Automatica 76 (2017) 378–382

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Effect of delayed impulses on input-to-state stability of nonlinear systems[☆]

Xiaodi Li [a,b], Xiaoli Zhang [a,b], Shiji Song [c]

[a] School of Mathematical Sciences, Shandong Normal University, Ji'nan, 250014, PR China
[b] Institute of Data Science and Technology, Shandong Normal University, Ji'nan, 250014, PR China
[c] Department of Automation, Tsinghua University, Beijing 100084, PR China

ARTICLE INFO

Article history:
Received 4 January 2016
Received in revised form
30 May 2016
Accepted 27 July 2016
Available online 7 December 2016

Keywords:
Nonlinear systems
Delayed impulses
Lyapunov methods
Input-to-state stability
Integral input-to-state stability

ABSTRACT

This paper studies the input-to-state stability (ISS) and integral input-to-state stability (iISS) of nonlinear systems with *delayed impulses.* By using Lyapunov method and the analysis technique proposed by Hespanha et al. (2005), some sufficient conditions ensuring ISS/iISS of the addressed systems are obtained. Those conditions establish the relationship between impulsive frequency and the time delay existing in impulses, and reveal the effect of delayed impulses on ISS/iISS. An example is provided to illustrate the efficiency of the obtained results.

1. Introduction

The theory of input-to-state stability (ISS) plays a central role in modern nonlinear control theory, in particular to robust stabilization of nonlinear systems, design of nonlinear observers, analysis of large-scale networks, etc. (Dashkovskiy, Ruffer, & Wirth, 2010; Freeman & Kokotovic, 2008; Jiang, Teel, & Praly, 1994; Sontag, 2001). The concept of ISS was introduced by Sontag (1989, 1998). In last years, many interesting results on ISS properties of various systems such as discrete systems, switched systems and hybrid systems have been reported, see, (Cai & Teel, 2005; Mancilla-Aguilar & Garcia, 2001; Nesic & Teel, 2004a,b; Pepe & Jiang, 2006).

Impulsive systems serve as basic models to study the dynamics of processes that are subject to sudden changes at certain moments in their states. They have been extensively studied in the literature (Haddad, Chellaboina, & Nersesov, 2006; Lu, Ho, & Cao, 2010; Naghshtabrizi, Hespanha, & Teel, 2008). Especially, due to the facts that impulsive systems with external inputs arise naturally from a number of applications such as in control systems with communication constraints, control algorithms of uncertain systems and network control systems with scheduling protocol, it is important to guarantee the impulsive system to be input-to-state stable when it is affected by some external inputs (Chen & Zheng, 2009; Dashkovskiy & Mironchenko, 2012; Hespanha, Liberzon, & Teel, 2008; Liu, Liu, & Xie, 2011). Hence, it is of great practical significance to investigate the ISS property of impulsive systems and it has become one of the hot issues in control theory. The concepts of ISS and iISS of impulsive systems was proposed by Hespanha, Liberzon, and Teel (2005) and Hespanha et al. (2008). They developed the Lyapunov method to impulsive systems and established some criteria for ISS properties by controlling the frequency of impulse occurrence. Chen and Zheng (2009) studied the ISS and iISS of nonlinear impulsive systems with time delays and presented several Razumikhin-type criteria. In Liu et al. (2011), Liu and Xie considered the ISS properties of impulsive and switching hybrid systems with time delays using the method of multiple Lyapunov–Krasovskii functionals and it can be applied to impulsive systems with arbitrarily large delays. Recently, Dashkovskiy and Mironchenko (2012) and Dashkovskiy, Kosmykov, Mironchenko, and Naujok (2012) further studied the ISS and iISS of nonlinear impulsive systems with or without delays via the generalized average dwell-time method, and especially a Lyapunov–Krasovskii-type ISS

☆ This work was jointly supported by National Natural Science Foundation of China (11301308, 61673247, 41427806), China PSFF (2014M561956, 2015T80737) and Research Fund for Excellent Youth Scholars of Shandong Province (ZR2016JL024). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor A. Pedro Aguiar under the direction of Editor André L. Tits.

