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Constrained model predictive manifold stabilization based on transverse normal forms☆

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ABSTRACT

The optimization-based stabilization of manifolds for nonlinear dynamical systems with constraints is investigated. Manifolds in the state or output space are considered for which the original system description can be transformed into a so-called transverse normal form. With this formulation, the motion of the system transverse and tangential to the manifold can be separately described. The transverse normal form is combined with a tailored model predictive control scheme to achieve the objectives of stabilizing the manifold, rendering it invariant, and imposing a desired motion on the manifold under due consideration of constraints. Furthermore, the stabilization of the manifold is prioritized over the movement on the manifold. Convergence of the model predictive control scheme is proven. The applicability of the proposed concept is demonstrated by an illustrative simulation example.

1. Introduction

The stabilization of manifolds or sets is a popular field in modern control theory. It can be seen as an extension of the classical task of set point stabilization as a set point constitutes a manifold of dimension zero. Typically, manifolds defined in the output or state space of a dynamical system are considered, see, e.g., Nielsen (2009). Roughly speaking, the goal of manifold stabilization is to ensure (asymptotic) convergence of the output or state of the system to the manifold. Moreover, the resulting controller is frequently supposed to achieve two further objectives. Firstly, the (tangential) movement on the manifold should be of a desired form, and the second objective is the so-called invariance property. Roughly speaking, it states that if at any given point in time the system is exactly on the manifold, or more precisely in a corresponding controlled invariant subset of the state space, then the manifold must never be left again in the nominal, undisturbed case. The invariance property is required to hold regardless of the tangential movement, cf. Nielsen, Fulford, and Maggiore (2010). In the following, unless stated otherwise, the term manifold stabilization not only refers to the stabilization itself but also includes the task of achieving a desired movement on the manifold.

One possibility is to tackle these tasks from a geometric point of view. For example, in Nielsen (2009) and Nielsen and Maggiore (2008) the stabilization of controlled invariant submanifolds of the state space of control-affine dynamical systems is investigated. To this end, the system description is transformed into a so-called transverse normal form (TNF). The TNF consists of two sets of coordinates describing the transverse and tangential motion with respect to the manifold. By rendering the transverse dynamics asymptotically stable the manifold is stabilized. The invariance property is fulfilled and the motion on the manifold can be influenced by controlling the tangential dynamics. However, the consideration of system constraints is not possible in a straightforward way.

There are many works dealing with the stabilization of invariant sets for passive systems. Typically, the sets to be stabilized are subsets of the zero level set of the storage function, see, e.g., El-Hawwary (2011) and Shiriaev (2000). In El-Hawwary and Maggiore (2010) passive systems in control-affine form are considered and the stabilization of open-loop positively invariant subsets of the zero level set of the storage function is investigated. The authors in Shiriaev and Fradkov (2000) deal with the stabilization of the zero level set of a nonnegative objective function under nonlinear system dynamics. It is shown that there exist controllers providing arbitrarily small input values for stabilizing the set, i.e., input constraints can in principle be respected by a suitable control law.

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In these works, the movement of the system in the set is not taken care of.

The stabilization of sets in the context of haptic simulators is investigated in Walker (2013). Similar to the works of Nielsen et al. the main ideas are based on projection techniques and TNFs. In Böck and Kugi (2014a) a method is presented for stabilizing manifolds in flat output spaces. The movement on the manifolds can be chosen freely and the invariance property is fulfilled. However, system constraints cannot be systematically respected.

The authors in Kellett and Teel (2000) show that uniform global asymptotic controllability to a closed set implies the existence of a locally Lipschitz control Lyapunov function (CLF). Based on this CLF a robust feedback law for stabilizing the set is constructed. In Albertini and Sontag (1999) time-varying systems are considered. Similar to Kellett and Teel (2000) it is shown that the existence of a continuous CLF with respect to a closed subset of the state space is equivalent to the global asymptotic controllability to that set. The authors in Nersesov, Ghorbanian, and Aghdam (2010) provide sufficient stability criteria for time-varying sets in the state space of nonlinear time-varying systems using vector Lyapunov functions. Based on these results a stabilizing feedback is developed for multi-agent dynamical systems and applied to multi-vehicle formation control. The asymptotic stabilization of subsets of the state space of systems with positive inputs is investigated in Imsland and Foss (2003). The proposed control law can also be applied to positive systems as a special case.

