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Determinability and state estimation for switched differential–algebraic equations[☆]

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ABSTRACT

The problem of state reconstruction and estimation is considered for a class of switched dynamical systems whose subsystems are modeled using linear differential–algebraic equations (DAEs). Since this system class imposes time-varying dynamic and static (in the form of algebraic constraints) relations on the evolution of state trajectories, an appropriate notion of observability is presented which accommodates these phenomena. Based on this notion, we first derive a formula for the reconstruction of the state of the system where we explicitly obtain an injective mapping from the output to the state. In practice, such a mapping may be difficult to realize numerically and hence a class of estimators is proposed which ensures that the state estimate converges asymptotically to the real state of the system.

1. Introduction

Switched differential–algebraic equations (DAEs) form an important class of switched systems, where the dynamics are not only discontinuous with respect to time, but also the state trajectories are constrained by certain algebraic equations which may also change as the system switches from one mode to another. Such dynamical models have found utility e.g. in the analysis of electrical power distribution (Gross, Trenn, & Wirsen, 2014) and of electrical circuits (Trenn, 2012). We consider switched DAEs of the following form

$$\begin{aligned} E_\sigma \dot{x} &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x \end{aligned} \tag{1}$$

where x, u, y denote the state (with dimension $n \in \mathbb{N}$), input (with dimension $\mathtt{u} \in \mathbb{N}$) and output (with dimension $\mathtt{y} \in \mathbb{N}$) of the system, respectively. The switching signal $\sigma : (t_0, \infty) \to \mathbb{N}$ is a piecewise constant, right-continuous function of time and in our notation it changes its value at time instants $t_1 < t_2 < \cdots$ called the switching times. For a fixed σ, the triplet (x, u, y) is used to denote signals satisfying (1). We adopt the convention that over the interval $[t_p, t_{p+1})$ of length $\tau_p := t_{p+1} - t_p$, the active mode is defined by the quadruple $(E_p, A_p, B_p, C_p) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times \mathtt{u}} \times \mathbb{R}^{\mathtt{y}\times n}$, $p \in \mathbb{N}$. Over the interval (t_0, t_1), it is assumed that the system has some past which may be described by (E_0, A_0, B_0, C_0). The solution concept for switched DAEs is studied in Trenn (2009) and a brief discussion is also included in Section 3.1.

Dynamical system with discontinuous, or constrained trajectories have gathered a lot of interest in the control community, as they form an important class of hybrid, or discontinuous dynamical systems, see e.g. Goebel, Sanfelice, and Teel (2009). One direction of research for these systems includes the study of structural properties that could be used for control design problems, and in this regard the problem of state reconstruction and estimation is of particular interest (Tanwani, 2011). In the current literature, one finds that the earlier work on observability and observers for switched/hybrid systems was aimed at using classical Luenberger observers for continuous dynamics and treat the switching or discontinuity as a perturbation that can be overcome by certain strong assumptions on system data. This line of work also requires that the underlying observability notion allows instantaneous recovery of the state from the measured output (Babaali & Pappas, 2005; Vidal, Chiuso, Soatto, & Sastry, 2003). Modified Luenberger observers have also been used for constrained dynamical systems using similar observability concepts: see Tanwani, Brogliato, and Prieur (2014) when the constraint sets are convex (or, mildly

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nonconvex), and Tanwani, Brogliato, and Prieur (2016) when the constraints result in impacts.

However, for switched dynamical systems, several different notions of observability can be defined. In Sun, Ge, and Lee (2002) and Xie and Wang (2003), a switched system comprising ordinary differential equations (ODEs) is called observable if there exists a switching signal that allows reconstruction of the initial state from the output measurements over an interval. This concept also appears in the observer construction (for continuous state and switching signal) proposed in Balluchi, Benvenuti, Di Benedetto, and Sangiovanni-Vincentelli (2003). However, in our recent work, a more relaxed notion of observability has been proposed for switched ODEs (Tanwani, Shim, & Liberzon, 2013) and switched DAEs (Tanwani & Trenn, 2012). The switching signal in this case is assumed to be known and fixed (i.e. playing the same role as the input u which is also assumed to be known and not influenced by the observer). By measuring the outputs and inputs over an interval, and using the data of subsystems activated during that interval, it is determined whether state reconstruction is possible or not. Several variants of this notion are also collected in the survey (Petreczky, Tanwani, & Trenn, 2015). A state estimation algorithm based on these generalized observability concept can be found in Shim and Tanwani (2014) and Tanwani et al. (2013); Tanwani, Shim, and Liberzon (2015) for switched ODEs. The main contribution of this paper is to address the observer design for switched DAEs which, apart from our preliminary work (Tanwani & Trenn, 2013), has not been addressed in this literature.

