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Adaptive output synchronization of heterogeneous network with an uncertain leader☆

Yuanqing Wu [a], Renquan Lu [a], Peng Shi [b,c], Hongye Su [d], Zheng-Guang Wu [d]

[a] School of Automation, and Guangdong Key Laboratory of IoT Information Processing, Guangdong University of Technology, Guangzhou 510006, China
[b] School of Electrical and Electronic Engineering, University of Adelaide, SA 5005, Australia
[c] College of Engineering and Science, Victoria University, VIC 8001, Australia
[d] National Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China

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ABSTRACT

In this paper, the output synchronization problem is investigated for a heterogeneous network. Specifically, the non-identical followers are affected by parameter perturbations and controlled to achieve output synchronization with an uncertain leader. We use the adaptive control theory and the robust output regulation theory to solve this problem. The proposed method includes two stages, which is similar to the separation principle. In the first stage, the designed distributed control law and adaptive control law can force that the outputs of the reference generators locally exponentially converge to the output of the leader. In the second stage, the robust output regulation control law is applied in a decentralized control model, to guarantee that the output of each non-identical follower robustly tracks the output of the corresponding reference generator. The main contributions of this paper are the constructions of adaptive reference generators in the first stage and robust regulators in the second stage. Examples are presented to show the effectiveness of the proposed design techniques.

1. Introduction

The synchronization problem of multi-agent systems (MASs) has attracted considerable attention due to its wide applications, see for example, Du, Li, and Shi (2012), Li, Duan, and Lewis (2014), Shi and Shen (2015), Yang, Meng, Dimarogonas, and Johansson (2014), Yu, Chen, and Lu (2009), Zhou, Shi, Lim, Yang, and Gui (2015) and Zhu, Jiang, and Feng (2014), and the references therein. In the leader–follower framework, the leader's motion is independent of all the followers and followed by them Ren and Beard (2007). The dynamics of the individual followers can be non-identical (Kim, Shim, & Seo, 2011; Wieland, Sepulchre, & Allgöwer, 2011) or identical (Ni & Cheng, 2010). For the case of non-identical followers, the output regulation theory is a valuable method to handle the synchronization problem (Huang, 2004; Isidori, Marconi, & Serrani, 2003). Note that the existing works are mainly based on the common assumption that the parameters of the leader are exactly known. However, such key assumption can hardly match the real world applications. For instance, if we handle the sinusoidal signal, we can easily design a robust controller to cope with the uncertainties on the amplitude and phase angle. Most importantly, the frequency at which the robust controller oscillates is necessary to exactly match the frequency of the exogenous signal or else it will result in a nonzero steady-state error. Therefore, the purpose of this paper is to investigate the output synchronization problem of a heterogeneous network subject to an uncertain leader.

The proposed controller for each follower includes two parts: adaptive reference generator and robust regulator. This method can be seen as a kind of separation principle. In the first stage, a set of identical reference generators are constructed to track the output of the uncertain leader. Specifically, the adaptive control law can guarantee that reference generators can copy the dynamics of the uncertain leader; the distributed control law can force the

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E-mail addresses: yqwuzju@163.com (Y. Wu), rqlu@gdut.edu.cn (R. Lu), peng.shi@adelaide.edu.au (P. Shi), hysu@iipc.zju.edu.cn (H. Su), nashwzhg@126.com (Z.-G. Wu).

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outputs of reference generators to achieve globally asymptotical and locally exponential synchronization with the output of the leader. The idea in the first stage is to synchronize a set of networked reference generators to reach a common steady state. In the second stage, each reference generator designed in the first stage is treated as the exosystem for the corresponding follower. As there exist uncertain parameters in followers, we can use the internal model to compensate for the plant uncertainties. The individual regulator for each follower includes a stabilizer and an internal model. The idea in this stage is to guarantee that the regulator has the internal model property with respect to the local exosystem in steady state. Although the leader and the non-identical followers are affected by parameter perturbations, the combination of these two parts can guarantee the locally exponential synchronization of the MASs.

The design method in this paper is based on Kim et al. (2011), Wieland et al. (2011) and Wu, Wu, and Su (2015). In our approach, we extend the results of such works in two ways: our method combine the adaptive control theory and the robust output regulation theory to deal with the parameter uncertainties in the leader and the followers. The adaptive control law is added to provide a precise locally exponential estimate of the uncertain parameters in the leader. The internal model is added to gain robustness versus follower parameter uncertainties. In this way, we can separate the design of the regulator from the synchronization purpose, and take full advantages of the adaptive control law in the first stage and the robust output regulation control law in the second stage. Compared with the work in Su and Huang (2013), our method can guarantee the local and exponential output synchronization, while the obtained synchronization in Su and Huang (2013) is asymptotic.

The rest of the paper is organized as follows. In Section 2, algebraic graph theory and problem formulation are introduced. By using the adaptive control law and distributed control law, the globally asymptotical and locally exponential synchronization about the set of reference generators (Stage 1) is presented in Section 3.1. By means of robust output regulation control law, the synchronization about the reference generators and non-identical followers (Stage 2) are considered in Section 3.2. Examples are given in Section 4 to illustrate the effectiveness of the obtained theoretical results, while Section 5 concludes the paper.