E-mail addresses: sodymath@163.com (X. Li), shijis@tsinghua.edu.cn (S. Song).

http://dx.doi.org/10.1016/j.automatica.2016.08.009

theorem and a Lyapunov–Razumikhin-type ISS theorem for single impulsive time-delay systems were established in Dashkovskiy et al. (2012).

With the development of impulsive control theory, increasing attention has been paid to the study of dynamics and controller design of impulsive systems in which the impulses involve time delays which are sometimes called delayed impulses, see (Chen, Wei, & Zheng, 2013; Chen & Zheng, 2011; Khadra, Liu, & Shen, 2009; Yang, Yang, & Nie, 2014). Such kind of impulses describe a phenomenon where impulsive transients depend on not only their current but also historical states of the system. For instance, Khadra et al. (2009) studied the stability problem of autonomous impulsive differential systems with linear delayed impulses and then applied it to synchronization control of two coupled chaotic systems. Chen and Zheng (2011) and Chen et al. (2013) considered the effects of delayed impulses on nonlinear time-delay systems and Takagi–Sugeno fuzzy systems, respectively. Recently, Yang et al. (2014) considered the exponential synchronization of discontinuous chaotic systems via delayed impulsive control. However, it seems that there have been few results that consider the effect of delayed impulses on ISS property for nonlinear systems, which still remains as an important and open problem.

In this paper, we shall study the ISS property of nonlinear systems with delayed impulses and external input affecting both the continuous dynamics and the state impulse map. We extend the analysis technique developed in Hespanha et al. (2008, 2005) to the systems with delayed impulses. Some sufficient conditions ensuring the ISS/iISS are obtained. The rest of this paper is organized as follows. In Section 2, the problem is formulated and some notations and definitions are given. In Section 3, we present the main results. An example is given in Section 4, and conclusions follow in Section 5.

2. Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ denote the set of nonnegative real numbers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ the n-dimensional and $n \times m$-dimensional real spaces, respectively, $\mathbb{Z}_+$ the set of positive integer numbers, $|\cdot|$ the Euclidean norm, and $\|\cdot\|_J$ the supremum norm on an interval $J \in \mathbb{R}$. Let $\alpha \vee \beta$ and $\alpha \wedge \beta$ denote the maximum and minimum value of α and β, respectively. Let $\mathcal{K}_\infty = \{\alpha \in C(\mathbb{R}_+, \mathbb{R}_+)|\ \alpha(0) = 0, \alpha(r)$ is strictly increasing in r, and $\alpha(r) \to +\infty$ as $r \to +\infty\}$, $\mathcal{KL} = \{\beta \in C(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+)|\beta(r,t)$ is in class $\mathcal{K}$ w.r.t. r for each fixed $t \geq 0$, and $\beta(r,t)$ is strictly decreasing to 0 as $t \to +\infty$ for each fixed $r \geq 0\}$.

Consider the system with delayed impulses of the form

$$\begin{cases}\dot{x}(t) = f(x(t), u(t)), & t \geq t_0 \geq 0,\ t \neq t_k,\\ x(t) = g(x(t^- - \tau), u(t^- - \tau)), & t = t_k,\ k \in \mathbb{Z}_+,\end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $\dot{x}(t)$ denotes the right-hand derivative of $x(t)$, $u \in \mathbb{R}^m$ is the external input which is right continuous, $\tau > 0$ is a delay constant, f and $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ satisfy f locally Lipschitz and $f(0,0) = g(0,0) = 0$. The impulse sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ satisfies $0 \leq t_0 < t_1 < \cdots < t_k \to +\infty$ as $k \to \infty$. We always assume that $t_k - \tau$ is non-impulsive point. Assume $x(t^+) = x(t)$, i.e., the solution of (1) is assumed to be right continuous. Given a sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ and a pair of time (t,s) satisfying $t > s \geq t_0$, let $N(t,s)$ denote the number of impulse times in the semi-open interval $[s,t)$.

