Automatica 78 (2017) 258–265

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Model predictive control with discrete actuators: Theory and application[☆]

James B. Rawlings, Michael J. Risbeck

Department of Chemical and Biological Engineering, University of Wisconsin–Madison, 1415 Engineering Drive, Madison, WI 53706, United States

ARTICLE INFO

Article history:
Received 7 December 2015
Received in revised form
29 August 2016
Accepted 2 December 2016
Available online 25 January 2017

Keywords:
Discrete actuators
Optimal control
Model-based control
Asymptotic stability
Periodic systems

ABSTRACT

Despite the presence of discrete actuators in many industrial processes, model predictive control (MPC) theory typically considers only continuous actuators, which requires discrete decisions to be removed from the MPC layer. However, if discrete inputs are chosen optimally, process performance may be greatly improved, and thus, discrete decisions should be treated directly in MPC theory. In this paper, we develop the idea that discrete actuators can be added to MPC theory without major modification, i.e., results established with sufficient generality for standard MPC with continuous actuators hold also for MPC with discrete actuators. First, we show that standard exponential stability for suboptimal MPC can be extended without modification to cover discrete actuators by avoiding restrictive assumptions about the geometry of the control set. Then, we prove stability results for tracking MPC applied to a time-varying periodic system. Finally, we demonstrate these results with two example systems.

1. Introduction

Discrete-valued actuators appear in nearly all large-scale industrial processes. The early MPC literature focused almost exclusively on continuous actuators, however, due in part to the computational burden imposed by discrete decision variables (García, Prett, & Morari, 1989; Mayne, Rawlings, Rao, & Scokaert, 2000; Rawlings & Mayne, 2009). In industrial practice, the discrete decisions are always removed from the MPC control layer and instead made at a different layer of the automation system using heuristics or other logical rules. However, with advances in computer performance and optimization software, it is now possible to include the discrete actuators within the control problem. We therefore wish to extend MPC theory to cover the case of mixed continuous/discrete actuators.

Within the literature, systems with both continuous and discrete actuators are sometimes referred to as "hybrid" systems (Bemporad & Morari, 1999; Camacho, Ramirez, Limon, de la Peña, & Alamo, 2010; Kobayshi, Shein, & Hiraishi, 2014). However, the term hybrid system is also often applied to systems without discrete actuators, e.g., when describing piecewise affine (Baotic, Christophersen, & Morari, 2006; Borrelli, Baotić, Bemporad, & Morari, 2005), or nonlinear switched systems (DeCarlo, Branicky, Pettersson, & Lennartson, 2000; El-Farra & Christofides, 2003), and when describing controllers that switch between multiple modes (Mhaskar, El-Farra, & Christofides, 2004). Here we reserve the term hybrid system for systems possessing both discrete- and continuous-time *dynamics*, e.g., as defined in Goebel, Sanfelice, and Teel (2012). Such general hybrid systems pose a number of control challenges (Sanfelice, 2013). In this paper, we restrict attention to nonlinear discrete-time systems with mixed continuous/discrete inputs. Note that this restricted class of systems does include piecewise affine systems (with or without discrete actuators) and switched systems.

Various results on stability of MPC with discrete actuators have appeared in the literature. In Bemporad and Morari (1999), convergence to the origin is shown for mixed-logical-dynamical systems based on certain positive-definiteness restrictions for the stage cost, although Lyapunov stability is not explicitly shown. For piecewise affine systems, Baotic et al. (2006) establish asymptotic stability for an infinite-horizon control law via Lyapunov function arguments. In Di Cairano, Heemels, Lazar, and Bemporad (2014), a hybrid Lyapunov function is directly embedded within the optimal control problem, enforcing cost decrease as a hard constraint. Boundedness of states can often be shown by treating discretization of inputs as a disturbance and deriving error bounds

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor Ian R. Petersen.

E-mail address: risbeck@wisc.edu (M.J. Risbeck).

http://dx.doi.org/10.1016/j.automatica.2016.12.024

with respect to the relaxed continuous-actuator system (Aguilera & Quevedo, 2013; Kobayshi et al., 2014; Quevedo, Goodwin, & De Doná, 2004). Finally, Picasso, Pancanti, Bemporad, and Bicchi (2003) show asymptotic stability for open-loop stable linear systems with only boundedness for open-loop unstable systems. All of these results are concerned with stability of an equilibrium point or steady state.

