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Brief paper

State estimation for discrete-time Markov jump linear systems with time-correlated measurement noise ☆

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ABSTRACT

In this paper, the state estimation problem for discrete-time Markov jump linear systems affected by time-correlated measurement noise is considered where the time-correlated measurement noise is described by a linear system model with white noise. As a result, two algorithms are proposed to estimate the state of the system under consideration based on a measurement sequence. The first algorithm is optimal in the sense of minimum mean-square error, which is obtained based on the measurement differencing method, Bayes' rule and some results derived in this paper. The second algorithm is a suboptimal algorithm obtained by using a lot of Gaussian hypotheses. The proposed suboptimal algorithm is finite-dimensionally computable and does not increase computational and storage load with time. Computer simulations are carried out to evaluate the performance of the proposed suboptimal algorithm.

1. Introduction

Discrete-time Markov jump linear systems (DTMJLSs) are discrete-time systems with parameters that evolve with discrete-time according to a finite-state Markov chain. In the last few decades, DTMJLSs have received wide attention because they can be used to describe a wide variety of practical systems (Logothetis & Krishnamurthy, 1999) with the behavior of physical processes subject to random abrupt changes in structure.

One of the most important problems for this class of systems is to estimate the system state based on a measurement sequence, which has received much research interest in recent years because several estimation problems in tracking of maneuvering object, signal processing, fault detection, telecommunication and manufacturing require estimating the state of DTMJLSs (Hsiao & Weng, 2012; Lee, Motai, & Choi, 2013; Li & Jia, 2012; Li & Jilkov, 2005; Ulker & Gunsel, 2012). Using different criteria, many research results on the state estimation problem of DTMJLSs were reported in the literature where most results are based on the minimum mean-square error (MMSE) criterion because, generally speaking, corresponding research results based on the criterion have better performance. Since optimal estimate of state in the sense of MMSE requires an exponential complexity of order N^k where k denotes the number of measurements and N denotes the possible realizations of a finite-state Markov chain (Ackerson & Fu, 1970; Bar-Shalom & Li, 1996), suboptimal algorithm had to be considered to limit the computational requirements. The generalized pseudo Bayesian (GPB) methods (Ackerson & Fu, 1970; Chang & Athans, 1978) and the interacting multiple-model (IMM) algorithm (Blom & Bar-Shalom, 1988) are the most popular suboptimal algorithms based on the MMSE criterion. Both the GPB and IMM algorithms were obtained by summing the weighted mode conditional estimates. The difference between the GPB and IMM algorithms is that these algorithms use different Gaussian hypotheses to compute the weights and mode conditional estimates. More precisely, at the kth step, the so-called GPB1 algorithm proposed in Ackerson and Fu (1970) assumed that the conditional probability density function of the system state at time $k-1$ given a sequence of the measurement up to $k-1$ is Gaussian. The GPB2 algorithm was achieved in Chang and Athans (1978) under the assumption that the conditional probability density function of the system state at time $k-1$ given a sequence of the measurement up to $k-1$ and any state of the Markov chain at time $k-1$ is Gaussian. The IMM algorithm was obtained by using hypotheses merging. Apart from using the Gaussian hypotheses in the GPB2 algorithm, the IMM algorithm also merges the assumption that the conditional probability

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density function of the system state at time $k-1$ given a sequence of the measurement up to $k-1$ and any state of the Markov chain at time k is Gaussian. Other suboptimal algorithms based on this criterion include simulation-based algorithms (Doucet & Andrieu, 2001; Doucet, Gordon, & Krishnamurthy, 2001), the variable structure IMM algorithm (Li & Bar-Shalom, 1996), the suboptimal adaptive algorithm (Tugnait, 1982a), the detection estimation algorithm (Tugnait, 1982b), etc. (Elliott, Dufour, & Sworder, 1996; Ho, 2011; Johnston & Krishnamurthy, 2001; Liu, Zhang, & Wang, 2011; Svensson & Svensson, 2010). The reweighted IMM algorithm proposed in Johnston and Krishnamurthy (2001) improved the IMM algorithm by computing the weights via MAP sequence estimation algorithm. Based on an alternative switching model, which forces the dynamic models to persist for at least a model-specific time, a state estimation algorithm was proposed in Svensson and Svensson (2010). In Ho (2011), using two IMM-EV algorithms via switching from one IMM-EV algorithm to the other, the estimation of the system state was obtained. By reducing the approximations used in the suboptimal adaptive algorithm, a truncated approximation based algorithm was proposed in Liu et al. (2011), which showed better performance than the suboptimal adaptive algorithm. Using the reference probability method and the change of measure in discrete time, finite-dimensional filters were obtained in Elliott et al. (1996). The state estimation problem for DTMJLSs based on other criteria was also studied (Alessandri, Baglietto, & Battistelli, 2010; Costa, 1994; Orguner & Demirekler, 2008; Zhang, 1999). In Costa (1994), the state estimation problem for DTMJLSs was investigated in the sense of linear MMSE. Using geometric arguments, a suboptimal algorithm, called the linear minimum mean square estimator, was proposed. Using an optimal control approach, a finite-dimensional recursive filter was developed in Zhang (1999), which is optimal in the sense of the most probable trajectory estimate. The state estimation problem based on the maximum likelihood criterion was considered in Alessandri et al. (2010) and a risk-sensitive multiple-model filtering algorithm was derived in Orguner and Demirekler (2008). The above mentioned research results were concerned with hidden Markov models, that is, the state of the Markov chain is unmeasured. More recently, the state estimation problem for DTMJLSs with delayed mode measurements has been dealt with in Liu and Hu (2014); Matei and Baras (2011); Matei, Martins, and Baras (2008) where the delayed mode measurement means that the state of the Markov chain cannot be measured instantaneously but it can be measured after a delay. However, all of the aforementioned literatures require the constraint that the measurement noise must be white.

