Automatica 76 (2017) 251–257

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed localization with mixed measurements under switching topologies[☆]

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ARTICLE INFO

Article history:
Received 5 February 2016
Received in revised form
23 July 2016
Accepted 11 October 2016

Keywords:
Distributed localization
Sensor network
Robot network

ABSTRACT

This paper investigates the distributed localization problem of sensor networks with mixed measurements. Each node holds a local coordinate system without a common orientation and is capable of measuring only one type of information (either distance, bearing, or relative position) to near-by nodes. Thus, three types of measurements are mixed in the sensor networks. Moreover, the communication topologies in the sensor networks may be time-varying due to unreliable communications. This paper develops a fully distributed algorithm called BCDL (Barycentric Coordinate based Distributed Localization) where each node starts from a random initial guess about its true coordinate and converges to the true coordinate via only local node interactions. The key idea in BCDL is to establish a unified linear equation constraints for the sensor coordinates by using the barycentric coordinates of each node with respect to its neighbors though the sensor nodes may have different types of measurements. Then a distributed iterative algorithm is proposed to solve the linear equations under time-varying communication networks. A necessary and sufficient graphical condition is obtained to ensure global convergence of the distributed algorithm.

1. Introduction

Localization is a fundamental problem in wireless sensor networks, which has enormous applications such as geographical data collection, target tracking, military surveillance, etc. (Hu & Evans, 2004; Mao, Fidan, & Anderson, 2007; Priyantha, 2005). The objective of localization is to determine the physical coordinates of all the sensor nodes in some common coordinate system.

In large-scale sensor networks, equipping each sensor node with a GPS may not be practical due to volume restrictions and expensive costs, let alone the environments where GPS does not work such as underwater deployments. On the other hand, there is usually no computing center that can get all the measurements in the network and computes all the sensors' locations in a centralized way (Cao, Anderson, & Morse, 2005; Ji & Zha, 2004; So & Ye, 2007; Thomas & Ros, 2005). Thus, it is more desirable to have a distributed localization algorithm, for which each node can run an algorithm locally, interacting only with neighboring nodes, such that all the nodes will converge to the true coordinates. Moreover, sensor nodes may not have the same type of measuring capability. Distance and bearing (angle of arrival, AOA) are two basic types of measurements depending on what kinds of technologies are used. Thus, there are three types of measurement information for the sensor nodes, namely, distance measurements, bearing measurements and relative position measurements (with both distance and bearing).

Refs. Anderson et al. (2009), Cao, Anderson, and Morse (2006), Cheng, Vandenberghe, and Yao (2009) and Simonetto and Leus (2014) consider distance-based localization from the optimization perspective. However, due to the non-convex property the algorithms are not guaranteed to be globally convergent. Recently, Diao, Lin, and Fu (2014) and Khan, Kar, and Moura (2009) propose linear algorithms, which ensure global convergence. Bearing measurements are used in Shames, Bishop, and Anderson (2013), Zhu and Hu (2014), Zhong, Lin, Chen, and Xu (2014) and Zhao and Zelazo (2016) for the localization, for which Shames et al. (2013)

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Claudio De Persis under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2016.11.005
0005-1098/

consider an optimization method and Zhu and Hu (2014), Zhong et al. (2014) and Zhao and Zelazo (2016) develop linear algorithms. Antonelli, Arrichiello, Caccavale, and Marino (2013), Barooah and Hespanha (2007), Barooah and Hespanha (2008), Lin, Fu, and Diao (2015) and Ravazzi, Frasca, Tempo, and Ishii (2013) address relative position based localization problems, which require less interactions between nodes since both distance and bearing information are used.

