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Brief paper

Adaptive decentralized control for a class of interconnected nonlinear systems via backstepping approach and graph theory ☆

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ABSTRACT

This paper is concerned with the adaptive decentralized control problem for a class of interconnected nonlinear systems, where the interconnections are assumed to be unknown and completely nonlinear. In addition, the interconnections and their bounds are allowed to contain the states of all subsystems. The main contribution is that, a strictly decentralized control scheme with compensation mechanism is developed to achieve the desirable tracking performance. More specifically, a smooth switching function is introduced to construct adaptive control laws, where the compensation mechanism is activated only if the immediate variable involved in the backstepping design exceeds a given constant, otherwise it will be turned-off. Furthermore, by combining graph theory and Lyapunov analysis method, it is proved that all the signals of the resulting closed-loop system are globally bounded, and the tracking errors of subsystems exponentially converge to a compact set, whose radius is adjustable by choosing different controller design parameters. Finally, the effectiveness of the proposed adaptive decentralized control scheme is illustrated with a simulated example.

1. Introduction

Interconnected systems have been used to model a wide variety of physical, natural, and artificial complex dynamical systems: such as power systems, computer and telecommunications networks, and aerospace systems, etc. Due to the complexity and the heavier computational burden of centralized control, it is often required to design decentralized controllers for interconnected systems based only on local information of subsystem.

Note that one of the main obstacles in decentralized control is how to address the interconnections. Under the match conditions, that is, all interconnections lie within the range space of the control vectors, the adaptive decentralized control problems were considered in Gavel and Siljak (1989) and Shi and Singh (1992) for large-scale systems subject to first order and high order interconnections. To remove such structural constraints on interconnections, the backstepping techniques (Krstic, Kanellakopoulos, & Kokotovic, 1995; Wen, 1994) have been widely used in the decentralized control of interconnected nonlinear systems. For example, the problems of decentralized tracking were addressed for large-scale output feedback nonlinear systems (Jiang, 2002; Krishnamurthy & Khorrami, 2003; Tong, Huo, & Li, 2014). In Chen and Li (2008) and Mehraeen, Jagannathan, and Crow (2011), adaptive neural output feedback controllers were designed for interconnected nonlinear systems in strict feedback form. The decentralized tracking control designs were further investigated in Ye (2011) for strict feedback interconnected systems with time delays. On the other hand, the corresponding decentralized control problems were also extended to stochastic nonlinear systems (Liu, Zhang, & Jiang, 2007; Xie & Xie, 2000; Zhou, Shi, Liu, & Xu, 2012). Especially, in Wen, Zhou, and Wang (2009), the backstepping-based decentralized adaptive control problem was studied for interconnected systems with interactions including subsystem inputs and outputs. In fact, a common feature of the above results is that the interconnections or their bounds for each subsystem contain its own states and the outputs of other subsystems, which are the so-called weak interconnections (Zhang & Lin, 2014).

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Recently, some interesting works have been done for coupled systems with strongly interconnections, where both the interconnections and their bounds are allowed to be the functions of the states of all subsystems. In Chen, Liu, Liu, and Lin (2009), Hovakimyan, Lavretsky, Yang, and Calise (2005) and Yoo and Park (2009), the neural networks are introduced to approximate the bounds of interconnections, and the semi-global stability of the tracking errors is ensured. The global stability problems were considered in Narenda and Oleng (2002) and Stankovic and Siljak (2009), while the interconnections or their bounding functions are in the linear form. Considering the more general strongly nonlinear interconnections, the decentralized state feedback controller is successfully constructed in Zhang and Lin (2014) via a Lyapunov function in a product integral manner. However, just as mentioned in the paper, the control laws design for each subsystem should share some prior information from other subsystems. To the best of our knowledge, no result has yet been reported about strictly decentralized control for coupled nonlinear systems with strongly interconnections. In such control scheme, each of the controllers $u_i,\ (i = 1, 2, \ldots, N)$ is assumed to have access only to the information of the corresponding subsystem, but not those of the other subsystems.

Motivated by the above analysis, this paper further investigates the adaptive decentralized control problem for a class of coupled nonlinear systems with strongly interconnections. The main innovation is that a strictly decentralized controller design approach, together with constructing a new compensation mechanism for the strongly interconnections, is developed to achieve the desired tracking performance. More specifically, a smooth switching function is introduced to redesign adaptive control laws, and the compensation mechanism is activated only if the immediate variable involved in the backstepping design exceeds a given constant, otherwise it will be turned-off. In this case, the singularity problem is also avoided. Furthermore, by combining a convex combination technique and Matrix Tree Theorem (Knuth, 1997), a global Lyapunov function is given such that all the signals of the resulting closed-loop system are proved to be globally bounded, and the tracking errors of each subsystem exponentially converge to an adjustable compact set, whose radius is related with the controller design parameters.

The following of this paper is organized as follows: some necessary preliminaries are presented in Section 2. The backstepping based strictly decentralized adaptive controller designs are summarized in Section 3. In Section 4, an example with comparison analysis is given to illustrate the effectiveness of the proposed methods. Finally, the conclusions are given in Section 5.

