Automatica 78 (2017) 119–122

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Non-augmented state estimation for nonlinear stochastic coupling networks☆

Wenling Li[a], Yingmin Jia[a], Junping Du[b]

[a] The Seventh Research Division, Beihang University (BUAA), Beijing 100191, China

[b] School of Computer Science and Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China

ARTICLE INFO

Article history:
Received 28 February 2016
Received in revised form
2 December 2016
Accepted 5 December 2016

Keywords:
State estimation
Complex network
Non-augmented approach

ABSTRACT

This paper considers the state estimation problem for discrete-time nonlinear stochastic coupling networks. A non-augmented filter is designed for each node to guarantee an optimized upper bound on the state estimation error covariance matrix despite nodes coupling as well as the linearization errors. Compared with the existing augmented filter, the cross-covariance matrices between coupling nodes are not required to be computed and the gain matrix can be obtained separately for each node by solving two Riccati-like difference equations.

1. Introduction

State estimation for complex networks has been widely studied in the theoretical research community and successfully applied in industry (Ata-ur-Rehman, Lyudmila, & Jonathon, 2016; Zia, Tucker, & Frank, 2005) This is partly due to the fact that state estimation is crucial not only because it helps understand the intrinsic structure of the networks but also because it is the first step to realize synchronization (Rodriguez-Angeles & Nijmeijer, 2004).

Compared with the state estimation for an isolated node, the state estimation problem for complex networks becomes more difficult due to the nodes coupling. Specifically, the states of the nodes are not only determined by themselves but also by their neighbors. To overcome this difficulty, many strategies have been proposed to develop filters for complex networks including uncertain stochastic complex networks with missing measurements and time-varying delay (Liang, Wang, & Liu, 2009; Liang, Wang, Liu, & Liu, 2014), H_∞ filters with uncertain coupling strength and incomplete measurements (Shen, Wang, Ding, & Shu, 2013; Shen, Wang, & Liu, 2011), uncertain complex networks with time-varying delays (Ding, Wang, Shen, & Shu, 2012), and time-varying complex networks with missing measurements (Hu, Wang, Liu, & Gao, 2016). It should be pointed out that all the existing work focus on developing filters using the augmented approach, i.e., all the state estimation errors of the nodes are formulated in a compact form and the gain matrices for all nodes are obtained simultaneously with respect to the overall covariance matrix of the estimation errors. Thus, the disadvantage of the augmented filters is that high computational cost is often required for large number of nodes.

In this paper, we attempt to develop a non-augmented filter for a class of discrete-time nonlinear stochastic coupling networks. By using the structure of the extended Kalman filter (EKF), a novel filter is developed by proposing the predicted and updated estimation error systems. To address the coupling features and the linearization errors, upper bound matrices are introduced for the corresponding covariance matrices so that the gain matrices can be derived by minimizing the trace of the upper bound matrix. A distinct feature of the proposed filter is that the cross-covariance matrices between coupling node are not required to be computed and the gain matrix can be derived separately for each node.

2. Problem statement

Consider the following nonlinear stochastic network

$$x_{i,k+1} = f(x_{i,k}) + \sum_{j=1}^{N} \omega_{ij} \Gamma x_{j,k} + w_{i,k} \tag{1}$$

$$z_{i,k} = h(x_{i,k}) + v_{i,k}, \quad i = 1, 2, \ldots, N \tag{2}$$

☆ This work was supported by the National Basic Research Program of China (973 Program, 2012CB821200, 2012CB821201), and the NSFC (61573031, 61134005, 61327807, 61473010, 61532006, 61520106010). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Carlo Fischione under the direction of Editor André L. Tits.