Notations: $\lambda_{\min}^{\{Q(\mu)\}}$ denotes the minimum eigenvalue of the matrix $Q(\mu)$, where μ ranges on a compact set $\mathcal{P}$. $\lambda_{\max}^{\{Q(\mu)\}}$ denotes the maximum eigenvalue of the matrix $Q(\mu)$. The vector $\mathbf{1}_N$ denotes a column vector with all entries equal to 1. $A \otimes B$ denotes the Kronecker product of the matrices A and B. The superscript "T" represents the transpose, and diag$\{\cdots\}$ stands for a block diagonal matrix. The $\|A\|$ denotes the Euclidean norm of a vector and its induced norm of the matrix A. Matrices I and 0 represent the identity matrix and the zero matrix, respectively. The $vec(A)$ is the vector obtained by stacking the columns of matrix A, left to right (Horn & Johnson, 2012). The $Comp[-s_0(t),\ -s_1(t),\ \ldots,\ -s_{w-1}(t)]$ denotes the matrix in companion form, which is

$$\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -s_0(t) & -s_1(t) & \cdots & -s_{w-1}(t) \end{bmatrix},$$

where scalars $s_0(t),\ s_1(t),\ \ldots,\ s_{w-1}(t)$ are continuous functions of time. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations (Wu, Shi, Shu, Su, & Lu, 2016).

2. Preliminaries

Before starting the problem formulation, we recall some basic communication graph terminologies.

2.1. Algebraic graph theory

The information exchange among the followers in the heterogeneous network is described by a graph $\mathcal{G}$. The definitions and properties of the adjacency matrix $\mathcal{A}$ and Laplacian matrix L for the time-invariant connected graph $\mathcal{G}$ can be referred to Su and Huang (2013). The matrix L satisfies $L\mathbf{1}_N = 0$. The set of neighbors for the node v_i is given as $\mathcal{N}_i = \{v_j \in \mathcal{V} : a_{ij} \neq 0\}$. The diagonal matrix $D = \text{diag}\{d_1, \ldots, d_k\}$, $k = 1, \ldots, N$ denotes the weights of the directed edges from the node 0 (which is the leader to be tracked) to the followers in $\mathcal{G}$. If $d_k > 0$, there exists a directed edge from node 0 to follower k. If $d_k = 0$, follower k cannot receive information from the leader directly. The graph $\bar{\mathcal{G}}$ is the induced graph from $\mathcal{G}$ and node 0, which consists of graph $\mathcal{G}$, node 0, and the directed edges from the node 0 to the followers in $\mathcal{G}$.

2.2. Problem formulation

The kth ($k = 1, \ldots, N$) non-identical follower in the heterogeneous network is described in the following form:

$$\begin{aligned} \dot{x}_k(t) &= A_k(\mu)x_k(t) + B_k(\mu)u_k(t) \\ y_k(t) &= C_k(\mu)x_k(t), \end{aligned} \tag{1}$$

where $x_k(t) \in \mathbb{R}^{n_k}$ is the state, $u_k(t) \in \mathbb{R}^{m_k}$ is the control input, $y_k(t) \in \mathbb{R}^p$ is the measured output. The matrices $A_k(\mu)$, $B_k(\mu)$ and $C_k(\mu)$ are matrices of continuous functions of μ, which is a vector of uncertain parameters ranging on a compact set $\mathcal{P}$, i.e., $\mu \in \mathcal{P}$. The dynamics of the leader depend on a vector ϱ of uncertain parameters, ranging on a compact set $\mathcal{Q}$, i.e., $\varrho \in \mathcal{Q}$, namely

$$\begin{aligned} \dot{w}_0(t) &= S(\varrho)w_0(t) \\ y_0(t) &= Qw_0(t), \end{aligned} \tag{2}$$

where $w_0(t) \in \mathbb{R}^{n_0}$, $y_0(t) \in \mathbb{R}$, $Q = \text{row}(1, 0, \ldots, 0)$ and $S(\varrho)$ is in companion form as

$$S(\varrho) = Comp[-s_0(\varrho), \ldots, -s_{n_0-1}(\varrho)]. \tag{3}$$

The scalars $s_0(\varrho),\ \ldots,\ s_{n_0-2}(\varrho)$ and $s_{n_0-1}(\varrho)$ are continuous functions of the vector ϱ and coefficients of the minimal polynomial of $S(\varrho)$.

Remark 1. Regardless of how the pair $(S(\varrho), Q)$ is written, the output $y_0(t)$ can be treated as the solution of the following homogeneous differential equation of order n_0,

$$y_0^{(n_0)}(t) + s_{n_0-1}(\varrho)y_0^{(n_0-1)}(t) + \cdots + s_0(\varrho)y_0(t) = 0.$$

Now, any $y_0(t)$ of this kind can always be expressed as the output of a system like (2), in which $S(\varrho)$ is in companion form and $Q = \text{row}(1, 0, \ldots, 0)$.

The objective of this paper is to design a controller for each non-identical uncertain follower such that outputs of all followers achieve globally asymptotical and locally exponential synchronization with the output of the uncertain leader, i.e., $y_k(t) - y_0(t) \to 0$ as $t \to \infty$ for each follower.

Remark 2. In the leader–follower framework, outputs of the non-identical followers (1) synchronize on a nontrivial trajectory, which is the output of the leader (2). This is equivalent to the output regulation theory (Huang, 2004; Isidori et al., 2003). However, in the classical output regulation theory, the matrices of the internal model are dependent on the precise values of