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Brief paper

Immersion and invariance stabilization of strict-feedback dynamics under sampling[☆]

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ABSTRACT

The paper deals with sampled-data stabilization of continuous-time dynamics in strict-feedback form via Immersion and Invariance. Starting from the characterization of the sampled-data target dynamics and its invariant manifold, a multi-rate control law is designed to achieve attractiveness and invariance of such a manifold. Simulations on an academic example and a practical case illustrate the performances.

1. Introduction

Stabilization of continuous-time (CT) strict-feedback dynamics has been widely investigated in the last decades. Several methodologies have been proposed exploiting the connected cascade structure. Among all, backstepping is certainly the most popular one and involves an iterative top-down Lyapunov-based procedure to compute the controller (Kokotović & Arcak, 2001). Strict-feedback structures can be assumed in a purely discrete-time (DT) context as well and similar top-down constructive procedures can be carried out for the design. However, several difficulties arise for the computation of the control solutions as they are only implicitly defined by nonlinear algebraic equations.

This last issue can be overcome in the sampled-data (SD) context where the discrete-time model represents the evolutions, at the sampling times, of the system under the action of piecewise constant control over the sampling intervals (Monaco & Normand-Cyrot, 2001). In that case, the strict-feedback structure is lost but, as it will be clarified in the sequel, the SD equivalent model inherits a nested structure which is useful for the design.

Several contributions discuss backstepping-like methods for SD dynamics (Burlion, Ahmed-Ali, & Lamnabhi-Lagarrigue, 2006; Nešić & Grüne, 2005; Postoyan, Ahmed-Ali, Burlion, & Lamnabhi-Lagarrigue, 2008). A SD Lyapunov-based adaptive control strategy was proposed in Postoyan et al. (2008) by exploiting the triangular structure. In a recent work by Tanasa, Monaco, and Normand-Cyrot (2016) Input-Lyapunov-Matching (ILM) was employed to design a multi-rate backstepping stabilizing controller.

Immersion and Invariance (I&I) has been introduced in continuous time as an alternative tool for nonlinear stabilization (Astolfi, Karagiannis, & Ortega, 2008; Astolfi & Ortega, 2003). It relies on the idea of driving the trajectories of a nonlinear system towards the ones of an a-priori defined stable target dynamics while preserving their boundedness. Such an approach qualifies for its robustness with respect to higher order dynamics, applicability to real cases and simplicity, as illustrated in several practical domains (Mannarino & Mantegazza, 2014; Rabai, Mnasri, Khaled, & Gasmi, 2013). A first extension to nonlinear discrete-time systems in strict-feedback form was provided by Yalcin and Astolfi (2011).

How to preserve I&I stabilization under digital control remains a challenging problem. In Mattei, Monaco, and Normand-Cyrot (2015), assuming part of the continuous-time dynamics stable, the sampled-data controller stretching the dynamics onto the

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associated continuous-time manifold guarantees its attractiveness and thus I&I stabilization.

In the present paper, we discuss the same problem for systems in strict-feedback form. In doing so, one has to keep in mind that the strict-feedback structure is lost under sampling and that the implementation of a piecewise constant controller via emulation of the continuous-time one is not satisfactory. Under the usual assumptions set on strict-feedback dynamics, we show that I&I stabilizability under sampling can be preserved by adequately redefining a sampled-data target system and its associated invariant manifold (both parameterized by the sampling period). The stabilizing design is then carried out via multi-rate feedback strategies of order equal to the number of cascade connections. This is first detailed for strict-feedback systems with two-cascade connections while the extension to the case of m cascades is sketched as it follows the same lines. Preliminary results on the one-cascade case are in Mattioni, Monaco, and Normand-Cyrot (2015b).

In conclusion, it is shown that the existence of a CT-I&I control for systems in strict-feedback form is sufficient to guarantee the existence of a m-rate SD-I&I feedback. The proof is constructive and the control solution admits an expansion in powers of the sampling period. In practice, only approximate solutions can be computed and implemented so affecting the overall performances. The stability properties of the closed-loop system under approximate controllers are discussed with respect to the length of the sampling period.

