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Robust economic Model Predictive Control using stochastic information[☆]

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ABSTRACT

In this paper, we develop a new tube-based robust economic MPC scheme for linear time-invariant systems subject to bounded disturbances with given distributions. By using the error distribution in the predictions of the finite horizon optimal control problem, we can incorporate stochastic information in order to improve the expected performance while being able to guarantee strict feasibility. For this new framework, we can provide bounds on the asymptotic average performance of the closed-loop system. Moreover, a constructive approach is presented in order to find an appropriate terminal cost leading to a slight degradation of the bound on the guaranteed average performance.

1. Introduction

In the last decade, *economic* Model Predictive Control (MPC) has received significant attention, both in theory and in application. In contrast to stabilizing MPC, where a positive definite stage cost is employed in order to stabilize a given set point, economic MPC focuses on the optimization of some general performance criterion. This can resemble the economics of the system, and hence, is of interest in many real world applications where economic goals are aspired, e.g., in process industry, in logistics, or in the energy sector. To this end, different settings and methods have been proposed in the literature (see, e.g., Amrit, Rawlings, & Angeli, 2011; Angeli, Amrit, & Rawlings, 2012; Diehl, Amrit, & Rawlings, 2011; Ellis, Durand, & Christofides, 2014; Müller, Angeli, & Allgöwer, 2013). In this paper, we consider linear systems only. These are of interest in many practical applications within economic MPC, for example in water supply networks (Limon, Pereira, Muñoz de la Peña, Alamo, & Grosso, 2014), climate control (Hovgaard, Larsen, & Jorgensen, 2011), or engine control (Broomhead, Manzie, Shekhar, Brear, & Hield, 2014), to name a few.

In real world applications, most systems are subject to disturbances. This can result in a degradation of performance and/or loss of feasibility. In order to overcome these problems and to deal with the disturbances, different concepts have been presented in the context of stabilizing MPC. In robust MPC, bounded disturbances are taken into consideration while aiming at the robust satisfaction of hard constraints, see, e.g., Chisci, Rossiter, and Zappa (2001) and Mayne, Seron, and Raković (2005). In stochastic MPC, disturbances of stochastic nature, i.e., with a given distribution, are considered. This stochastic information can be used in order to improve performance (see, e.g., Chatterjee, Hokayem, & Lygeros, 2011; Muñoz de la Peña, Bemporad, & Alamo, 2005). Moreover, probabilistic constraints are typically considered instead of hard constraints, e.g., Cannon, Kouvaritakis, and Ng (2009), Lorenzen, Allgöwer, Dabbene, and Tempo (2015), and Primbs and Sung (2009).

Even though disturbances occur in many applications, only few results can be found on the intersection of robust and economic MPC. In Huang, Biegler, and Harinath (2012), a stability result for robust economic MPC is presented which is based on a robust tracking of an a priori determined optimal nominal trajectory. An approach for stabilizing an economically optimal steady-state despite disturbances is presented in Broomhead et al. (2014) and extended in Broomhead, Manzie, Shekhar, and Hield (2015) by the same authors to periodic disturbance and cost functions leading to periodic terminal conditions. A scenario tree based approach for economic MPC is studied in Lucia, Andersson, Brandt, Diehl, and Engell (2014). In Hovgaard et al. (2011), a linear system and a linear objective are considered minimizing energy consumption and taking also probabilistic constraints into account. In Bayer, Müller, and Allgöwer (2014), it is shown that simply transferring robust MPC ideas into an economic framework might not lead to the best

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performance. Namely, in stabilizing MPC the disturbances have to be counteracted in order to achieve the desired goal of (robustly) stabilizing a given setpoint. On the other hand, in economic MPC the objective is to optimize some global performance criterion, and using information about the influence of disturbances on the system can be beneficial to achieve a better performance. In order to consider the influence of the disturbances on the performance, in Bayer et al. (2014) the cost function is modified by averaging the cost over all possible states within some invariant set and a robust MPC framework is employed to guarantee robust constraint satisfaction. Since no further assumptions on the disturbances are imposed other than boundedness, this averaging is done by weighting all states in the invariant set equally.

In this paper, it is shown how additional stochastic information on the disturbance, if available, can be used to improve closed-loop performance in robust economic MPC. Here, we consider a robust MPC framework to guarantee robust constraint satisfaction while employing additional stochastic information within the cost function. To this end, we compute the exact prediction of the error set at each open-loop time step and use the robust MPC approach presented in Chisci et al. (2001) to guarantee robust feasibility. Moreover, we compute the distribution of the error over these sets using the given distribution of the disturbance, and employ this information within the finite horizon optimal control problem by taking the expected value of the cost. We show that for a particular assumption on the terminal cost, bounds on the average performance of the closed loop can be derived which resemble known results from both nominal economic MPC and previous concepts on robust economic MPC. We provide two constructive approaches for finding an appropriate quadratic terminal cost. Both lead to a slight degradation of the original average performance statement. While the first approach is computationally less demanding, the second leads to a smaller degradation of the average performance statement. The proposed robust economic MPC scheme using stochastic information in general results in a better closed-loop performance than the robust economic MPC scheme in Bayer et al. (2014). On the other hand, the online computational complexity is slightly larger, since different (time-varying) constraints are needed. As an additional contribution, we propose an intermediate version of the two approaches presented in Bayer et al. (2014) and in this paper. To this end, we modify the stage cost by integrating over the whole invariant set, but take additionally the distribution over the invariant set into account. Finally, we provide a detailed discussion for a numerical example.