Definition 1 (*Hespanha et al., 2008*). For the prescribed sequence $\{t_k\}_{k\in\mathbb{Z}_+}$, the system (1) is said to be input-to-state stable (ISS) if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ such that for every initial condition (t_0, ϕ) and input u, the corresponding solution of (1) satisfies

$$|x(t)| \leq \beta(\|\phi\|_\tau, t - t_0) + \gamma(\|u\|_{[t_0,t]}),\ t \geq t_0,$$

where $\|\phi\|_\tau = \sup_{[t_0-\tau,t_0]} |\phi|$. It is said to be *uniformly ISS* over a given class $\mathcal{H}$ of admissible sequences of impulse times if the ISS property expressed by the above inequality holds for every sequence in $\mathcal{H}$, with functions β and γ that are independent of the choice of the sequence.

Definition 2 (*Hespanha et al., 2008*). For the prescribed sequence $\{t_k\}_{k\in\mathbb{Z}_+}$, the system (1) is said to be integral-input-to-state stable(iISS) if there exist functions $\beta \in \mathcal{KL}$ and $\alpha, \gamma \in \mathcal{K}_\infty$ such that for every initial condition (t_0, ϕ) and every input u, the corresponding solution of (1) satisfies

$$\alpha(|x(t)|) \leq \beta(\|\phi\|_\tau, t - t_0) + \int_{t_0}^{t} \gamma(|u(s)|)ds + \sum_{k\in N(t,t_0)} \gamma(|u(t_k^- - \tau)|),\ \ t \geq t_0.$$

The notion of *uniform iISS* over a given class S of impulse time sequences is defined in the same way as for ISS.

Definition 3. A function $V : \mathbb{R}^n \to \mathbb{R}_+$ is said to be an exponential ISS-Lyapunov function for (1) with rate coefficients $c, d \in \mathbb{R}$ if V is locally Lipschitz, positive definite, radially unbounded, and satisfies

$$\nabla V(x) \cdot f(x,u) \leq -cV(x) + \mathcal{X}(|u|), \qquad \forall x\ a.e., \forall u \tag{2}$$

$$V(g(x,u)) \leq e^{-d}V(x) + \mathcal{X}(|u|), \qquad \forall x, u \tag{3}$$

for some $\mathcal{X} \in \mathcal{K}_\infty$. In this paper, in order to consider the effect of delayed impulses, we only consider the case of $c, d \in \mathbb{R}_+$.

Definition 4. Given constants $\sigma, \eta, d \in \mathbb{R}_+$, let $\mathcal{U}_\sigma^{\eta,d} = \{\mu \in C(\mathbb{R}_+, \mathbb{R})\}$ be a set of functions that satisfy

(i) $\mu(r_1) + \mu(r_2) \leq \mu(r_1 + r_2) + \eta$, for any $r_1, r_2 \in \mathbb{R}_+$;

(ii) $2\eta - d < \mu(\tau)$;

(iii) $\mu(r) + \sigma r \leq d$ and tends to $-\infty$ as $r \to +\infty$.

Note that the set of functions $\mathcal{U}_\sigma^{\eta,d}$ which is composed of conditions (i)–(iii) includes some different classes of functions. For example, consider the case of delay $\tau = 1$, it is easy to check that $\mu(t) = 1 - t \in \mathcal{U}_{0.5}^{1,3}$, $\mu(t) = 2 - (\ln(e/2))t - \ln(1+t) \in \mathcal{U}_{\ln(e/2)}^{2,4}$, and $\mu(t) = 1 - t + e^{-t} \in \mathcal{U}_{0.8}^{3,6}$.

3. Main results

In this section, we will establish some criteria which provide sufficient conditions for ISS/iISS of system (1). The idea is inspired by the work of Hespanha, Liberzon and Teel in Hespanha et al. (2008, 2005).

Theorem 1 (*Uniformly ISS*). *Let V be a candidate exponential ISS-Lyapunov function of system (1) with rate coefficients $c \in \mathbb{R}_+$ and $d \in (0, \tau c)$. For any constants $\sigma, \eta \in \mathbb{R}_+$ and function $\mu \in \mathcal{U}_\sigma^{\eta,d}$, let $\mathcal{F}(\mu)$ denote the class of impulse time sequences $\{t_k\}_{k\in\mathbb{Z}_+}$ satisfying*

$$(\tau c - d)N(t,s) - c(t-s) \leq \mu(t-s) \quad \forall t \geq s \geq t_0. \tag{4}$$

Then system (1) is uniformly ISS over $\mathcal{F}(\mu)$.