The paper is organized as follows. After some recalls and introductory concepts in Section 2, the main results are discussed in Sections 3 and 4. Constructive aspects and extensions are detailed in Section 5. In Section 6, examples and simulations are carried out.

2. Recalls and basic facts

2.1. Assumptions and notations

Maps and vector fields are assumed smooth (i.e., infinitely differentiable of class C^∞) and forward complete to guarantee the existence of solutions and prevent from finite escape time. The sampling period $\delta \in]0, T^*[$ is assumed regular. $T^* > 0$ denotes the maximum allowable sampling period (MASP, Tanasa et al., 2016). Given a vector field $f : \mathbb{R}^n \to T_x\mathbb{R}^n$, L_f denotes the associated Lie derivative operator, $L_f = \sum_{i=1}^n f_i(\cdot)\frac{\partial}{\partial x_i}$. e^{L_f} denotes the associated Lie series operator, $e^{L_f} = 1 + \sum_{i\geq 1} \frac{L_f^i}{i!}$. When no ambiguity is possible, $L_f L_g$ stands for $L_f \circ L_g$. For any real valued function h on $\mathbb{R}^n$, one gets $e^{L_f} h(x) = e^{L_f} h\big|_x = h(e^{L_f} x)$, where $e^{L_f} x$ stands for $e^{L_f} Id\big|_x$ and I_d is the identity function over $\mathbb{R}^n$. The evaluation of a function at time $t = k\delta$ ($\big|_{t=k\delta}$) is omitted when it is clear from the context. The subscript $\big|_k$ is omitted as well when no confusion arises. A function $R(x, \delta)$ is said to be of order δ^p, $p \geq 1$ ($R(x, \delta) = O(\delta^p)$) if whenever R is defined it can be written as $R(x, \delta) = \delta^{p-1}\tilde{R}(x, \delta)$ and $\exists \theta \in \mathcal{K}_\infty$ and $\delta^* \geq 0$, such that for each $\delta \leq \delta^*$, $\tilde{R}(x, \delta) \leq \theta(\delta)$.

2.2. Problem statement

In this paper we consider strict-feedback continuous-time dynamics (Khalil, 2002) in the general m-cascade form

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_{21}\\
\dot{x}_{2j} &= f_{2j}(x_1, x_{21}, \ldots, x_{2j}) + g_{2j}(x_1, x_{21}, \ldots, x_{2j})x_{2j+1}\\
\dot{x}_3 &= u
\end{aligned} \tag{1}$$

where $x_1 \in \mathbb{R}^p$, $x_2 = (x_{21}, \ldots, x_{2m-1})^\top \in \mathbb{R}^{m-1}$, $x_3 = x_{2m}$, x_{2j}, $u \in \mathbb{R}$ for $j = 1, \ldots, m-1$. We assume that $g_{2j}(\cdot) \neq 0$ (globally) and that the origin is the unique equilibrium of (1). From now on, the stabilizability of the x_1-dynamics via fictitious feedback $x_{21} = \gamma(x_1)$ is assumed.

Assumption 2.1. There exist functions $\gamma(\cdot) : \mathbb{R}^p \to \mathbb{R}$ with $\gamma(0) = 0$ and proper[1] $W(\cdot) : \mathbb{R}^p \to \mathbb{R}^+$, such that $(L_{f_1} + \gamma L_{g_1})W(x_1) < 0$ for all $x_1 \in \mathbb{R}^p/\{0\}$.

Accordingly, *I&I stabilizability* of (1) can be proven in the sense of Definition 1 in Astolfi and Ortega (2003).