E-mail addresses: lwlmath@buaa.edu.cn (W. Li), ymjia@buaa.edu.cn (Y. Jia), junpingdu@126.com (J. Du).

http://dx.doi.org/10.1016/j.automatica.2016.12.033
0005-1098/© 2016 Elsevier Ltd. All rights reserved.

where $x_{i,k} \in \mathbb{R}^n$ is the state vector of the ith node and $z_{i,k} \in \mathbb{R}^p$ is the measurement vector of the ith node. $f(\cdot)$ and $h(\cdot)$ are known nonlinear functions that are assumed to be twice continuously differentiable. Γ is a matrix and $W = [\omega_{ij}]_{N\times N}$ is the coupling configuration matrix with $\omega_{ij} \geq 0$. The process noise $w_{i,k}$ and the measurement noise $v_{i,k}$ are assumed to be mutually uncorrelated zero-mean white Gaussian with known covariance matrices $Q_{i,k}$ and $R_{i,k}$, respectively.

The structure of the EKF is adopted to design the following filter for the ith node

$$\bar{x}_{i,k} = f(\hat{x}_{i,k}) + \sum_{j=1}^{N} \omega_{ij}\Gamma\hat{x}_{j,k} \tag{3}$$

$$\hat{x}_{i,k+1} = \bar{x}_{i,k} + K_{i,k+1}[z_{i,k+1} - h(\bar{x}_{i,k})] \tag{4}$$

where $\bar{x}_{i,k}$ and $\hat{x}_{i,k+1}$ denote the predicted and the updated estimates at time instant $k+1$, respectively. $K_{i,k+1}$ is the gain matrix to be determined for the ith node at time instant $k+1$. The updated estimation error and its corresponding covariance matrix are defined as

$$e_{i,k+1} = x_{i,k+1} - \hat{x}_{i,k+1} \tag{5}$$

$$P_{i,k+1} = \mathbb{E}\{e_{i,k+1}e_{i,k+1}^T\}. \tag{6}$$

The aim of this paper is to design a filter described by (3)–(4), such that there exists a sequence of positive-definite matrices $\Phi_{i,k+1}$ satisfying

$$P_{i,k+1} \leq \Phi_{i,k+1}. \tag{7}$$

The bound $\Phi_{i,k+1}$ is obtained by solving a Riccati difference equation and the gain matrix $K_{i,k+1}$ is derived by minimizing the trace of the upper bound matrix $\Phi_{i,k+1}$ at each time instant. It should be pointed out that the notation $X \geq Y$ (respectively, $X > Y$) means that $X-Y$ is positive semidefinite in the sense of the corresponding quadratic forms (respectively, positive definite).

Remark 1. In Hu et al. (2016), the augmented approach has been used to derive the state estimation of the stochastic complex networks (1)–(2) with missing measurements. To be specific, the augmented estimation errors of all nodes and the corresponding covariance matrix are defined as

$$\tilde{e}_{k+1} = [e_{1,k+1}^T, \ldots, e_{N,k+1}^T]^T \tag{8}$$

$$\tilde{P}_{k+1} = \mathbb{E}\{\tilde{e}_{k+1}\tilde{e}_{k+1}^T\}. \tag{9}$$

The filter is designed by defining a sequence of positive-definite matrices $\tilde{\Phi}_{k+1}$ satisfying

$$\tilde{P}_{k+1} \leq \tilde{\Phi}_{k+1}. \tag{10}$$

The gain matrix is derived by minimizing the trace of the upper bound matrix $\tilde{\Phi}_{k+1}$ at each time instant. It can be seen that the dimension of the matrix $\tilde{\Phi}_{k+1}$ becomes larger as the number of the nodes increases and therefore high computational costs are often required to derive the gain matrix by using the augmented approach.

3. Non-augmented filter

The following lemma is adopted from the literature to derive the non-augmented filter.

Lemma 1 (Xie, Soh, & Souza, 1994)**.** *Given matrices A, B, C and D with appropriate dimensions such that $CC^T \leq I$. Let U be a symmetric positive definite matrix and $a > 0$ be an arbitrary positive constant such that $a^{-1}I - DUD^T > 0$. Then the following inequality holds*

$$\begin{aligned}&(A + BCD)U(A + BCD)^T \\ &\quad \leq A(U^{-1} - aD^TD)^{-1}A^T + a^{-1}BB^T.\end{aligned} \tag{11}$$

