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Brief paper

Robust observed-state feedback design for discrete-time systems rational in the uncertainties ☆

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ABSTRACT

Design of controllers in the form of a state-feedback coupled to a state observer is studied in the context of uncertain systems. The classical approach by Luenberger is revisited. Results provide a heuristic design procedure that mimics the independent state-feedback/observer gains design by minimizing the coupling of observation error dynamics on the ideal state-feedback dynamics. The proposed design and analysis conditions apply to linear systems rationally-dependent on uncertainties defined in the cross-product of polytopes. Convex linear matrix inequality results are given thanks to the combination of a descriptor multi-affine representations of systems and the S-variable approach. Stability and H_∞ performances are assessed by multi-affine parameter-dependent Lyapunov matrices.

1. Introduction

The goal of the paper is to investigate, in the robust control context, the two step design strategy proposed by Luenberger (1971): *"The first phase is design of the control law assuming that the state vector is available. The second phase is the design of a system that produces an approximation to the state vector. This system is called an observer, or Luenberger observer"*. At the difference of the case of systems without uncertainties, this ideal separation principle does not hold in the robust case. Stability of the closed-loop cannot any more be guaranteed by independent choices of stabilizing state-feedback and observer gains. A heuristic systematic procedure can nevertheless be constructed with the aim of minimizing the inevitable coupling between the state-feedback and observer dynamics.

Controllers with observed-state feedback structure form a sub-class of dynamic output feedback controller models. Moreover, the observed-state feedback controllers build with identity Luenberger observers are special representations of dynamic controllers of the same order as the plant (full-order controllers). The issue of designing full-order controllers has convex LMI-based solutions as long as the systems are not affected by uncertainties (Ebihara, Peaucelle, & Arzelier, 2015; Scherer, Gahinet, & Chilali, 1997). Unfortunately, as soon as the systems are affected by uncertainties, the design problem is either non-convex (Geromel, Korogui, & Bernussou, 2007; Kanev, Scherer, Verhaegen, & De Schutter, 2004), conservative (Lien & Yu, 2008; Peaucelle & Arzelier, 1998), or needs to compute and manipulate the parameter-dependent coefficients of the characteristic polynomial (Chesi, 2014). Heuristics can handle the non-convexity. Our strategy is one of such.

The plants we aim at stabilizing are in discrete-time, with uncertainties θ, and described in state-space by $x_{k+1} = A_r(\theta)x_k + B_r(\theta)u_k$ and $y_k = Cx_k$. The final goal is to design a Luenberger like identity observer with dynamics $\hat{x}_{k+1} = A_o\hat{x}_k + B_o u_k + L(C\hat{x}_k - y_k)$ associated to a observed-state feedback $u_k = K\hat{x}_k$.

A first design step in the proposed procedure is, as in the classical Luenberger strategy, the design of a robust static state-feedback control K. This topic has been studied in the past by many authors, see Boyd, El Ghaoui, Feron, and Balakrishnan (1994) and de Oliveira, Bernussou, and Geromel (1999) for example. The result we provide for this step is a variation on results from de Oliveira

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et al. (1999), extended to systems rational in the uncertainties using properties exposed in Ebihara et al. (2015).

The second design step is observer design. This problem has diverse solutions in the literature for uncertain systems. Two types of results can be distinguished. A first category tackles the observer problem as part of the output filtering issue. Geromel and de Oliveira (2001) gives for that problem an LMI formulation in the case of systems with structured polytopic uncertainties. As discussed in that paper, the significant feature of robust filtering (which is usually considered as dual with respect to state-feedback) is that it needs to optimize over more decision variables than just one gain. At the difference of state-feedback where only the gain K is designed, the methodology of Geromel and de Oliveira (2001) involves the design of both an observer-like gain L and of the state matrix A_o. The same conclusions hold for results of Scherer and Köse (2008) in the context of IQCs. The second category of results tackles directly the observer design. These, at the difference of filter-design results, have the main advantage not to assume open-loop stability of the plant. Only the error $e_k = x_k - \hat{x}_k$ between the plant states and the observer states needs to be stable. Surprisingly, robust observer results do not consider the upper formulated issue about having the state matrix A_o as a design variable. For example recent results of Abbaszadeh and Marquez (2009) and Mondal, Chakraborty, and Bhattacharyya (2010) consider only the design of the L matrix while A_o is chosen a priori to be the one of the nominal system ($A_o = A_r(0)$). In Lien and Yu (2008), the assumption of a fixed A_o is alleviated, but results are restricted to unstructured norm-bounded uncertainties. Our result considers both matrices A_o and B_o as free to design variables. As discussed in Polyak, Nazin, Durieu, and Walter (2004), this problem is a difficult one, and we do not claim to provide a final answer.

The closed-loop dynamics in terms of plant state x_k and observation error e_k are driven by the following closed-loop state matrix:

$$A_c(\theta) = \begin{bmatrix} A_r(\theta) + B_r(\theta)K & -B_r(\theta)K \\ \Delta_A(\theta) + \Delta_B(\theta)K & A_o + LC - \Delta_B(\theta)K \end{bmatrix}$$

where $\Delta_A(\theta) = A_r(\theta) - A_o$ and $\Delta_B(\theta) = B_r(\theta) - B_o$. From this formula, it is trivial that if $\Delta_A = 0$ and $\Delta_B = 0$ then the closed loop dynamics depend only on the choice of stabilizing gains K and L which could be done independently. This is the separation, which is no more achievable in the robust context. A way around this difficulty would be to minimize the norms of $\Delta_A(\theta)$ and $\Delta_B(\theta)$ independently of any other consideration such as stability or performance. It is not the choice we adopt. The proposed procedure is to minimize the effects of $\epsilon_k = Ke_k$ on the state-feedback dynamics (due to the upper-right block in $A_c(\theta)$) seen as outputs of the error dynamics perturbed by $(\Delta_A(\theta) + \Delta_B(\theta)K)x_k$ (lower-left block in $A_c(\theta)$). We show, that formalized in this way, the observer design problem has convex LMI formulations and one can via multi-objective methodology manage the inevitable tradeoff between good precision and low transient peaks (Khalil, 2008).