With reference to standard arguments, one defines the target dynamics as $\dot{\xi}(t) = f_1(\xi) + g_1(\xi)\gamma(\xi)$ and the immersion mapping as $\pi(\xi) = col(\xi, \gamma_1(\xi), \ldots, \gamma_m(\xi))$ with, for $i = 1, \ldots, m-1$

$$\begin{aligned}
\gamma_1(\xi) &= \tilde{\gamma}_1(\xi) = \gamma(\xi)\\
\gamma_{i+1}(\xi) &= \tilde{\gamma}_{i+1}(\xi, \gamma_1(\xi), \ldots, \gamma_i(\xi))\\
&= g_{2i}^{-1}(\xi, \gamma_1(\xi), \ldots, \gamma_i(\xi))(\dot{\gamma}_i(\xi) - f_{2i}(\xi, \gamma_1(\xi), \ldots, \gamma_i(\xi))).
\end{aligned} \tag{2}$$

According to (2), $\dot{\gamma}_m(\xi) = c(\xi)$ defines the control constraining the state evolutions of (1) over the target. Setting

$$\begin{aligned}
z_1 &= \phi_1(x_1, x_{21}) = x_{21} - \tilde{\gamma}_1(x_1)\\
z_j &= \phi_j(x_1, x_{21}, \ldots, x_{2j}) = x_{2j} - \tilde{\gamma}_j(x_1, x_{21}, \ldots, x_{2j})\\
z_m &= \phi_j(x_1, x_2, x_3) = x_3 - \tilde{\gamma}_m(x_1, x_2)
\end{aligned} \tag{3}$$

for $j = 2, \ldots, m-1$, GAS of the equilibrium of (1) is achieved under the feedback

$$u_c = \psi(x, z) = -K(x)z + \dot{\tilde{\gamma}}_m(x), \quad K(x) > 0 \tag{4}$$

which guarantees manifold attractivity and trajectory boundedness of the extended dynamics over $\mathbb{R}^{p+2m}$

$$\begin{aligned}
\dot{z}_j &= g_{2j}(x_1, z_1 + \tilde{\gamma}_1(x_1), \ldots, z_j + \tilde{\gamma}_j(x_1, x_{21}, \ldots, x_{2j}))z_{j+1}\\
\dot{z}_m &= u - \dot{\tilde{\gamma}}_m(x)\\
\dot{x}_1 &= f_1(x_1) + g_1(x_1)(z_1 + \tilde{\gamma}_1(x_1))\\
\dot{x}_{2j} &= g_{2j}(x_1, x_{21}, \ldots, x_{2j})z_{j+1} + \dot{\tilde{\gamma}}_j(x_1, x_{21}, \ldots, x_{2j})\\
\dot{x}_3 &= u
\end{aligned} \tag{5}$$

for $j = 1, \ldots, m-1$.

Remark 2.1. Mappings $\tilde{\gamma}_i : \mathbb{R}^p \times \mathbb{R}^{i-1} \to \mathbb{R}$ are instrumental to define the off-manifold components z. In the sequel, with a slight abuse of notation, we will use γ_i instead of $\tilde{\gamma}_i$ when no confusion arises.

In the following, we discuss the problem of preserving I&I stabilizability when the control variable $u(t)$ is piecewise constant; i.e. $u(t) = u_k$ for $t \in [k\delta, (k+1)\delta[$, $k \geq 0$. For this purpose, it is instrumental to redefine I&I stabilizability for nonlinear DT systems (Monaco & Normand-Cyrot, 2015) of the form

$$x_{k+1} = F(x_k, u_k) \tag{6}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ and x^* is the equilibrium.

Definition 2.1. The DT dynamics (6) is said to be I&I stabilizable if there exist mappings

$$\begin{aligned}
&\alpha(\cdot) : \mathbb{R}^p \to \mathbb{R}^p; \quad \pi(\cdot) : \mathbb{R}^p \to \mathbb{R}^n; \quad c(\cdot) : \mathbb{R}^p \to \mathbb{R}\\
&\phi(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n-p}; \quad \psi(\cdot, \cdot) : \mathbb{R}^{n\times(n-p)} \to \mathbb{R}
\end{aligned}$$

such that the following conditions hold:

[1] $W : \mathbb{R}^n \to \mathbb{R}$ is *proper* if $\forall\, r > 0$, $W^{-1}([0, r]) = \{x \in \mathbb{R}^n : W(x) \geq r\}$ is compact.