally, examples are given to illustrate the effectiveness of the proposed results.

1. Introduction

A Boolean network (BN) is a kind of logical systems with binary state variables. In BNs, 1 (or 0) corresponds to the on (or off) state of the Boolean variable. Furthermore, each Boolean variable updates its state according to a Boolean function (Kauffman, 1969). In recent years, the study on BNs has gained increasing research attention in a variety of fields. For example, Drossel, Mihaljev, and Greil (2005) and Zou and Zhu (2015) have studied the topological structures of BNs. BNs with binary inputs are called Boolean control networks (BCNs). Various control problems of BCNs, such as controllability and stability, have been investigated these years (Chen & Sun, 2013; Cheng & Qi, 2009; Fornasini & Valcher, 2013a; Laschov & Margaliot, 2011; Li & Song, 2014; Li & Sun, 2011; Li & Wang, 2012; Liu, Chen, Lu, & Wu, 2015; Liu, Lu, & Wu, 2014; Lu, Zhong, Ho, Tang, & Cao, 2016; Zhang & Zhang, 2013; Zhong, Lu, Liu, & Cao, 2014).

It is well known that stability, synchronization and partial stability are important issues in control theory and a considerable number of results have been obtained, for instance, see Suo and Sun (2015), Tang, Gao, Zhang, and Kurths (2015); Tang, Qian, Gao, and Kurths (2014), Wu, Yang, Shi, and Su (2015), Yang and Wang (2013) and Zhang and Boukas (2009). In addition, the study of stability, synchronization and partial stability for BNs is meaningful and important in practice. For example, the study of synchronization will provide information on the coevolution of several biological species (Morelli & Zanette, 2001). Some interesting results about the stability and synchronization of BNs have been provided, see e.g. Cheng, Qi, Li, and Liu (2011) and Fornasini and Valcher (2013b).

The study of set stability and set stabilization is meaningful in the real world. In some cases, we need to study whether a system or a collection of interconnected systems can converge to or be stabilized to a subset of the state space, instead of to a single point, which is the basic idea of set stability and set stabilization. Hence, set stability and set stabilization of BCNs have drawn much research attention. It has been pointed out by Guo, Wang, Gui, and Yang (2015) that synchronization, stability, and partial stability are the special cases for set stability and set stabilization of BNs. Hence, it is quite important to investigate the set stability and set stabilization of BNs.

Although the set stabilization of BCNs has been considered in Guo et al. (2015), it should be noted that the problem of the set stabilization for a switched Boolean control network (SBCN) has not been addressed before. In fact, a switched system which appears with different subsystems due to environmental changes and control requirements, etc., constituents a significant component in control theory. The research on the dynamics of a SBCN is of great significance due to its wide application in practical situations. For example, a SBCN is constructed for the acute myeloid leukemia (AML) signaling network (Hwang & Lee, 2010). The research of SBNs has attracted much attention,

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see e.g., controllability, output controllability and disturbance decoupling of SBNs (Chen & Sun, 2014; Li & Wang, 2015; Li, Wang, Xie, & Cheng, 2014). In the literature, it is noted that the stabilization for a SBCN has been investigated in Ding, Guo, Xie, Yang, and Gui (2015). A feedback control mechanism has been presented to make a SBCN stabilized to a fixed point. However, set stabilization for a SBCN has not been considered, although it is a generalization of stabilization and has been manifested wide applications. Thus, it is meaningful for us to study the set stabilization for a SBCN. The study of set stabilization will generalize the results of set stabilization for BNs, which will enrich the control theory of a logical system. Furthermore, the investigation on set stabilization and control design for a SBCN will provide a feasible way to design a controller if we want to achieve the stabilization or synchronization for SBCNs.

Motivated by the above discussions, this paper studies the set stabilization for a SBCN. The main purpose of this paper is to design two kinds of controllers in which control inputs are independent of and dependent on the switching signals, respectively. The main difficulties of extending the results on set stabilization of BCNs to SBCNs lie in the following two aspects: (i) The dynamics of a SBCN is much more complicated than a conventional BCN as it switches among several BCNs according to a switching rule. In addition, it is well known that the study on control problems of a switched system is a difficult and challenging task (Lin & Antsaklis, 2009; Shorten et al., 2007). Hence, the problem of set stabilization of a SBCN is important and shows its difficulty in mathematical analysis, since a SBCN not only has the properties of a switched system but also preserves the characteristics of a Boolean network. (ii) In our paper, the switching signal is arbitrary, which makes the derivation of necessary and sufficient conditions for set stabilization of SBCNs much more difficult. To tackle these difficulties, we should calculate the largest control invariant set for SBCNs and compute the set consisting of all the states that can be steered to the largest control invariant set. However, the study on these two problems mentioned above for SBCNs is not straightforward and we have to develop a new method to deal with these two problems. In Guo et al. (2015), the controllability matrix theory is used to calculate these sets for BCNs without system switching. Nevertheless, we cannot use this approach directly, since there are no results on the controllability matrix theory for SBCNs under arbitrary switching signals, to the best of our knowledge.