In Findeisen (2004) sampled-data open-loop feedbacks are considered. For using this type of feedback, convergence conditions of the state of a nonlinear system to a compact set are derived. Constraints on inputs and states are taken into account. The class of sampled-data open-loop feedbacks includes model predictive control (MPC). MPC relies on solving an optimal control problem (OCP) at each sampling instant to determine the control input for the system, see, e.g., Allgöwer, Badgwell, Qin, Rawlings, and Wright (1999) and Rawlings and Mayne (2009). The convergence conditions obtained in Findeisen (2004) are applied to MPC as well. In contrast to this work, the movement of the system in the set is not taken care of and the target set is defined as compact subset of the state space. In this paper, manifolds in the state and output space are considered and other cases are in principle also possible. Furthermore, here the MPC is based on transformed system coordinates.

In recent years, the stabilization of one-dimensional manifolds or curves has gained more and more interest. In this context, the curves are usually called paths and therefore the corresponding control scheme is often named path following control. Trajectory tracking control also deals with the stabilization of curves. However, there the curves are equipped with a time-parameterization which turns them into trajectories. In contrast, for path following control no a priori time-parameterization of the curves is given. It has to be inherently determined by the path following controller. The paths are typically defined in the output (Faulwasser & Findeisen, 2010; Nielsen et al., 2010) or state space (Faulwasser, Kern, & Findeisen, 2009; Skjetne, Teel, & Kokotović, 2002; Yu, Li, Chen, & Allgöwer, 2012) of a dynamical system. Path following control is closely related to manifold stabilization as a path can be seen as a one-dimensional manifold, see, e.g., Skjetne et al. (2002) where this fact is used for controller design. Therefore, path following control is included in the framework presented in this paper as a special case. Nevertheless, there are many works in literature focusing on path following control based on different control approaches and tailored to different applications.

If the paths are given in the output space of the system, the corresponding zero path error manifold in the state space can be calculated and stabilized. This is done, e.g., in El-Hawwary (2011) for a unicycle and special types of paths. An interesting point in this work is the existence of a control law such that saturation constraints for the inputs can be considered. The same approach of determining and stabilizing the zero path error manifold is employed, e.g., in Akhtar, Waslander, and Nielsen (2012), Nielsen and Maggiore (2004), and Nielsen et al. (2010). There, the invariance property is fulfilled but system constraints are not taken into account.

Other approaches to path following control are given by hybrid control strategies as well as Lyapunov and backstepping techniques, see, e.g., Dačić, Nešić, and Kokotović (2007), Encarnação and Pascoal (2001), and Skjetne, Fossen, and Kokotović (2004). The utilization of MPC for path following control is investigated, e.g., in Faulwasser and Findeisen (2010), Lam, Manzie, and Good (2010), and Yu et al. (2012). Real-time capable MPC schemes for path following are presented, e.g., in Böck and Kugi (2014b) and Lam et al. (2010). MPC offers the possibility to systematically account for system constraints. However, often the invariance property is not fulfilled.

In summary, a controller for manifold stabilization or path following is expected to not only stabilize the respective manifold or path but it ideally also ensures the invariance property and a desired tangential movement. Moreover, usually system constraints have to be respected. Particularly in view of these constraints, a prioritization of the movement to the manifold over the tangential movement is of interest as well. To the best of the authors' knowledge, no other existing control scheme in literature for manifold stabilization and path following is able to simultaneously and systematically account for all these issues. The research presented in this paper aims at closing this gap. To this end, a novel tailored MPC scheme is proposed which relies on existing concepts of manifold stabilization transforming the original system description into coordinates of a TNF. Therefore, this work is also an extension of Böck and Kugi (2014a). Depending on the underlying concept, manifolds in the state and output space can inter alia be considered. The existing approaches for the transformation to TNF do not consider system constraints. The extension with MPC presented in this paper allows to systematically incorporate such constraints. Furthermore, in contrast to other MPC approaches to manifold stabilization and path following in literature, the proposed MPC structure achieves the above mentioned prioritization of the movement to the manifold over the tangential movement. Reaching the manifold is the primary target and no compromise between convergence to the manifold and movement on the manifold has to be taken. For practical applications, the prioritization is regarded important because imposing a desired tangential movement generally does not make sense if the manifold or path is not reached. On the other hand, it is essential for ensuring the invariance property under due consideration of the system constraints which is often not guaranteed by existing classical MPC schemes in literature. For these schemes, if the constraints are used to full capacity, usually a compromise situation occurs. In general, it provokes that the manifold or path is left and, hence, the invariance property is not fulfilled. Besides remedying this issue, the presented scheme allows to independently tune the convergence to the manifold and the movement on the manifold, which constitutes another difference to existing MPC approaches in literature.

This work is organized as follows. In Section 2 the TNFs are introduced which serve as the basis for the proposed MPC framework. The considered problem is stated in Section 3. Section 4 introduces the MPC scheme and in Section 5 its convergence properties are investigated. The theoretical results are applied to an illustrative example in Section 6 and some conclusions are drawn in Section 7.