Discrete actuators are commonly used in industrial processes to cycle a system through a desired periodic operation. For constrained linear systems, a stabilizing MPC controller can be constructed by choosing periodically time-varying terminal sets as the maximum positive invariant sets for the unconstrained periodic LQR (Gondhalekar & Jones, 2011). Control laws can also be determined from a set of linear matrix inequalities (Böhm, Raff, Reble, & Allgöwer, 2009), and this technique can be generalized to nonlinear systems on bounded sets (Reble, Böhm, & Allgöwer, 2009), although the approach is conservative. To handle reachability issues and setpoint changes, a periodic tracking signal can be found during online optimization (Limon et al., 2012). For nonlinear systems, a terminal constraint can be employed that requires the predicted trajectory to terminate on a periodic reference trajectory (Falugi & Mayne, 2013).

For our chosen class of nonlinear discrete time systems with mixed continuous/discrete inputs, the goal of this paper is to see how far we can develop the following motivating idea.

Theorem 1 (*Folk Theorem*)**.** *Any result that holds for standard MPC holds also for MPC with discrete actuators.*

We will establish a sampling of precise versions of the folk theorem while reusing standard MPC results. As we will discuss, the key is to start with standard MPC results for continuous-valued inputs that do *not* assume that the feasible input set has an interior. Although from a historical perspective this requirement eliminates the vast majority of previous MPC results, a sufficient body of recent results exists to undertake this new development.

2. Results

2.1. Nominal stability of suboptimal MPC

The simplest MPC problem to consider is nominal stability of an equilibrium point, which we assume without loss of generality is the origin. Note that we subsume classic optimal MPC with the form of suboptimal MPC to be defined here. The main motivation for suboptimal MPC is to replace an intractable problem (global solution of a nonconvex nonlinear program) with a tractable problem that can be readily solved for real-time implementation of the controller (Scokaert, Mayne, & Rawlings, 1999).

We reconsider a standard MPC result given in Pannocchia, Rawlings, and Wright (2011). The discrete-time system is

$$x^+ = f(x, u) \tag{1}$$

in which $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and the input at a given time, while $x^+ \in \mathbb{R}^n$ is the successor state. Both state and input are subject to constraints $x(k) \in \mathbb{X}$ and $u(k) \in \mathbb{U}$ for all $k \in \mathbb{I}_{\geq 0}$. As is standard in MPC, take an integer N (referred to as the finite horizon) and an input sequence $\mathbf{u}$ of length N, $\mathbf{u} = (u(0), u(1), \ldots, u(N-1))$. Let $\phi(k; x, \mathbf{u})$ denote the solution of (1) at time k for a given initial state $x(0) = x$. To define the MPC problem we require the following three sets:

$$\begin{aligned} \mathbb{Z}_N &:= \{(x, \mathbf{u}) \mid u(k) \in \mathbb{U},\ \phi(k; x, \mathbf{u}) \in \mathbb{X} \\ &\qquad \text{for all } k \in \mathbb{I}_{0:N-1},\ \phi(N; x, \mathbf{u}) \in \mathbb{X}_f\} \\ \mathcal{X}_N &:= \{x \in \mathbb{R}^n \mid \exists \mathbf{u} \in \mathbb{U}^N \text{ such that } (x, \mathbf{u}) \in \mathbb{Z}_N\} \\ \mathcal{U}_N(x) &:= \{\mathbf{u} \mid (x, \mathbf{u}) \in \mathbb{Z}_N\}, \quad x \in \mathcal{X}_N \end{aligned} \tag{2}$$

in which $\mathbb{X}_f \subseteq \mathbb{X}$ is the terminal region. For any state $(x, \mathbf{u}) \in \mathbb{R}^n \times \mathbb{U}^N$, the cost function is

$$V_N(x, \mathbf{u}) := \sum_{k=0}^{N-1} \ell(\phi(k; x, \mathbf{u}), u(k)) + V_f(\phi(N; x, \mathbf{u}))$$

and one then has the standard finite horizon optimal control problem

$$\mathbb{P}_N(x) : \min_{\mathbf{u}} V_N(x, \mathbf{u}) \quad \text{s.t. } \mathbf{u} \in \mathcal{U}_N(x)$$

which is feasible for all $x \in \mathcal{X}_N$. Next comes a standard set of assumptions that guarantee closed-loop stability.

