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A multiobjective and fixed elements based modification of the evolutionary structural optimization method

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Abstract

In 1993, Y.M. Xie and G.P. Steven introduced an approach called *evolutionary structural optimization* (ESO). ESO is based on the simple idea that the optimal structure (maximum stiffness, minimum weight) can be produced by gradually removing the ineffectively used material from the design domain. The design domain is constructed by the FE method, and furthermore, external loads and support conditions are applied to the element model. Considering the engineering aspects, ESO seems to have some attractive features: the ESO method is very simple to program via the FEA packages and requires a relatively small amount of FEA time. On the other hand, different constraints cannot be added into the problem. In the ESO optimization the results supposedly approach truss-like, fully stressed topologies, which have the maximum stiffness with respect to the volume. Generally, these types of structures correspond to least-weight trusses.

Although ESO is not capable of handling general stress or displacement constraints, the design problems are often such that these constraints do not need to be included in the topology optimization, especially if the design optimization task is divided into two stages. In the first stage, only the overall geometry is outlined, and for that reason, the actual constraints do not have to be activated. In the second stage, the sizing optimization is performed. It can be concluded that ESO is well suited to solve the first stage optimization problems.

In some design problems it may occur that the structure cannot attain the fully stressed state because of geometrical constraints. It follows that the topology having the maximum stiffness with respect to the volume does not necessarily produce the least-weight structure when the stress constraints are applied in the second stage optimization. The geometrical constraints may force some structural components to be subject to a understressed state, i.e. to carry some "waste material". As a consequence, the aim of this paper was to study whether ESO can be modified so that some geometrical constraints can be taken into account already in the first stage topology optimization. The modification was based on the assumption that if the stress level of otherwise understressed structural components can be increased during the *compliance-volume* product minimization, a lighter topology may be obtained. This new approach, the multiobjective and fixed elements based modification of the evolutionary structural optimization (MESO) utilizes a new optimization objective in which the overall stiffness of the structure and the loading of some parts of the structure are increased simultaneously. The gradient vector of the MESO objective function was determined by the FE method. Some of the partial derivatives involved were first presolved and then approximated. This approach was justified by large savings in the analysis time. Yet, MESO cannot take the general constraints into account.

To study the performance of the MESO optimization, two numerical examples were evaluated. The main purpose of these examples was to study whether MESO can produce structures lighter than the ESO results for problems having both stress and geometrical constraints.

These example were based on the two-stage optimization approach. In both example the MESO truss turned out to be lighter than the corresponding ESO truss. However, the ESO truss was the stiffest one also having a smaller overall stress value. In the question of shallow structures the deflection criteria may be predominant, and as a consequence, the ESO optimization may yield lighter structures than MESO.

Keywords: Topological optimization; Evolutionary structural optimization; ESO; MESO

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1. Introduction

1.1. Structural optimization

Many elegant methods for structural optimization are presented in the literature. Short surveys of this field can be found in [47,27,41,46]. Structural optimization is often divided into three classes: *sizing*, *geometrical*, and *topology optimization* [36]. The differences between these classes can be explained by a truss example:

In *sizing optimization*, the cross-sectional dimensions can be chosen as the design variables. The values of the design variables are not allowed to go to zero, in other words, individual structural members cannot be removed.
In *geometrical optimization*, the coordinates of the joints can be chosen as the design variables.
In *topology optimization*, the number of structural members and the connectivity of the members is optimized.

In the case of plate-type structures, the term *shape* or *fixed-topology optimization* may be used instead of geometrical optimization. In these cases, the boundary line of a structure may be varied during the optimization process. If additional holes are introduced into the structure, the terms *variable-topology* and *generalized shape optimization* may be used [17].

It should be noted that there may be cases in which the above classification does not apply. The terminology concerning the structural optimization classes also varies in the literature. For simplicity, the optimization scheme including all three classes of optimization is also called topology optimization in this paper.

It is characteristic for sizing and geometrical optimization that the topology of a structure cannot be altered during the optimization process. When applying these methods, it is not guaranteed that the structure obtained is the best or even a good one: another initial topology might produce a remarkably better solution for the minimization problem. Since topology optimization also looks for the best overall topology satisfying the problem constraints, the optimization process cannot be misled as easily by a poor initial guess. For this reason, only by using topology optimization it is possible to produce the best overall structure.

1.2. Topology optimization

1.2.1. General

Topology optimization was pioneered by [26], who studied statically determinate trusses for a number of loading and support conditions. His analytical results, so-called *Michell trusses*, have an infinite number of members of varying length. In Michell trusses, each bar is subject to a constant strain (stress). It has also been analytically proved that the Michell truss cannot have any greater compliance to the given load than any other truss using the same amount of material (for linearly elastic material compliance equals twice the work done by the external forces or twice the total strain energy of the structure) [40]. Since the Michell trusses have an infinite number of structural components, they are rather impractical in the engineering applications.

In the 1960s, topology optimization was remarkably improved when the so-called *ground structure approach* was first introduced [12]. In the ground structure approach, the design domain is formed by a finite number of truss members, and each member is a potential part of the optimal truss. By applying numerical optimization methods, the additional bars can be removed from the design domain, and as a result, the remaining truss members represent the optimal topology.

Originally, ground structure problems were solved by using direct optimization methods, i.e the *mathematical programming* (*MP*) *algorithms*. However, they were, and still are, inefficient in solving large optimization problems. On the other hand, the MP algorithms are well suited for handling all kinds of objective functions and constraints. To solve a realistic optimization problem a rather large design domain has to be employed, and consequently, the MP algorithms may limit the use of the ground structure approach. Instead of the MP algorithms, indirect optimization methods, i.e. *optimality criteria* (*OC*) *algorithms*, can be employed to solve structural problems. In the OC optimization, it is necessary to determine an appropriate criterion on which the optimality of the solution is based. The criterion may be related, for instance, to the structural stresses: it is often assumed that for the least-weight truss each bar is subject to the corresponding allowable stress value. The approach based on the above criteria is also called *the fully stressed design* (*FSD*) *method* [14]. In the fully stressed state each structural component is subject to its maximum/minimum allowable stress value. The allowable stress values may be different for each component. The allowable stresses may also be different in tension and compression, if, for instance, buckling is considered. If the limiting stresses are equal for every structural component, the resulting FSD structure is also equally stressed. Most often in the literature, the equally stressed state is also called fully stressed. Typically, the OC algorithms consist of consecutive, iterative redesign loops, in order to produce, for instance, a fully stressed state. Prager's contribution to the OC-based topology optimization, starting from the late 1960s, should be especially acknowledged [30]. Compared with the MP algorithms, the OC methods are efficient in large optimization problems, but lack generality in various kinds of minimization problems. OC algorithms are also discussed by Save and Prager [40].

Topology optimization of trusses using both MP and OC algorithms is studied, for instance, in [29,33,37–39].

In the literature, topology optimization is most often applied to truss ground structures based design domains. However, optimization procedures have been developed to deal with general layout optimization problems in which