We close this section by noting that a preliminary version of parts of our results has appeared in the conference paper (Bayer, Lorenzen, Müller, & Allgöwer, 2015). The main novelties of this paper compared to Bayer et al. (2015) are the following: First, we present the proposed robust economic MPC framework in a more comprehensive manner (including proofs of all of our results, which were partly missing in the conference version). In particular, we present a detailed performance analysis for the case when using a quadratic approximation of the terminal cost, and we also show how a less conservative approximation of the terminal cost can be found. Moreover, we develop an intermediate approach combining the schemes developed in Bayer et al. (2014) and in this paper. Finally, we provide a convergence proof of the error distribution on its robust invariant set, and a more detailed elaboration of the numerical example.

The remainder of this paper is structured as follows. In Section 2, we introduce the problem setup. The finite horizon optimal control problem is presented and discussed in Section 3. A bound on the closed-loop asymptotic average performance is derived in Section 4, and in Section 5, we provide two constructive approaches for finding an appropriate quadratic terminal cost. A different robust economic MPC scheme based only on information of the error distribution over the robust positive invariant set is presented in Section 6. Both setups are applied to a numerical example in Section 7, and the paper is concluded in Section 8.

Notation: We denote by $\mathbb{I}_{\geq 0}$ the set of all non-negative integers and by $\mathbb{I}_{[a,b]}$ the set of all integers in the interval $[a, b] \subseteq \mathbb{R}$. For sets $X, Y \subseteq \mathbb{R}^n$, the Minkowski set addition is defined by $X \oplus Y := \{x + y \in \mathbb{R}^n : x \in X, y \in Y\}$; the Pontryagin set difference is defined as $X \ominus Y := \{z \in \mathbb{R}^n : z + y \in X, \forall y \in Y\}$.

2. Problem setup

In this paper, we consider discrete-time LTI systems of the form

$$x(t + 1) = Ax(t) + Bu(t) + w(t), \quad x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the system state and $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ is the input at time $t \in \mathbb{I}_{\geq 0}$, respectively. For the states and inputs, we consider pointwise-in-time constraints of the form $(x(t), u(t)) \in \mathbb{Z}$, for all $t \in \mathbb{I}_{\geq 0}$, where $\mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U}$ is a compact set. We assume that (A, B) is stabilizable.

The unknown disturbance $w(t)$ at time t satisfies the following assumption.

Assumption 1. For each $t \in \mathbb{I}_{\geq 0}$, the disturbance satisfies

$$w(t) \in \mathbb{W} \subset \mathbb{R}^n, \tag{2}$$

where $\mathbb{W}$ is a compact and convex set containing the origin in its interior. Furthermore, w is distributed over $\mathbb{W}$ according to some given probability density function (PDF) $\rho_{\mathbb{W}} : \mathbb{R}^n \to [0, \infty]$, which has bounded support $\mathbb{W}$. All disturbances are independent and identically distributed (i.i.d.) and have zero mean.

For the input u, we employ an affine parametrization of the form

$$u(t) = Kx(t) + c(t), \tag{3}$$

where $c(t) \in \mathbb{R}^m$ is the manipulated input at time $t \in \mathbb{I}_{\geq 0}$ and $K \in \mathbb{R}^{m\times n}$ is a state feedback, determined such that $A_{\mathrm{cl}} = A + BK$ is a stable matrix.

Thus, system (1) can equivalently be written as

$$x(t + 1) = A_{\mathrm{cl}}x(t) + Bc(t) + w(t). \tag{4}$$

For A_{cl}, we introduce the following assumption.

Assumption 2. The system matrix A_{cl} is invertible.

Remark 1. In principle, this assumption could be relaxed. However, for ease of presentation, we restrict ourselves to system matrices which are invertible. Note that this is not a major restriction. In fact, if (A, B) is controllable, the eigenvalues of A_{cl} can be placed arbitrarily inside the unit disc. Note that the state feedback K is used later in order to prevent the prediction errors from growing exponentially.

Our objective in the following is to find a feasible control input to system (1) minimizing the asymptotic average cost

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \ell(x(t), u(t)), \tag{5}$$

where $\ell : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ can be some general stage cost function which is assumed to be continuous. Due to the disturbances, it is difficult to find a general solution for this problem. Thus, we develop an economic MPC scheme such that a priori bounds for the expected value of (5) can be derived.