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Survey Paper

Recent developments on the stability of systems with aperiodic sampling: An overview[☆]

Laurentiu Hetel[a], Christophe Fiter[a], Hassan Omran[b], Alexandre Seuret[c], Emilia Fridman[d], Jean-Pierre Richard[a,e], Silviu Iulian Niculescu[f]

[a] CNRS CRISTAL UMR 9189, École Centrale de Lille / Univ. Lille 1, 59651 Villeneuve d'Ascq, France
[b] ICube lab., UMR 7357 University of Strasbourg/CNRS, France
[c] CNRS, LAAS, 7 avenue du Colonel Roche, F–31400 Toulouse, France
[d] School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel
[e] Non-A, INRIA Lille-Nord Europe, France
[f] Laboratoire des Signaux et Systèmes (L2S, UMR CNRS 8506), CNRS - Centrale SUPELEC - Univ. Paris Sud, 3 rue Joliot Curie, 91192 Gif-sur-Yvette, France

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ABSTRACT

This article presents basic concepts and recent research directions about the stability of sampled-data systems with aperiodic sampling. We focus mainly on the stability problem for systems with arbitrary time-varying sampling intervals which has been addressed in several areas of research in Control Theory. Systems with aperiodic sampling can be seen as time-delay systems, hybrid systems, Input/Output interconnections, discrete-time systems with time-varying parameters, etc. The goal of the article is to provide a structural overview of the progress made on the stability analysis problem. Without being exhaustive, which would be neither possible nor useful, we try to bring together results from diverse communities and present them in a unified manner. For each of the existing approaches, the basic concepts, fundamental results, converse stability theorems (when available), and relations with the other approaches are discussed in detail. Results concerning extensions of Lyapunov and frequency domain methods for systems with aperiodic sampling are recalled, as they allow to derive constructive stability conditions. Furthermore, numerical criteria are presented while indicating the sources of conservatism, the problems that remain open and the possible directions of improvement. At last, some emerging research directions, such as the design of stabilizing sampling sequences, are briefly discussed.

1. Introduction

The last decade has witnessed an enormous interest in the study of networked and embedded control systems (Chen, Johansson, Olariu, Paschalidis, & Stojmenovic, 2011; Hespanha, Naghshtabrizi, & Xu, 2007; Hristu-Varsakelis & Levine, 2005; Zhang, Branicky, & Phillips, 2001). This interest is mainly due to the ubiquitous presence of embedded controllers in relevant application domains and the growing demand in industry on systematic methods to model, analyse and design systems where sensor and control data are transmitted over a digital communication channel. The study of systems with aperiodic sampling emerged as a modelling abstraction which allows to understand the behaviour of Networked Control Systems (NCS) with sampling jitters, packet drop-outs or fluctuations due to the inter-action between control algorithms and real-time scheduling protocols (Antsaklis & Baillieul, 2007; Astolfi, Nesic, & Teel, 2008; Zhang et al., 2001). With the emergence of event-based and self-triggered control techniques (Årzén, 1999; Åström & Bernhardsson, 1999; Heemels, Johansson, & Tabuada, 2012; Velasco, Fuertes, & Marti, 2003), the study of aperiodic sampled-data systems constitutes nowadays a very popular research topic in control. In this survey, we focus on questions arising in the control of systems with arbitrary

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E-mail addresses: laurentiu.hetel@ec-lille.fr (L. Hetel), christophe.fiter@univ-lille1.fr (C. Fiter), hassan.samir.omran@gmail.com (H. Omran), aseuret@gmail.com, aseuret@laas.fr (A. Seuret), emilia@eng.tau.ac.il (E. Fridman), Jean-Pierre.Richard@ec-lille.fr (J.-P. Richard), Silviu.Niculescu@lss.supelec.fr (S.I. Niculescu).

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time-varying sampling intervals. Important practical questions such as the choice of the sampling frequency, the evaluation of necessary computational and energetic resources or the robust control synthesis are mainly related to stability issues. These issues often lead to the problem of estimating the Maximum Sampling Interval (MSI) for which the stability of a closed-loop sampled-data system is ensured.

The study of aperiodic sampled-data systems has been addressed in several areas of research in Control Theory. Systems with aperiodic sampling can be seen as particular *time-delay systems*. Sampled-and-hold in control and sensor signals can be modelled using *hybrid systems* with impulsive dynamics. Aperiodic sampled-data systems have also been studied in the *discrete-time* domain. In particular, Linear Time Invariant (LTI) sampled-data systems with aperiodic sampling have been analysed using discrete-time *Linear Parameter Varying (LPV) models*. The effect of sampling can be modelled using *operators* and the stability problem can be addressed in the framework of Input/Output interconnections as typically done in modern *Robust Control*. While significant advances on this subject have been presented in the literature, problems related to both the fundamentals of such systems and the derivation of constructive methods for stability analysis remain open, even for the case of linear system. The objective of the article is to present in a unified and structured manner a collection of significant results on this topic.

The core of the article is dedicated to the analysis of systems with arbitrary varying sampling intervals. We will only consider the deterministic aspects of the problem. The case when sampling intervals are random variables given by a probability distribution will not be discussed here. After presenting some generalities and motivations concerning sampled-data systems with aperiodic sampling (in Section 2), some basic qualitative results are recalled in Section 3. Section 4 presents the main stability analysis approaches. At last, in Section 5, we briefly discuss some emerging research problems, such as the design of stabilizing sampling sequences. We indicate the main challenges, the relations with the arbitrary sampling problem and some perspectives on which the current approaches and tools for aperiodic sampled-data systems may be useful in the future.