The state x_k is seen in the observer design phase as some input perturbation but it is not just any bounded signal. For this reason, our proposed procedure includes an analysis step between the state-feedback design and the observer design steps. This analysis provides information on expected trajectories of x_k for known state-feedback gain K. Not only this analysis step allows to improve the quality of the observer design but it also allows in the end to assess closed-loop stability with small-gain theorem argument. This small-gain argument happens to be conservative. Hence if it is not positive, a closed-loop analysis LMI test is also proposed to analyze stability and performance of the global observed-state feedback system.

The overall design procedure is exposed for systems rationally-dependent on uncertainties θ. Uncertainties are modeled as lying in the cross-product of independent polytopes, which is an equivalent formulation to multi-simplex modeling in Morais, Braga, Oliveira, and Peres (2013). For this type of models, we propose a transformation called descriptor multi-affine representation (DMAR for short) which is an alternative to linear-fractional representations (Hecker & Varga, 2004). The DMAR is directly inspired by results from Coutinho, Trofino, and Fu (2002) and Masubuchi, Akiyama, and Saeki (2003) and has smaller dimensions than the conventional linear-fractional representations. The rational dependence is converted into a multi-affine dependence at the expense of rewriting the plant in descriptor form. As exposed in de Oliveira and Skelton (2001) and Ebihara et al. (2015) this descriptor structure happens to be well adapted for deriving S-variable LMI results. We highlight this fact by separating modeling issues (lemmas of Section 2), and the S-variable technique (Theorems of Section 3).

The outline of the paper is as follows. Section 2 is dedicated to the exposure of the descriptor multi-affine representation of rationally-dependent uncertain systems. The four LMI results for state-feedback design, state-feedback loop analysis, observer design and observed-state feedback loop analysis are given in Section 3. The heuristic procedure with two design steps and two analysis steps is presented in Section 4. A numerical example illustrates the results in Section 5. Conclusions are drawn in the final section.

Notation:

A^T is the transpose of the matrix A. $\{A\}^{\mathcal{S}}$ stands for the symmetric matrix $\{A\}^{\mathcal{S}} = A+A^T$. $\mathrm{diag}\left(F_1, \ldots \quad F_i, \quad \ldots\right)$ stands for a block-diagonal matrix whose diagonal blocks are the F_i matrices. $A \succ B$ is the matrix inequality stating that $A - B$ is symmetric positive definite. The terminology *"congruence operation of A on B"* is used to denote A^TBA. A matrix inequality of the type $N(X) \succ 0$ is said to be a linear matrix inequality (LMI for short), if $N(X)$ is affine in the decision variables X. $\Xi_{\bar{v}} = \{\xi_{v=1\ldots\bar{v}} \geq 0, \ \sum_{v=1}^{\bar{v}} \xi_v = 1\}$ is the unit simplex in $\mathbb{R}^{\bar{v}}$. The elements ξ of unitary simplexes are used to describe polytopic type uncertainties. For a discrete-time signal $v_{k\geq 0}$, $\|v\|_2^2 = \sum_{k=0}^{\infty} v_k^T v_k$ is the squared l_2 norm and $\|v\|_{2,\bar{k}}^2 = \sum_{k=0}^{\bar{k}} v_k^T v_k$ stands for the truncated squared norm. $\|v\|_p = \sup_{k\geq 0}(v_k^T v_k)^{1/2}$ denotes the peak of the euclidean norm over time.

2. Descriptor multi-affine modeling of rationally dependent uncertain systems

We shall consider in this paper parameter-dependent systems such as

$$\begin{aligned} x_{k+1} &= A_r(\theta)x_k + B_r(\theta)u_k + B_{rw}(\theta)w_k \\ z_k &= C_{rz}(\theta)x_k + D_{rzu}(\theta)u_k + D_{rzw}(\theta)w_k \\ y_k &= Cx_k \end{aligned} \quad (1)$$

where all the matrices except C are rational and continuous in uncertain parameters gathered in the notation θ. The parameters θ are assumed to lie in a set Θ defined as the cross product of $\bar{p}$ sets $\theta \in \Theta = \left\{(\theta_1, \ldots, \theta_{\bar{p}}) \in \Theta_1 \times \cdots \times \Theta_{\bar{p}}\right\}$. The $\bar{p}$ components of θ are independent vectors of $\mathbb{R}^{m_p}$. Each set Θ_p is assumed to be a polytope with $\bar{v}_p$ vertices from the set $\mathcal{V}_p = \left\{{\theta_p}^{[1]}, \ldots, {\theta_p}^{[\bar{v}_p]}\right\}$. Θ_p is the convex hull of the vertices, or equivalently, each θ_p writes as the weighted sum of vertices with weight from unitary simplexes $\Theta_p = Co(\mathcal{V}_p) = \left\{\theta_p = \sum_{v_p=1}^{\bar{v}_p} \xi_{p,v_p}{\theta_p}^{[v_p]} : \xi_p \in \Xi_{\bar{v}_p}\right\}$. In the following, $\mathcal{V} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_{\bar{p}}$ is the set of all extremal values of the parameters. A generic element of $\mathcal{V}$ will be denoted $\theta^{[v]}$ with $v = (v_1, \ldots, v_{\bar{p}})$ the vector of indices of vertices for each component. $\mathcal{I}$ is the set of all vectors of indices v. $\theta^{[v]}$ is the