Automatica 76 (2017) 65–69

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Global output regulation for a class of lower triangular nonlinear systems: A feedback domination approach☆

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ARTICLE INFO

Article history:
Received 22 December 2015
Received in revised form
8 August 2016
Accepted 30 September 2016

Keywords:
Lower triangular nonlinear systems
Output regulation
Finite-time control
Feedback domination

ABSTRACT

An alternative but new nonlinear output regulation approach is proposed for a class of lower triangular nonlinear systems by using the tool of adding a power integrator. The invariant manifold is solved and represented in terms of various derivatives of disturbances and references. As such, it is unnecessary to suppose that the external disturbances being governed by certain exosystems anymore. The nonlinearities are not restricted to be sufficiently smooth due the utilization of feedback domination approach. It is shown that the time taken to reach the desired invariant manifold from any initial states under external disturbances is guaranteed to be finite time. The control law is concise in structure for implementation since the feedback domination is implemented in each of the recursive design step.

1. Introduction

Output regulation is one of the central design problems in nonlinear control theory (Ding, 2001; Huang & Chen, 2004; Isidori & Byrnes, 1990; Pavlov, van de Wouw, & Hijmeijer, 2006). It aims to achieve precise reference tracking and disturbance rejection while maintaining system stability (Chen & Huang, 2014; Ding, 2013; Huang, 2004). The remarkable properties of most output regulation approaches are highlighted as follows: (1) the external disturbances are generally periodic signals supposed to be governed by some neutral stable exosystems (Ding, 2013; Isidori & Byrnes, 1990; Pavlov et al., 2006); (2) the nonlinearities are usually assumed to be sufficiently smooth with respect to their arguments (Chen & Huang, 2014); and (3) globally/semi-globally/locally asymptotical output regulation is a general control design goal which indicates that the time taken to achieve the target of precise tracking is actually infinite (Chen & Huang, 2014; Ding, 2013; Isidori & Byrnes, 1990).

In this communique, we aim to address the output regulation problem in a new perspective for a class of lower triangular nonlinear system subject to general disturbances depicted by Chen and Huang (2014)

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\bar{x}_i, w), \quad i \in \mathbb{N}_{1:n-1} \\ \dot{x}_n &= u + f_n(\bar{x}_n, w) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i \triangleq [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are system state, control input and controlled output, respectively. $w(t) \in \mathbb{D} \subset \mathbb{R}^p$ denotes the external disturbance. The regulation error is defined as $\varepsilon(t) = y(t) - y_r(t)$ where $y_r(t)$ is the desired reference signal.

The development of the new output regulation approach is divided into the following two steps. First, without resorting to any specific exosystems, the desired invariant manifold is solved and depicted by different time derivatives of both reference and disturbance signals. Second, to relaxing the assumption on sufficient smoothness of nonlinearities, the new output regulation law is developed via a feedback domination approach by means of the tool of adding a power integrator (Lin & Qian, 2000; Polendo & Qian, 2007). It is further discussed that the finite time output regulation is achievable with some mild assumption on the homogeneous degree of the system. Since many useful estimators and differentiators like disturbance observers (Kim, Rew, & Kim, 2010) and higher-order sliding mode differentiators (Levant, 2003) provide adequate means to get the different time derivatives of disturbances, the only assumption on solvability of the new output

☆ This work is supported in part by National Natural Science Foundation of China (Nos. 61573099 and 61633003), Natural Science Foundation of Jiangsu Province (No. BK2012327), and Fundamental Research Funds for the Central Universities (Nos. 2242016R30011 and 2242016K41067). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor A. Pedro Aguiar under the direction of Editor André L. Tits.

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http://dx.doi.org/10.1016/j.automatica.2016.11.008
0005-1098/

regulation problem is that the external disturbance shall be measured by sensors. This assumption is well received in certain circumstances among control society community, for example, see full information output regulation (Huang, 2004) and feedforward control (Bequette, 2002) largely in process industries. In a summary, the distinct features of the new output regulation approach are concluded as follows:

(1) *there is no need to suppose that the disturbances are governed by certain neutral stable exosystems, which relaxes the restrictions on disturbances;*
(2) *the nonlinear items in the system is not required to be sufficiently smooth but only needed to satisfy some homogeneous growth condition;*
(3) *the time taken to reach the desired invariant manifold from any initial states is guaranteed to be finite rather than infinite.*