The state estimation problem for systems with time-correlated measurement noise has been widely studied where the time-correlated noise is often described by a linear system model with white noise. When the measurement noise is the output of a linear system driven by white noise, the measurement differencing method is effective to remove the time-correlated portion of the measurement (Bryson & Johansen, 1965). Using this method, the state estimation problem of discrete-time linear systems in the sense of linear MMSE was studied in Bryson and Henrikson (1968) and Petovello, O'Keefe, Lachapelle, and Cannon (2009). The above results were extended in Liu (2015) to consider multiplicative noises. More recently, the Gaussian approximation smoothing estimation for nonlinear systems with time-correlated measurement noise has been investigated in Wang, Liang, Pan, Zhao, and Yang (2015), and an optimal filtering algorithm for discrete-time linear systems with time-correlated multiplicative measurement noises has been developed in Liu (2016). However, all these results about time-correlated noise do not consider the case of DTMJLSs. To the best of our knowledge, the state estimation problem for DTMJLSs with time-correlated measurement noise described by a linear system model with white noise has not been investigated. In fact, the case of white measurement noise is hard to be satisfied and the white measurement noise is only an ideal case. For examples, in wireless communication and global navigation satellite systems, the measurement noises are generally time-correlated (Mihaylova, Angelova, Bull, & Canagarajah, 2011; Petovello et al., 2009; Wang, Li, & Rizos, 2012). When these characteristics of correlated noise are not fully accounted for in the model, unacceptable performance can often result. Since the time-correlated noise is often described by a linear system model with white noise, it is necessary to consider this kind of correlated noise that appears in DTMJLSs.

In this paper, we consider the state estimation problem for DTMJLSs with time-correlated measurement noise which is the output of a discrete-time linear system with white noise. The main aim of this paper is to design state estimation algorithms for the system under consideration. The measurement differencing method has been used in dynamic systems with time-correlated measurement noise to tackle the state estimation problem (Bryson & Johansen, 1965; Bryson & Henrikson, 1968; Liu, 2015; Petovello et al., 2009; Wang et al., 2015). However, the method is not applied to the state estimation problem of DTMJLSs due probably to its mathematical complexity. We point out that the measurement differencing method can be used in DTMJLSs with time-correlated measurement noise to solve the state estimation problem. The main contributions of this paper can be highlighted as follows: (1) An optimal algorithm for state estimation of the system under consideration is proposed in the sense of MMSE. (2) Since the proposed optimal algorithm requires exponentially increasing computational and storage load with time, using some Gaussian hypotheses, a suboptimal algorithm is developed to limit the computational requirements. More specifically, at the kth step, the suboptimal algorithm uses the assumption that the conditional probability density function of the system state at time $k-1$ given a sequence of a new measurement up to $k-1$ and any state of the Markov chain at time $k-1$ is Gaussian where the new measurement is obtained from the measurement differencing method. (3) The developed suboptimal algorithm has time-independent complexity and is suitable for online applications. It is worth mentioning that, in order to obtain the two new algorithms, an equality about the conditional mean of system state is proved. More precisely, we prove an equality between the conditional mean of system state given an original measurement sequence and the conditional mean of system state given another new measurement sequence where the new measurement is the difference between the original measurement at the present time and the product of a matrix and the original measurement at the previous time.

This paper is organized as follows. In Section 2, the problem under consideration is formulated. An optimal algorithm is proposed in Section 3, and a suboptimal algorithm is developed in Section 4. In Section 5, using an example of maneuvering target tracking whose state measurement is corrupted by time-correlated noise, the performance of the proposed suboptimal algorithm is evaluated by comparing with the IMM and GPB2 algorithms based on three criteria, namely RMS x–y position and velocity errors defined in this section as well as the probability of each mode over time. The conclusion is provided in Section 6.

Notation: The n-dimensional real Euclidean space is denoted by R^n. For a matrix A, A^T, A^{-1} and $|A|$ represent its transpose, inverse and determinant, respectively. For two random vectors x and y, the conditional mean and covariance matrix of x given y are denoted by $E[x|y]$ and $\mathrm{Var}(x|y)$, respectively.