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Brief paper

Strict Lyapunov functions for time-varying systems with persistency of excitation☆

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ABSTRACT

We study the stability of the origin for a class of linear time-varying systems with a drift that may be divided in two parts. Under the action of the first, a function of the trajectories is guaranteed to converge to zero; under the action of the second, the solutions are restricted to a periodic orbit. Hence, by assumption, the system's trajectories are bounded. Our main results focus on two generic case studies that are motivated by common nonlinear control problems: model-reference adaptive control, control of nonholonomic systems, tracking control problems, to name a few. Then, based on the standing assumption that the system's dynamics is persistently excited, we construct a time-dependent Lyapunov function that has a negative definite derivative. Our main statements may be regarded as off-the-shelf tools of analysis for linear and nonlinear time-varying systems.

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1. Introduction

Lyapunov's first method is useful to approximate, locally, the nonlinear dynamics of a model by a linear differential equation. Similarly, if we are interested in establishing stability properties in the large (or even global), one may find it useful to rewrite a nonlinear ordinary differential equation as a linear time-varying system by replacing the states with the trajectories.

Regarding nonlinear systems as linear time-varying is not unusual. It is useful, for instance, in the analysis of adaptive control systems (Khalil, 1996, pp. 626–627), in the stabilization of parameterized time-varying systems (Tsinias, 2005), in the analysis and design of observers for bilinear systems (Besançon, 1999; Zhang, 2002), and in dynamic feedback stabilization, notably of chain-form systems (Samson, 1995).

Establishing uniform asymptotic stability of the origin for linear time-varying systems, however, is a difficult task in general. For instance, eigen-value analysis is generally inconclusive, even for boundedness of the solutions. A recurrent property found in the literature on analysis of time-varying systems is persistency of excitation, which was originally introduced in the context of systems identification (Åstrom & Bohn, 1965) and it is known to be necessary and sufficient for uniform exponential stability of certain linear time-varying systems, see *e.g.* Morgan and Narendra (1977). It also implies uniform global asymptotic stability for some nonlinear systems (Narendra & Annaswamy, 1989), notably in the context of model-reference adaptive control. For linear time-varying systems it is well-known that persistency of excitation is equivalent to the so-called uniform complete observability. Then, under this property and an output-injection argument, uniform global exponential stability follows (Anderson et al., 1986).

For nonlinear time-varying systems, the notion of persistency excitation is intrinsically related to detectability (Artstein, 1982; Lee, 2003). However, since persistency excitation was introduced in the context of linear systems, as a property of functions that depend only on time, much of the literature on the subject does not apply directly to the analysis of nonlinear systems. Indeed, in general, the uniform character of asymptotic stability is lost (Karafyllis & Tsinias, 2003), unless one explicitly assumes that persistency of excitation holds uniformly in the initial conditions (Loría & Panteley, 2002; Tsinias, 2005). In contrast to this, persistency of excitation is not necessary for weaker forms of stability such as bounded-input-bounded-state (Goodwin & Teoh, 1983; Middleton & Goodwin, 1988).

Besides the question of stability lies that of stabilization and, at a more fundamental level, that of controllability. A general definition of (non-uniform) asymptotic controllability, for

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nonlinear systems with time-varying parameters, is given for instance in Tsinias (2005). Namely, it is established that under persistency of excitation of the input gain matrix (along the trajectories) one can always construct a control input to steer the trajectories to the origin with non-uniform rate of convergence. Systems with time-varying input gain appear naturally, *e.g.* in the stabilization of spacecrafts with magnetic actuators—see *e.g.*, Sukumar and Akella (2011).

In stabilization problems, persistency of excitation also appears naturally when there is a structural impediment to use autonomous smooth feedback, as in the case of chain-form systems (Samson, 1995). In this reference, under a change of coordinates and a preliminary feedback, the closed-loop system is transformed into a so-called skew-symmetric system. Then, following the design rationale from Samson (1995), in Loría, Panteley, and Melhem (2002) uniform global asymptotic stability was established for the closed-loop systems using controllers with persistency of excitation.

A common drawback of most works on persistency of excitation in the literature is that the methods of proof are based on tools such as Barbălat's lemma, properties of spaces of integrable functions, output-injection, *etc.*, as opposed to Lyapunov's direct method. Yet the benefits of the latter cannot be overestimated. Having a strict Lyapunov function (with negative definite derivative) is an essential step in Lyapunov redesign to achieve robustness to unmodelled dynamics (Narendra & Annaswamy, 1987).

In this paper we present Lyapunov functions for certain linear time-varying systems with persistency of excitation. We broach two different case-studies motivated by scenarios reminiscent of model-reference adaptive control, stabilization of non-holonomic systems, and systems with time-varying input gain. The common thread between these case-studies is, as *e.g.* in Morgan and Narendra (1977), neutral stability; more precisely, we assume that we know a non-strict Lyapunov function that ensures (non-asymptotic) stability. Our standing assumptions to guarantee attractivity are stated in terms of persistency of excitation. Our Lyapunov-function constructions are much inspired by Malisoff and Mazenc (2009) and Mazenc, de Queiroz, and Malisoff (2009). However, to the best of our knowledge, for the classes of systems that we study, there does not exist equivalent results in the literature.

In next section we describe the class of systems that we study. Our main statements are presented in Section 3. Some concluding remarks are provided in Section 4.

2. Problem formulation and motivation

We address the question of stability, via Lyapunov's direct method, for a wide class of linear time-varying systems, defined by

$$\dot{x} = [\mathcal{A}_o(t) + \mathcal{A}_s(t)]x, \quad x \in \mathbb{R}^n \tag{1}$$

where $\mathcal{A}_o$ and $\mathcal{A}_s$ are bounded differentiable mappings $\mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times n}$.