2. Notations and some useful lemmas

Notations: The symbols $\mathbb{R}^{+}_{\rm odd}$ and $\mathbb{R}^{\geq 1}_{\rm odd}$ denote the set of ratio of positive odd integers and set of ratio of positive odd integers and greater than 1, respectively. For integers j and k satisfying $0 \leq j \leq k$, let $\mathbb{N}_{j:k} \triangleq \{j, j+1, \ldots, k\}$ be a set of nonnegative integers. The symbol $\mathcal{C}^i$ denotes the set of all differentiable functions whose first ith time derivatives are continuous. For $\pi_i \in \mathbb{R}$, $\eta_i \in \mathbb{R}$, $w \in \mathbb{R}^p$ and $y_r \in \mathbb{R}$, denote $\overline{\pi}_i \triangleq [\pi_1, \pi_2, \ldots, \pi_i]^T$, $\overline{\eta}_i \triangleq [\eta_1, \eta_2, \ldots, \eta_i]^T$, $\overline{w}_i \triangleq \{w, \dot{w} \cdots, w^{[i-2]}\}$, $\overline{y}_i^r \triangleq \{y_r, \dot{y}_r, \ldots, y_r^{[i-1]}\}$. The symbols f_i^x and f_i^π are shorts for $f_i(\overline{x}_i, w)$ and $f_i(\overline{\pi}_i, w)$, respectively. Let $\|w(t)\|_\infty = \max_{i\in\mathbb{N}_{1:p}} \{\sup_{t\geq 0} |w_i(t)|\}$ be the infinity norm of vector $w(t)$. The symbol $\mathcal{L}_\infty$ represents the set of all signals whose infinity-norms are bounded. Define a $\mathcal{C}_0$ function as $\mathrm{sig}^\alpha(\cdot) \triangleq |\cdot|^\alpha \mathrm{sgn}(\cdot)$ with $\mathrm{sgn}(\cdot)$ denoting the standard signum function.

The following two lemmas are instrumental for the development of the main results of the paper.

Lemma 1 (*Polendo & Qian, 2007*). *For $x \in \mathbb{R}$, $y \in \mathbb{R}$, $\ell \geq 1$ is a constant, the following inequalities hold $|x+y|^\ell \leq 2^{\ell-1}|x^\ell + y^\ell|$, $(|x|+|y|)^{\frac{1}{\ell}} \leq |x|^{\frac{1}{\ell}} + |y|^{\frac{1}{\ell}} \leq 2^{\frac{\ell-1}{\ell}}(|x|+|y|)^{\frac{1}{\ell}}$. If $\ell \in \mathbb{R}^{\geq 1}_{\rm odd}$, then $|x-y|^\ell \leq 2^{\ell-1}|x^\ell - y^\ell|$ and $|x^{\frac{1}{\ell}} - y^{\frac{1}{\ell}}| \leq 2^{\frac{\ell-1}{\ell}}|x-y|^{\frac{1}{\ell}}$.*

Lemma 2 (*Lin & Qian, 2000*). *For $x \in \mathbb{R}$, $y \in \mathbb{R}$, $c > 0$, $d > 0$ and any real-valued function $\gamma(x, y) > 0$, the following inequality holds $|x|^c|y|^d \leq \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}(c+d)}(x,y)|y|^{c+d}$.*

3. Main results

3.1. System pretreatment

With some brief derivations, the steady-state of the system is given by $x_{1s} = \pi_1(\overline{y}_1^r)$, $x_{is} = \pi_i(\overline{w}_i, \overline{y}_i^r)$, $u_s = \pi_u(\overline{w}_{n+1}, \overline{y}_{n+1}^r)$ with π_i and π_u determined by the following recursive calculations

$$\pi_1 = y_r, \qquad \pi_i = \frac{d\pi_{i-1}}{dt} - f_{i-1}^\pi, \qquad \pi_u = \frac{d\pi_n}{dt} - f_n^\pi \tag{2}$$

for $i \in \mathbb{N}_{2:n}$. Clearly, the set $\Omega_{x,u} \triangleq \{(x, u) \in (\overline{\pi}_n, \pi_u)\}$ is an invariant manifold for the system (1).

Defining auxiliary variables $\eta_i = x_i - \pi_i$ ($i \in \mathbb{N}_{1:n}$), the dynamic system (1) is governed by

$$\begin{aligned} \dot{\eta}_i &= \eta_{i+1} + f_i^x - f_i^\pi, \quad i \in \mathbb{N}_{1:n-1} \\ \dot{\eta}_n &= u - \pi_u + f_n^x - f_n^\pi \end{aligned} \tag{3}$$

where π_i and π_u (the solutions of output regulation Eqs. (2)) are in terms of the disturbances, the reference signal and their higher-order derivatives, rather than represented by exosystems in Huang and Chen (2004).

The requirements on disturbances and system nonlinearities are presented and discussed as follows.