In the above references, it is assumed that all the sensor nodes must have the same and single type of measurements. For example, Diao et al. (2014) consider distance measurements for all the nodes. In this paper, we do not assume that all the nodes have the same type of measurements. Instead, we assume that some nodes may measure only distances, some nodes may measure only bearing angles, while some other nodes may measure both distances and bearing angles. Thus, the localization problem becomes totally different from the one studied in Diao et al. (2014). To the best of our knowledge, only Eren (2011) in recent years considers distributed localization using mixed distance and bearing measurements. Nevertheless, the algorithm in Eren (2011) is a sequential based approach, which is limited to certain network structures since it starts from a set of anchor nodes whose absolute locations are known and computes the coordinates of other nodes one by one or group by group (Anderson et al., 2009; Eren et al., 2004; Fang, Cao, Morse, & Anderson, 2009). Alternatively, another method is called the concurrent method, for which every node is able to iteratively solve its coordinate in a distributed manner using the measurements about its neighbors and the estimated coordinates of its neighbors (Diao et al., 2014; Khan et al., 2009; Zhao & Zelazo, 2016).

In this paper, our objective is to develop a concurrent method for distributed localization with mixed measurements in a sensor network. As first step towards this general distributed localization problem, we assume that all the nodes to be localized lie strictly inside the convex hull spanned by the anchors. Moreover, we assume that the sensor nodes do not have a common sense of direction as equipping sensors with compass may not be necessary. In addition, communications between nodes may fail from time to time, and thus node-to-node interaction may indeed be time-varying. This paper develops a fully distributed algorithm called BCDL (Barycentric Coordinate based Distributed Localization) where each node starts from a random initial guess about its true coordinate and converges to the true coordinate via only local node interactions. The key idea in BCDL is to establish unified linear equation constraints for the sensor coordinates in the global coordinate system by using the barycentric coordinate of each node with respect to its neighbors. For all the cases of mixed measurements, we develop detailed procedures of computing the barycentric coordinate of each node by using only locally available information. Then, a linear distributed algorithm is proposed for each node to iteratively solve its absolute coordinate under switching communication topologies by solving the linear equations constructed using the barycentric coordinates. Furthermore, a necessary and sufficient graphical condition is obtained to ensure global convergence of the algorithm.

Notation. $\mathbb{R}$ represents the set of real numbers. $\mathbf{1}_n$ denotes the n-dimensional vector of ones and I_n denotes the identity matrix of order n. The symbol $\otimes$ represents the Kronecker product. Denote $\mathrm{co}\{x_1, \ldots, x_m\}$ as the convex hull spanned by $x_1, \ldots, x_m \in \mathbb{R}^2$.

2. Preliminaries and problem statement

2.1. Graph theory

A *directed graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a vertex set $\mathcal{V}$ of elements called *nodes* and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ of ordered pairs of nodes called *edges*. For each $i \in \mathcal{V}$, $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ denotes the set of its *neighbors*.

For a graph $\mathcal{G}$, define the *Laplacian matrix* $L \in \mathbb{R}^{n\times n}$ as

$$L(i,j) = \begin{cases} -w_{ij} & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i \\ 0 & \text{if } i \neq j \text{ and } j \notin \mathcal{N}_i \\ \sum\limits_{k\in\mathcal{N}_i} w_{ik} & \text{if } i = j \end{cases}$$

where $w_{ij} > 0$ is the weight associated with edge (j, i).

A *configuration* in $\mathbb{R}^2$ of n nodes is defined by their coordinates in $\mathbb{R}^2$, denoted as $p = [p_1^\top, \ldots, p_n^\top]^\top \in \mathbb{R}^{2n}$, where each $p_i \in \mathbb{R}^2$. A *framework* in $\mathbb{R}^2$ is a graph $\mathcal{G}$ equipped with a configuration p, denoted as $(\mathcal{G}, p)$. Two frameworks $(\mathcal{G}, p)$ and $(\mathcal{G}, q)$ are *similar* if $p_i - p_j = \gamma A(q_i - q_j),\ \forall i, j \in \mathcal{V}$, where $\gamma > 0$ is a scaling factor and A is a unitary matrix, and we write $(\mathcal{G}, q) \sim (\mathcal{G}, p)$.