Now, we define the predicted estimation error and its covariance matrix

$$\bar{e}_{i,k} = x_{i,k+1} - \bar{x}_{i,k} \tag{12}$$

$$\bar{P}_{i,k} = \mathbb{E}\{\bar{e}_{i,k}\bar{e}_{i,k}^T\}. \tag{13}$$

As shown in Hu et al. (2016), by using the Taylor series expansion technique, the predicted estimation error and the updated estimation error can be represented as follows

$$\bar{e}_{i,k} = (F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)e_{i,k} + \sum_{j=1}^{N}\omega_{ij}\Gamma e_{j,k} + w_{i,k} \tag{14}$$

$$\begin{aligned}e_{i,k+1} &= (I - K_{i,k+1}H_{i,k+1} - K_{i,k+1}L_{i,k}^h\Omega_{i,k}^h)\bar{e}_{i,k} \\ &\quad - K_{i,k+1}v_{i,k+1}\end{aligned} \tag{15}$$

where $F_{i,k} = \frac{\partial f(x)}{\partial x}|_{x=\hat{x}_{i,k}}$ and $H_{i,k+1} = \frac{\partial h(x)}{\partial x}|_{x=\bar{x}_{i,k}}$. $L_{i,k}^f$ and $L_{i,k}^h$ are problem-dependent scaling matrices. $\Omega_{i,k}^f$ and $\Omega_{i,k}^h$ denote the unknown time-varying matrix accounting for the linearization errors satisfying $\Omega_{i,k}^f(\Omega_{i,k}^f)^T \leq I$ and $\Omega_{i,k}^h(\Omega_{i,k}^h)^T \leq I$, respectively (Giuseppe, 2005). In this paper, as in Giuseppe (2005), the scaling matrices $L_{i,k}^f$ and $L_{i,k}^h$ are employed to account for the linearization errors. For more details we refer the reader to Appendix C of Giuseppe (2005).

Then, the predicted estimation error covariance can be derived with respect to (14)

$$\begin{aligned}\bar{P}_{i,k} &= (F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)P_{i,k}(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)^T + Q_{i,k} \\ &\quad + \sum_{j=1}^{N}\omega_{ij}\mathbb{E}\{(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)e_{i,k}e_{j,k}^T\Gamma^T \\ &\quad + \Gamma e_{j,k}e_{i,k}^T(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)^T\} \\ &\quad + \sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ij}\omega_{il}\mathbb{E}\{\Gamma e_{j,k}e_{l,k}^T\Gamma^T\}.\end{aligned} \tag{16}$$

By using the inequality $xy^T + yx^T \leq xx^T + yy^T$, an upper bounded matrix can be derived for the third term on the right hand side of (16)

$$\begin{aligned}&\sum_{j=1}^{N}\omega_{ij}\mathbb{E}\{(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)e_{i,k}e_{j,k}^T\Gamma^T \\ &\quad + \Gamma e_{j,k}e_{i,k}^T(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)^T\} \\ &\leq \sum_{j=1}^{N}\omega_{ij}[(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)P_{i,k}(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)^T \\ &\quad + \Gamma P_{j,k}\Gamma^T] \\ &= \bar{\omega}_i(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)P_{i,k}(F_{i,k} + L_{i,k}^f\Omega_{i,k}^f)^T \\ &\quad + \sum_{j=1}^{N}\omega_{ij}\Gamma P_{j,k}\Gamma^T\end{aligned} \tag{17}$$

where $\bar{\omega}_i = \sum_{j=1}^{N}\omega_{ij}$.

Similarly, an upper bounded matrix can be derived for the fourth term on the right hand side of (16)

$$\begin{aligned}&\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ij}\omega_{il}\mathbb{E}\{\Gamma e_{j,k}e_{l,k}^T\Gamma^T\} \\ &\leq \frac{1}{2}\sum_{j=1}^{N}\sum_{l=1}^{N}\omega_{ij}\omega_{il}(\Gamma P_{j,k}\Gamma^T + \Gamma P_{l,k}\Gamma^T) \\ &= \bar{\omega}_i\sum_{j=1}^{N}\omega_{ij}\Gamma P_{j,k}\Gamma^T.\end{aligned} \tag{18}$$