Assumption 1. The external disturbances and reference signal satisfy $w(t) \in \mathcal{C}^n$, $y_r(t) \in \mathcal{C}^n$, $\frac{d^i w(t)}{dt^i} \in \mathcal{L}_\infty$ and $\frac{d^i y_r(t)}{dt^i} \in \mathcal{L}_\infty$ bounded by

$$\left\| \frac{d^i w(t)}{dt^i} \right\|_\infty \leq \gamma_w^+, \qquad \left\| \frac{d^i y_r(t)}{dt^i} \right\|_\infty \leq \gamma_r^+ \tag{4}$$

for $i \in \mathbb{N}_{0:n}$, where γ_r^+ and γ_w^+ are some positive constants.

Assumption 2. There exists a constant τ (a ratio of an even and an odd integer, which is called homogeneous degree) and an $\mathcal{C}^0$ function $\gamma_i(\overline{\eta}_i) > 0$ such that for all $\overline{x}_i \in \mathbb{R}^i$, $\overline{\pi}_i \in \mathbb{R}^i$ and $w \in \mathbb{D} \subset \mathbb{R}^p$ with $\mathbb{D}$ being a compact set, the following inequality holds

$$|f_i^x - f_i^\pi| \leq \gamma_i(\overline{\eta}_i) \sum_{j=1}^{i} |\eta_j|^{\frac{r_i+\tau}{r_j}}, \quad i \in \mathbb{N}_{1:n} \tag{5}$$

with $r_1 = 1$, $r_i + \tau = r_{i+1} \in \mathbb{R}^+_{\rm odd}$.

Remark 1. The disturbances satisfying Assumption 1 represent quite a broad class of exogenous signals. For example, the smooth disturbances governed by neutral stable linear/nonlinear exosystem system (Ding, 2013; Huang, 2004) and general smooth non-periodic disturbances are special cases of this assumption.

Remark 2. Assumption 2 is called homogeneous growth condition (Polendo & Qian, 2007), which covers a large class of nonlinear functions. Taking any $\mathcal{C}^1$ nonlinear function $f_i(\overline{x}_i, w)$ as an example, it can be shown that there exists a homogeneous degree $-1 < \tau \leq 0$ and a $\mathcal{C}^0$ function $\gamma_i(\overline{\eta}_i) > 0$ such that the inequality (5) is satisfied. In addition, for certain $\mathcal{C}^0$ but nonsmooth nonlinearities, the inequality (5) is also satisfied by choosing an appropriate homogeneous degree τ. For example, the nonlinearity $f_1^x = w_1 x_1^{1/3}$ satisfies (5) with $\tau = -2/3$ and $\gamma_1 > 2^{2/3}|w_1|$.

3.2. New approach to output regulation

The new approach is developed by using a basic concept of invariant manifold as well as an enhanced version of backstepping approach, namely adding a power integrator (Lin & Qian, 2000). The detailed derivation of the new output regulation approach consists of three steps described as follows:

Step 1: Let $\sigma, \rho \in \mathbb{R}^+_{\rm odd}$ satisfy $\sigma \geq \max_{i\in\mathbb{N}_{1:n}}\{r_i\}$, $\rho \geq \max_{i\in\mathbb{N}_{1:n}}\{r_i + \tau, \sigma\}$. Design a positive definite and proper Lyapunov function candidate

$$V_1 = \int_0^{\eta_1} \left(s^{\frac{\sigma}{r_1}} - \eta_1^{*\frac{\sigma}{r_1}} \right)^{\frac{2\rho - r_2}{\sigma}} ds \tag{6}$$

where η_1^* is the first step virtual control law assigned as $\eta_1^* = 0$. By virtue of the inequality (5) in Assumption 2, the time derivative of V_1 along (3) is

$$\begin{aligned} \dot{V}_1 &= \eta_1^{\frac{2\rho - r_2}{r_1}} \left(\eta_2 + f_1^x - f_1^\pi \right) \\ &\leq \eta_1^{\frac{2\rho - r_2}{r_1}} \left(\eta_2 - \eta_2^* + \eta_2^* \right) + \gamma_1(\eta_1)\eta_1^{\frac{2\rho}{r_1}}. \end{aligned} \tag{7}$$

The virtual control law η_2^* is designed as

$$\eta_2^* = -k_1(\eta_1)\xi_1^{\frac{r_2}{\sigma}} \tag{8}$$

with $\xi_1 = \eta_1^{\frac{\sigma}{r_1}} - \eta_1^{*\frac{\sigma}{r_1}}$ and $k_1(\eta_1)$ is the control gain assigned such that $k_1(\eta_1) \geq n + \gamma_1(\eta_1)$. Substituting the control law (8) into (7) gives

$$\dot{V}_1 \leq \eta_1^{\frac{2\rho - r_2}{r_1}} \left(\eta_2 - \eta_2^* \right) - n\eta_1^{\frac{2\rho}{r_1}} \tag{9}$$