The system (1) covers several systems from the literature. For instance, if $\mathcal{A}_s$ is constant and Hurwitz and $\mathcal{A}_o(t)$ is skew-symmetric we recover the class of systems studied in Morgan and Narendra (1977). If, on the other hand, $\mathcal{A}_o \equiv 0$ and $\mathcal{A}_s(t)$ is negative semidefinite we recover the systems studied in Morgan and Narendra (1977). A particular case of the latter are "gradient-type" adaptive systems, which are often written in the form $\dot{x} = -\Psi(t)\Psi(t)^\top x$, with $\Psi : \mathbb{R}_{\geq 0} \to \mathbb{R}^{m\times n}$, $m \geq n$. For such systems, with Ψ bounded and with bounded derivative, it is well known that the origin is uniformly globally exponentially stable if and only if Ψ is persistently exciting (Åstrom & Bohn, 1965; Narendra & Annaswamy, 1989), that is, if there exist $\mu > 0$ and $T > 0$ such that

$$\int_t^{t+T} \Psi(s)\Psi(s)^\top ds \geq \mu I \quad \forall\, t \geq 0. \tag{2}$$

There are various proofs of this fact in the literature; see for instance (Morgan & Narendra, 1977; Narendra & Annaswamy, 1989), as well as Brockett (2000) and Loría and Panteley (2002) where tight stability bounds are also established.

Generally speaking, the model (1) has two essential constituting parts: the so-called *oscillating* drift $\mathcal{A}_o(t)x$ and the *steering* drift $\mathcal{A}_s(t)x$. Under the action of the oscillating drift, the trajectories of (1) tend to oscillate while under the action of the steering drift there exists a function of the state and time, $(t, x) \mapsto h$ (*e.g.*, part of the state), that vanishes. These properties are captured in the following hypothesis and they are illustrated by Example 1 farther below.

Assumption 1. (1) There exist two bounded smooth functions P_s and Q_s taking values from $\mathbb{R}_{\geq 0}$ to $\mathbb{R}^{n\times n}$ such that, for all $t \geq 0$, $P_s(t)$ is symmetric positive definite, $Q_s(t)$ is symmetric positive semi-definite, $Q_s(t) \not\equiv 0$, and

$$\mathcal{A}_s(t)^\top P_s(t) + P_s(t)\mathcal{A}_s(t)^\top + \dot{P}_s(t) = -Q_s(t). \tag{3}$$

(2) There exists a smooth bounded function $P_o : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times n}$ such that, for all $t \geq 0$, $P_o(t)$ is symmetric positive definite and

$$\mathcal{A}_o(t)^\top P_o(t) + P_o(t)\mathcal{A}_o(t)^\top + \dot{P}_o(t) = 0. \tag{4}$$

Remark 1. In particular, the system $\dot{x} = \mathcal{A}_o(t)x$ with $\mathcal{A}_o$ satisfying item (2) above is called neutrally stable. •

Example 1 (*Motivation*)**.** Let a and b be bounded persistently exciting functions and let

$$\mathcal{A}_s(t) := \begin{bmatrix} -a(t) & 0 \\ 0 & 0 \end{bmatrix}, \qquad \mathcal{A}_o(t) := \begin{bmatrix} 0 & -b(t) \\ b(t) & 0 \end{bmatrix}. \tag{5}$$

Now, consider the system (1), (5) with $x = [x_1,\ x_2]^\top \in \mathbb{R}^2$. This system satisfies Assumption 1 with $P_s = P_o = I$. Hence, the derivative of $V(x) = x^\top P_s x = |x|^2$ along the trajectories of (1), (5) satisfies $\dot{V} \leq -a(t)x_1^2 \leq 0$. Integrating the latter we obtain that $|x(t)|$ is bounded for all $t \geq 0$ and $\sqrt{a}x_1 \in \mathcal{L}_2$. Therefore, using standard arguments based on Barbălat's lemma, we may conclude that $h(t, x) = \sqrt{a(t)}x_1$ converges to zero (Khalil, 1996).

Now, for the sake of argument, let us regard $h(t, x) = \sqrt{a(t)}x_1$ as a converging output. Then, x_2 also converges if the system is observable with respect to h, which holds if and only if b is persistently exciting (Anderson et al., 1986). For general classes of time-varying systems, the weaker property of detectability has also been linked to persistency of excitation—see Lee (2003).

For the system defined by (1) and (5), however, persistency of excitation of b does not suffice, in general, to establish the convergence of the state trajectories because a varies with time too. For instance, let a and b be piece-wise constant periodic functions taking non-negative values, hence persistently exciting. More specifically, for each integer $n \geq 0$, let $J_n := (\pi/\bar{b})[2n+1,\ 2n+2]$ and let $\mathcal{J} := \cup_{n\geq 0} J_n$. Then, let $\bar{a}, \bar{b} > 0$ and, for all $t \in \mathcal{J}$, let $a(t) := 0$ and $b(t) := \bar{b}$ while, for all $t \notin \mathcal{J}$ let $a(t) := \bar{a}$ and $b(t) := 0$. Hence, $a(t)b(t) \equiv 0$. For this particular choice of a and b, the trajectories $x_1(t)$ and $x_2(t)$ generated by (1), (5), with $\bar{a} = \bar{b} = 1$ and initial condition $t_\circ = 0$, $x_1(0) = 0$, and $x_2(0) = 1$, do not converge to zero, but oscillate. □

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