Notations: Throughout the paper, $\mathbb{R}_+$ denotes the set $\{\lambda \in \mathbb{R}, \lambda \geq 0\}$, $\|x\|$ represents any norm of the vector x and $\|x\|_p$, $p \in \mathbb{N}$, the p norm of a vector x. For a matrix M, M^T denotes the transpose of M and $M^\star$, its conjugate transpose. For square symmetric matrices M, N, $M \succeq N$ (resp. $M \succ N$) means that $M - N$ is a positive (resp. definite positive) matrix. $*$, in a symmetric matrix represents elements that may be induced by symmetry. $\|M\|_p$, $p \in \mathbb{N}$ denotes the induced p-norm of a matrix M. $\bar{\sigma}(M)$ denotes the maximum singular value of M. $\mathcal{C}^0(X, Y)$, for two metric spaces X and Y, is the set of continuous functions from X to Y. $\mathcal{L}_p^n(a, b)$, $p \in \mathbb{N}$ denotes the space of functions $\phi : (a, b) \to \mathbb{R}^n$ with norm $\|\phi\|_{\mathcal{L}_p} = \left[\int_a^b \|\phi(s)\|^p ds\right]^{\frac{1}{p}}$, and $\mathcal{L}_{2e}^n[0, \infty)$ is the space of functions $\phi : [0, \infty) \to \mathbb{R}^n$ which are square integrable on finite intervals.

2. Generalities

2.1. System configuration

In this paper we study the properties of sampled-data systems consisting of a plant, a digital controller, and appropriate interface elements. A general configuration of such a sampled-data system is illustrated by the block diagram of Fig. 1. In this configuration, $y(t)$ is a continuous-time signal representing the plant output (the plant variables that can be measured). This signal is represented as a function of time t, $y : \mathbb{R}_+ \to \mathbb{R}^p$.

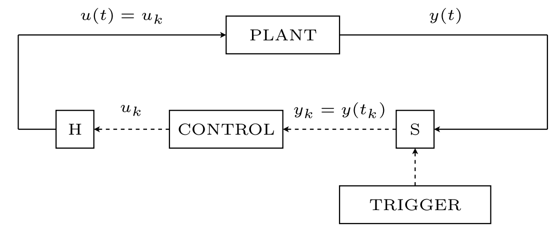

Fig. 1. Classical sampled-data system configuration.

The digital controller is usually implemented as an algorithm on an embedded computer. It operates with a sampled version of the plant output signal, $\{y_k\}_{k\in\mathbb{N}}$, obtained upon the request of a sampling trigger signal at discrete *sampling instants* t_k and using an analog-to-digital converter (the sampler block, S, in Fig. 1). This trigger may represent a simple clock, as in the classical periodic sampling paradigm, or a more complex scheduling protocol which may take into account the sensor signal, a memory of its last sampled values, etc. The sampling instants are described by a monotone increasing sequence of positive real numbers $\sigma = \{t_k\}_{k\in\mathbb{N}}$ where

$$t_0 = 0, \qquad t_{k+1} - t_k > 0, \qquad \lim_{k\to\infty} t_k = \infty. \tag{1}$$

The difference between two consecutive sampling times $h_k = t_{k+1} - t_k$ is called the kth *sampling interval*. Assuming that the effect of quantizers may be neglected, the sampled version of the plant output is the sequence $\{y_k\}_{k\in\mathbb{N}}$ where $y_k = y(t_k)$.

In a sampled-data control loop, the digital controller produces a sequence of control values $\{u_k\}_{k\in\mathbb{N}}$ using the sampled version of the plant output signal $\{y_k\}_{k\in\mathbb{N}}$. This sequence is converted into a continuous-time signal $u(t)$, where $u : \mathbb{R}_+ \to \mathbb{R}^m$ (corresponding to the plant input) via a digital-to-analog interface. We consider that the digital-to-analog interface is a zero-order hold (the hold block, H, in Fig. 1). Furthermore, we assume that there is no delay between the sampling instant t_k and the moment the control u_k (obtained based on the kth plant output sample, y_k) is effectively implemented at the plant input. Then the input signal $u(t)$ is a piecewise constant signal $u(t) = u(t_k) = u_k$, $\forall t \in [t_k, t_{k+1})$.

In this survey, we will consider that the plant is modelled by a finite dimensional ordinary differential equation of the form

$$\begin{cases} \dot{x} = F(t, x, u), \\ y = H(t, x, u), \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$ represents the plant state-variable. Here $F : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ with $F(t, 0, 0) = 0, \forall t \geq 0$, and $H : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$. It is assumed that for each constant control and each initial condition $(t_0, x_0) \in \mathbb{R}_+ \times \mathbb{R}^n$ the function F describing the plant model (2) is such that a unique solution exists for an interval $[t_0, t_0 + \epsilon)$ with ϵ large enough with respect to the maximum sampling interval. The discrete-time controller is considered to be described by an ordinary difference equation of the form

$$\begin{cases} x_{k+1}^c = F_d^c\left(k, x_k^c, y_k\right), \\ u_k = H_d^c\left(k, x_k^c, y_k\right), \end{cases} \tag{3}$$

where $x_k^c \in \mathbb{R}^{n_c}$ is the controller state. Here, $F_d^c : \mathbb{N} \times \mathbb{R}^{n_c} \times \mathbb{R}^p \to \mathbb{R}_c^n$ and $H_d^c : \mathbb{N} \times \mathbb{R}^{n_c} \times \mathbb{R}^p \to \mathbb{R}^m$. We will use the denomination *sampled-data system* for the interconnection between the continuous-time plant (2) with the discrete-time controller (3) via the relations

$$y_k = y(t_k), \qquad u(t) = u_k, \quad \forall t \in [t_k, t_{k+1}), \ \forall k \in \mathbb{N}, \tag{4}$$

under a sequence of sampling instants $\sigma = \{t_k\}_{k\in\mathbb{N}}$ satisfying (1).