2. Problem statement and preliminaries

2.1. Preliminaries

A directed graph or digraph $\mathcal{G} = (V, E)$ contains a set $V = 1, 2, \ldots, N$ of vertices and a set E of arcs (edges) (i, j) leading from initial vertex i to terminal vertex j. A subgraph $\mathcal{H}$ of $\mathcal{G}$ is said to be spanning if $\mathcal{H}$ and $\mathcal{G}$ have the same vertex set. A digraph $\mathcal{G}$ is weighted if each arc (j, i) is assigned a non-negative weight l_{ij}. The weight $w(\mathcal{H})$ of a subgraph $\mathcal{H}$ is the product of the weights on all its arcs.

A directed path $\mathcal{P}$ in $\mathcal{G}$ is a subgraph with distinct vertices $i_1, i_2, \ldots, i_m$ such that its set of arcs is $\{(i_k, i_{k+1}) : k = 1, 2, \ldots, m-1\}$. If $i_m = i_1$, we call $\mathcal{P}$ a directed cycle. A connected subgraph $\mathcal{T}$ is a tree if it contains no cycles, directed or undirected. A tree $\mathcal{T}$ is rooted at vertex i, called the root, if i is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph $\mathcal{Q}$ is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle.

Given a weighted digraph $\mathcal{G}$ with N vertices, define the weight matrix $\Lambda = (l_{ij})^{N\times N}$ whose entry l_{ij} equals the weight of arc (j, i). For our purpose, we denote a weighted digraph as $(\mathcal{G}, \Lambda)$, and it is used in this paper to model the topology of all links in the networks. A digraph $\mathcal{G}$ is strongly connected if, for any pair of distinct vertices, there exists a directed path from one to the other. A weighted digraph $(\mathcal{G}, \Lambda)$ is strongly connected if and only if the weight matrix Λ is irreducible. The Laplacian matrix of $(\mathcal{G}, \Lambda)$ is defined as

$$L = \begin{bmatrix} \sum_{k\neq 1} l_{1k} & -l_{12} & \cdots & -l_{1N} \\ -l_{21} & \sum_{k\neq 2} l_{2k} & \cdots & -l_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -l_{N1} & -l_{N2} & \cdots & \sum_{k\neq N} l_{Nk} \end{bmatrix}. \quad (1)$$

2.2. Problem statement

In this paper, we consider an interconnected nonlinear systems built on the digraph $\mathcal{G}$ by assigning each vertex its own internal dynamics and then coupling these vertex dynamics based on directed arcs in $\mathcal{G}$. Assume that each vertex dynamics is described by the following strict-feedback system:

$$\begin{aligned} \dot{x}_{i,m}(t) &= f_{i,m}(\bar{x}_{i,m}) + g_{i,m}(\bar{x}_{i,m})x_{i,m+1} \\ &\quad + h_{i,m}(\bar{x}_{1,m}, \bar{x}_{2,m}, \ldots \bar{x}_{N,m}), \\ \dot{x}_{i,n}(t) &= f_{i,n}(\bar{x}_{i,n_i}) + g_{i,n_i}(\bar{x}_{i,n_i})u_i \\ &\quad + h_{i,n}(\bar{x}_{1,n}, \bar{x}_{2,n}, \ldots \bar{x}_{N,n}), \\ y_i(t) &= x_{i,1}, \quad m = 1, 2, \ldots, n-1, \end{aligned} \quad (2)$$

where $x_i = [x_{i,1}, x_{i,2}, \ldots x_{i,n}]^T \in R^n$, $u_i \in R$ and $y_i \in R$ are the system state, control input, and output of the subsystem i, $i = 1, 2, \ldots, N$, respectively. $f_{i,m}$, $g_{i,m}$, are known smooth functions, and $h_{i,m}$, $m = 1, 2, \ldots, n$ are unknown interconnection terms. Throughout this paper, we denote $\bar{x}_{i,m} = [x_{i,1}, x_{i,2}, \ldots x_{i,m}]^T$.

Remark 1. Since the interconnections depend on the states of all subsystems, the system (2) can be used to describe many state space models of interconnected nonlinear systems, and the details can be found in Niculescu (2001), Spooner and Passino (1999) and Tang, Tomizuka, Guerrero, and Montemayor (2000), etc.

Moreover, the controller design also requires the following assumptions:

Assumption 1. The digraph $\mathcal{G}$ is strongly connected.

Assumption 2. There exist positive constants $Y_0, Y_1, \ldots, Y_{n-1}$ such that the desired trajectory $y_{i,d}(t)$ and its time derivatives satisfy $|y_{i,d}(t)| \le Y_0$, $|\dot{y}_{i,d}(t)| \le Y_1$, $|\ddot{y}_{i,d}(t)| \le Y_2, \ldots |y_{i,d}^{(n-1)}(t)| \le Y_{n-1}$.

Assumption 3. The functions $g_{i,m}$, $i = 1, 2 \cdots N$, $m = 1, 2, \ldots, n$, are known, and there exist positive constants $\bar{g}_{i,m}$ and $\underline{g}_{i,m}$ such that $\bar{g}_{i,m} \ge |g_{i,m}(\bar{x}_{i,m})| \ge \underline{g}_{i,m} > 0$.

Not that if the interconnected functions between any adjacent subsystems are not zero, then Assumption 1 holds according to the definition given in Section 2.1. In addition, Assumptions 2–3 have been widely used in the existing backstepping control design, such as Chen et al. (2009), Krstic et al. (1995) and Tee, Ge, and Tay (2009).

Now, the considered problem is formulated as follows.