Already in the context of nonswitched systems several challenges arise in the study of DAEs. The difference basically arises due to the presence of algebraic equations (static relations) in the description of the system because of which the state trajectories can only evolve on the sets defined by the algebraic equations of the active mode. Observer designs have been studied for (nonswitched) DAEs since 1980's, e.g. Dai (1989) and Fahmy and O'Reilly (1989). Unlike ODEs, the observer design in DAEs requires additional structural assumptions and, furthermore, the order of the observer may depend on the design method. Because of these added generalities, observer design for nonswitched DAEs is still an active research field (Bobinyec, Campbell, & Kunkel, 2011; Darouach, 2012), and the recent survey articles summarize the development of this field (Berger & Reis, submitted for publication; Bobinyec & Campbell, 2014).

In studying switched DAEs, our modeling framework allows for time-varying algebraic relations. The changes in algebraic constraints due to switching introduce jumps in the state of the system, and because of the possibility of a higher-index DAE, these jumps may get differentiated and generate impulsive solutions. The notion of observability studied in this paper takes into account the additional structure due to algebraic constraints, and the added information from the outputs in case there are impulses observed in the measurements. This observability concept is then used to construct a mapping from the output space to the state space, which allows us to theoretically reconstruct the state. The key element of constructing this mapping is to show how the structure of a linear DAE is exploited to recover the information about the state in individual subsystems. This structural decomposition is then combined with the expressions used for evolution of states in switched DAEs to accumulate all possible recoverable information from past measurements about the state at one time instant. The construction then yields a systematic procedure for writing the value of the state at a time instant in terms of outputs measured over an interval for which the system is observable.

The theoretical mappings constructed in the process are often not realizable in practice, but the derivation is used to describe a general class of state observers that generate asymptotically converging state estimates. The main result states that if the observable components of the individual subsystems can be estimated

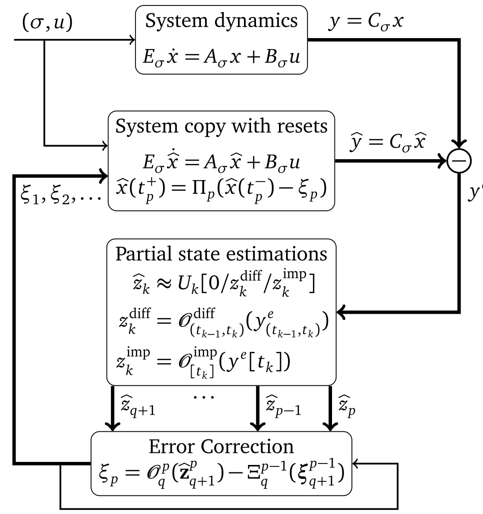

Fig. 1. A schematic representation of the state estimator.

well-enough, and the required observability assumption persistently holds with time, then the estimates converge asymptotically. Our initial work on observers for switched DAEs (Tanwani & Trenn, 2013), and even the observers proposed for switched ODEs (Shim & Tanwani, 2014; Tanwani et al., 2013) can be seen as a special case of the general class of state estimators studied in this paper.

2. Contribution and layout

This section provides a summary of all the technical results that are developed in this article, and a coherent view of how the different components are connected together to solve the state estimation problem for switched DAEs. The reader may also refer to this section as an index for finding the appropriate section for technical terms. The main contribution of this article is to provide a systematic procedure for designing observers for system class (1) which generate asymptotically convergent state estimates. The flow-diagram which shows the working of the proposed observer is given in Fig. 1.

Our proposed observer basically comprises two layers. In the first layer, a system copy with variable $\widehat{x}$ (also the state estimate) is simulated with same time scale as the actual plant, and $\widehat{x}$ is reset at some switching times t_p by an error correction vector ξ_p. The second layer comprises an algorithm to compute ξ_p in very short (negligibly small) time. This article (in Sections 4–6) develops the theoretical tools and numerical certificates that guarantee efficient implementation of this algorithm. The central idea is that ξ_p closely approximates the estimation error prior to the reset times $e(t_p^-) := \widehat{x}(t_p^-) - x(t_p^-)$ by using the output measurements over the interval $(t_q, t_p]$, $q < p$. If this approximation can be achieved to a desired accuracy, and these sufficiently rich approximates are injected into the estimate's dynamics by resetting $\widehat{x}$ often enough, then our algorithms ensure that $\widehat{x}(t)$ converges to $x(t)$ as t tends to infinity. There are three primary mechanisms involved in achieving this desired goal:

- Determinability notion in Section 4 captures the property that, by measuring the output of (1) over an interval $(t_q, t_p]$, we must be able to recover the state of the system $x(t_p^+)$ exactly; and Theorem 1 provides conditions on system data to achieve this property.
- In Section 5, under the assumption that system (1) is determinable over the interval $(t_q, t_p]$, we build the theoretical engine to compute ξ_p. This involves looking at the observable