For a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, a node v is said to be *reachable* from a set $\mathcal{R} \subset \mathcal{V}$ if there exists a path from a node in $\mathcal{R}$ to node v. Moreover, $\mathcal{R}$ is said to be *closed* if any node in $\mathcal{R}$ is not reachable from $\mathcal{V} \setminus \mathcal{R}$.

A *dynamic graph* $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ represents a graph whose edge set changes over time. For a time interval $[t_1, t_2]$ the *union graph* is defined as $\mathcal{G}([t_1, t_2]) = (\mathcal{V}, \bigcup_{t\in[t_1,t_2]} \mathcal{E}(t))$. A node v is *uniformly jointly reachable* from $\mathcal{R} \subset \mathcal{V}$ if there exists T such that for every t, v is reachable from $\mathcal{R}$ in the union graph $\mathcal{G}([t, t + T])$.

A square matrix $E \in \mathbb{R}^{n\times n}$ is *nonnegative* if all its entries are nonnegative. Moreover, E is called *(row) stochastic* if it is nonnegative and every row sum equals 1.

2.2. Barycentric coordinate representation

Consider a point, say i with its coordinate p_i, and other three points, say j, k and l with their coordinates p_j, p_k and p_l. The barycentric coordinate of point i with respect to j, k and l is $\{a_{ij}, a_{ik}, a_{il}\}$ that satisfies $p_i = a_{ij}p_j + a_{ik}p_k + a_{il}p_l$ and $a_{ij} + a_{ik} + a_{il} = 1$, where a_{ij}, a_{ik} and a_{il} can be expressed by

$$a_{ij} = \frac{S_{\triangle ikl}}{S_{\triangle jkl}}, \qquad a_{ik} = \frac{S_{\triangle jil}}{S_{\triangle jkl}}, \qquad a_{il} = \frac{S_{\triangle jki}}{S_{\triangle jkl}}, \tag{1}$$

where $S_{\triangle ikl}, S_{\triangle jil}, S_{\triangle jki}$ and $S_{\triangle jkl}$ are the signed areas of the corresponding triangles $\triangle ikl$, $\triangle jil$, $\triangle jki$ and $\triangle jkl$. The sign of $S_{\triangle jkl}$ is positive if node j is on the left-hand side when one moves from node k to node l, and negative otherwise. In particular, a_{ij}, a_{ik} and a_{il} are all positive if and only if i lies strictly in the convex hull spanned by j, k and l. This is because geometrically $i \in \mathrm{co}\{j, k, l\}$ implies that i can be expressed by a convex combination of j, k and l and vice versa. In general, the barycentric coordinate for point i with respect to more than three points is $\{a_{i1}, \ldots, a_{in}\}$ that satisfies $p_i = a_{i1}p_1 + a_{i2}p_2 + \cdots + a_{in}p_n$ and $a_{i1} + a_{i2} + \cdots + a_{in} = 1$.

2.3. Problem statement

The objective of the paper is to develop a distributed scheme for computing the coordinates of all nodes in a network. We consider a sensor network consisting of n nodes. Assume that there are m ($m \geq 3$) *anchor nodes* labeled from 1 to m, whose absolute positions are already known. Other nodes are called *sensor nodes* labeled from $m + 1$ to n, which are to be localized. The goal is to let each node iteratively solve its own absolute coordinate in the global coordinate system Σ_g.

Suppose each node i holds its own local coordinate system Σ_i. For a node i, let p_i denote its coordinate in Σ_g and let p_i^j denote its coordinate in Σ_j. Note that $p_i^i = 0$. We use a graph $\bar{\mathcal{G}} = (\mathcal{V}, \bar{\mathcal{E}})$ to model the sensing and communication topology, where $\mathcal{V} = \mathcal{A} \cup \mathcal{S}$ with $\mathcal{A} = \{1, \ldots, m\}$ denoting the set of anchors and $\mathcal{S} = \{m + 1, \ldots, n\}$ representing the set of sensor nodes. Let $\bar{\mathcal{N}}_i$ be the set