Assumption 1. The functions $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ and $V_f : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ are continuous, $f(0, 0) = 0$, $\ell(0, 0) = 0$, and $V_f(0) = 0$.

Assumption 2. The set $\mathbb{U}$ is compact and contains the origin. The sets $\mathbb{X}$ and $\mathbb{X}_f$ are closed and contain the origin in their interiors, $\mathbb{X}_f \subseteq \mathbb{X}$.

Assumption 3. For any $x \in \mathbb{X}_f$, the set

$$\kappa_f(x) := \{u \in \mathbb{U} \mid f(x, u) \in \mathbb{X}_f \text{ and } V_f(f(x, u)) + \ell(x, u) \leq V_f(x)\}$$

is nonempty.

Assumption 4. There exist positive constants a, a_1', a_2', a_f and $\bar{r}$, such that the cost functions satisfy the inequalities

$$\begin{aligned} \ell(x, u) &\geq a_1' |(x, u)|^a && \text{for all } (x, u) \in \mathbb{X} \times \mathbb{U} \\ V_N(x, \mathbf{u}) &\leq a_2' |(x, \mathbf{u})|^a && \text{if } |(x, \mathbf{u})| \leq \bar{r} \\ V_f(x) &\leq a_f |x|^a && \text{for all } x \in \mathbb{X}. \end{aligned}$$

Suboptimal MPC. Rather then solving $\mathbb{P}_N(x)$ *exactly*, Pannocchia et al. (2011) consider using any (unspecified) suboptimal algorithm having the following properties. Define the set

$$\mathcal{U}_N^r(x) := \{\mathbf{u} \in \mathcal{U}_N(x) \mid V_N(x, \mathbf{u}) \leq V_f(x) \text{ if } x \in r\mathbb{B}\}. \tag{3}$$

Let $\mathbf{u} \in \mathcal{U}_N^r(x)$ denote the (suboptimal) control sequence for the initial state x, and let $\tilde{\mathbf{u}}$ denote a *warm start* for the successor initial state $x^+ = f(x, u(0; x))$, obtained from $(x, \mathbf{u})$ by

$$\tilde{\mathbf{u}} := (u(1; x), u(2; x), \ldots, u(N-1; x), u_f) \tag{4}$$

in which $u_f \in \kappa_f(\phi(N; x, \mathbf{u}))$ as defined in Assumption 3. We observe that the warm start satisfies $\tilde{\mathbf{u}} \in \mathcal{U}_N(x^+)$. Then, the suboptimal input sequence for any given $x^+ \in \mathcal{X}_N$ is defined as *any* $\mathbf{u}^+ \in \mathbb{U}^N$ that satisfies:

$$\mathbf{u}^+ \in \mathcal{U}_N(x^+) \tag{5a}$$

$$V_N(x^+, \mathbf{u}^+) \leq V_N(x^+, \tilde{\mathbf{u}}) \tag{5b}$$

$$V_N(x^+, \mathbf{u}^+) \leq V_f(x^+) \qquad \text{when } x^+ \in r\mathbb{B} \tag{5c}$$

in which r is a positive scalar sufficiently small so that $r\mathbb{B} \subseteq \mathbb{X}_f$ (with $\mathbb{B}$ the unit ball in $\mathbb{R}^n$).

The advantage of the suboptimal control algorithm is that, instead of having to solve a general mixed-integer nonlinear program to global optimality at each step, one need construct only a new warm start as in (4) and can spend any remaining computational time improving on the warm start. Note that if the terminal control law is in explicit form, then u^+ is readily obtained by evaluating κ_f; alternatively, u^+ can be computed via solving $\mathbb{P}_1$, i.e., a 1-stage optimization, which is typically much easier than the full N-stage optimization.

Generating the initial input sequence $\mathbf{u}$ is the most computationally demanding step in the algorithm, but a suitable $\mathbf{u}$ could