In order to solve these problems, we first give the algebraic form of SBCNs and find the largest control invariant set for SBCNs according to the special feature of the logical matrices and SBCNs. Then, we calculate the state consisting of all the states that can be steered to the largest control invariant set based on the algebraic form of SBCNs. In the process of the calculation, we can obtain the control sequence. Finally, some feedback control design algorithms are also presented according to the obtained control sequence. The main contributions of this paper lie in the following three aspects: (i) The algorithms of calculating the largest control invariant set and the set consisting of all the states that can be steered to the largest control invariant sets are provided for SBCNs under arbitrary switching signals, in which control inputs hinge on switching signals or not, respectively. (ii) Necessary and sufficient conditions are presented for set stabilization of for SBCNs under arbitrary switching signals. (iii) The switching-signal-dependent and switching-signal-independent feedback controllers have been presented systematically through a constructive method.

The rest of this paper is organized as follows. Section 2 contains some preliminary results. Section 3 gives the main results of this paper. Illustrative examples are given in Section 4. A brief conclusion is provided in Section 5.

2. Preliminaries

In this section, some necessary preliminaries are given.

First, we will introduce the definition of semi-tensor product of matrices.

Definition 2.1 (*Cheng, Qi, & Li, 2011*)**.** For $M \in \mathbb{R}^{m\times n}$ and $N \in \mathbb{R}^{p\times q}$, their semi-tensor product (STP), denoted by $M \ltimes N$, is defined as follows:

$$M \ltimes N := (M \otimes I_{s/n})(N \otimes I_{s/p}),$$

where s is the least common multiple of n and p, and $\otimes$ is the Kronecker product.

Remark 2.1. It is noted that all the fundamental properties of conventional matrix product remain true. Based on this, the symbol "$\ltimes$" can be omitted if no confusion raises. There are also some basic properties of STP, for detail, see Cheng, Qi, and Li (2011).

Next, some necessary notations are provided in the following.

(1) Denote by $\mathbb{N}$ the natural number set.
(2) $\mathcal{D} := \{1, 0\}$.
(3) $\Delta_k := \{\delta_k^i | 1 \le i \le k\}$, where δ_k^i is the ith column of the identity matrix I_k. For compactness, $\Delta_2 = \Delta$.
(4) We denote the ith column of matrix A by $\mathrm{Col}_i(A)$ and denote the set of columns of matrix A by $\mathrm{Col}(A)$.
(5) Denote by $(M)_{i,j}$ the (i, j)th element of a matrix M.
(6) Split a matrix A into equal dimension blocks and denote by $\mathrm{Blk}_i(A)$ the ith block of matrix A.
(7) Denote by $\mathcal{M}_{m\times n}$ the set of all $m \times n$ matrices. A matrix $A \in \mathcal{M}_{m\times n}$ is called a logical matrix, if the columns of A are elements of Δ_m. Denote the set of logical matrices by $\mathcal{L}$; the set of $n \times s$ logical matrices is defined by $\mathcal{L}_{n\times s}$.
(8) A matrix $B \in \mathcal{M}_{n\times s}$ is called a Boolean matrix, if its entries $b_{ij} \in \mathcal{D}$, for every i, j. The set of $n \times s$ Boolean matrices is denoted by $\mathcal{B}_{n\times s}$.
(9) Assume that there is a matrix $M = [\delta_n^{i_1}, \delta_n^{i_2}, \dots, \delta_n^{i_s}]$, for notational compactness, $M := \delta_n[i_1, i_2, \dots, i_s]$.

Denote by True $= 1 \sim \delta_2^1$, False $= 0 \sim \delta_2^2$, then for $A(t) \in \mathcal{D}$, $A(t)$ can be seen as a value taking from $\Delta = \{\delta_2^1, \delta_2^2\}$, where $p \sim q$ denotes the logical equivalence of p and q.

Finally, we will give the following lemma which is fundamental for the matrix expression of the logical function.

Lemma 2.1 (*Cheng, Qi, & Li, 2011*)**.** *Any logical function $L(A_1, \dots, A_n)$ with logical arguments $A_1, \dots, A_n \in \Delta$ can be expressed in a multi-linear form as*

$$L(A_1, \dots, A_n) = M_L A_1 A_2 \cdots A_n,$$

where $M_L \in \mathcal{L}_{2\times 2^n}$ is unique, called the structure matrix of L.

3. Main results

In this section, we will study the set stabilization in which control inputs relying on the switching signals or not, respectively.