Automatica 74 (2016) 63–70

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Observer-based input-to-state stabilization of networked control systems with large uncertain delays ☆

Anton Selivanov, Emilia Fridman

School of Electrical Engineering, Tel Aviv University, Israel

ARTICLE INFO

Article history:
Received 18 November 2015
Received in revised form
25 June 2016
Accepted 8 July 2016

Keywords:
Predictor-based control
Observer-based control
Networked control systems
Event-triggered control

ABSTRACT

We consider output-feedback predictor-based stabilization of networked control systems with large unknown time-varying communication delays. For systems with two networks (sensors-to-controller and controller-to-actuators), we design a sampled-data observer that gives an estimate of the system state. This estimate is used in a predictor that partially compensates unknown network delays. We emphasize the purely sampled-data nature of the measurement delays in the observer dynamics. This allows an efficient analysis via the Wirtinger inequality, which is extended here to obtain exponential stability. To reduce the number of sent control signals, we incorporate the event-triggering mechanism. For systems with only a controller-to-actuators network, we take advantage of continuously available measurements by using a continuous-time predictor and employing a recently proposed switching approach to event-triggered control. For systems with only a sensors-to-controller network, we construct a continuous observer that better estimates the system state and increases the maximum output sampling, therefore, reducing the number of required measurements. A numerical example illustrates that the predictor-based control allows one to significantly increase the network-induced delays, whereas the event-triggering mechanism significantly reduces the network workload.

1. Introduction

In networked control systems (NCSs), which are comprised of sensors, controllers, and actuators connected through a communication medium, transmitted signals are sampled in time and are subject to time-delays. Most existing papers on NCSs study robust stability with respect to small communication delays (see, e.g., Antsaklis & Baillieul, 2004, Fridman, Seuret, & Richard, 2004, Gao, Chen, & Lam, 2008, Liu & Fridman, 2012a). To compensate large transport delays, a predictor-based approach can be employed. This was done in Karafyllis and Krstic (2012) for sampled-data state-feedback control of nonlinear systems and in Karafyllis and Krstic (2015) for an output-feedback control with approximate predictors. Sampled-data predictor-based state-feedback control of linear systems under continuous-time measurements has been considered in Mazenc and Normand-Cyrot (2013). Nonlinear systems under sampled-data measurements and continuous output-feedback control have been studied in Ahmed-Ali, Karafyllis, and Lamnabhi-Lagarrigue (2013) and Karafyllis, Krstic, Ahmed-Ali, and Lamnabhi-Lagarrigue (2014).

All the aforementioned works deal with *known constant* network-induced delays. Predictor-based networked control under *uncertain time-varying* delays has been considered in Selivanov and Fridman (2016b), where a *state*-feedback controller has been studied. In this paper, we propose a predictor-based dynamic *output*-feedback controller for NCSs with *uncertain time-varying* delays. We present a new model of a closed-loop observer-based NCS in the framework of the time-delay approach. In such a model, several delays appear due to sampling and network-induced delays. We emphasize the purely sampled-data nature of measurement delays in the observer dynamics. This allows an efficient analysis via the Wirtinger inequality, which is extended here to obtain exponential stability.

We start by considering the case of two networks: sensors-to-controller and controller-to-actuators (Section 2). Both networks introduce large time-varying delays. We assume that the messages sent from the sensors are time stamped (Zhang, Branicky, & Phillips, 2001). This allows the controller to calculate the sensors-to-controller delay. The controller-to-actuators delay is assumed

☆ This work is supported by Israel Science Foundation (Grant No. 1128/14). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Dimos V. Dimarogonas under the direction of Editor Christos G. Cassandras.

E-mail addresses: antonselivanov@gmail.com (A. Selivanov), emilia@eng.tau.ac.il (E. Fridman).

http://dx.doi.org/10.1016/j.automatica.2016.07.031
0005-1098/

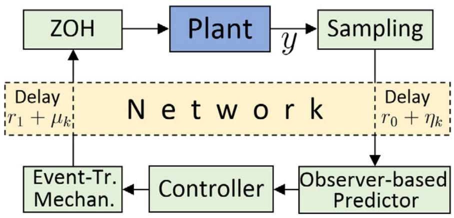

Fig. 1. NCS with two networks.

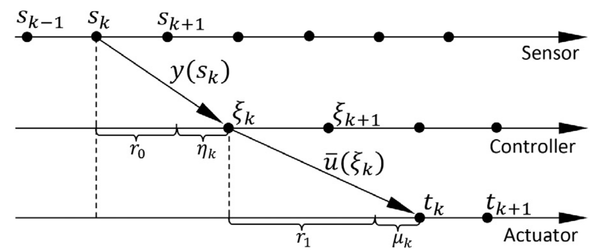

Fig. 2. Time-delays and updating times.

to be unknown but belongs to a known delay interval. We design an observer that is calculated on the controller side and gives an estimate of the system state. This estimate is used in a predictor, which partially compensates both delays. To reduce the workload of the controller-to-actuators network, we incorporate the event-triggering mechanism (Tabuada, 2007).

In Section 3, we proceed to NCSs with continuous measurements and controller-to-actuators networks, where we demonstrate that a recently proposed switching approach to event-triggered control (Selivanov & Fridman, 2016a) takes advantage of continuously available measurements and further reduces the number of sent control signals. For the case of continuous control and sampled measurements, we construct a continuous observer that better estimates the system state and increases the maximum output sampling, therefore, reducing the number of required measurements (Section 4). All the results are demonstrated in Section 5 by an example borrowed from Zhang et al. (2001).

First, we present an extension of the Wirtinger inequality (Liu, Suplin, & Fridman, 2010, Lemma 3.1).

Lemma 1 (*Wirtinger Inequality*). *Let $a, b, \alpha \in \mathbb{R}$, $0 \le W \in \mathbb{R}^{n\times n}$, and $f:[a,b] \to \mathbb{R}^n$ be an absolutely continuous function with a square integrable first derivative such that $f(a) = 0$ or $f(b) = 0$. Then*

$$\int_a^b e^{2\alpha t} f^T(t)Wf(t)\,dt \le e^{2|\alpha|(b-a)}\frac{4(b-a)^2}{\pi^2}\int_a^b e^{2\alpha t}\dot{f}^T(t)W\dot{f}(t)\,dt.$$

Proof is based on an idea from Gelig and Churilov (1998, Lemma A.18). If $\alpha \ge 0$, we have

$$\begin{aligned}\int_a^b e^{2\alpha t} f^T(t)Wf(t)\,dt &\le e^{2\alpha b}\int_a^b f^T(t)Wf(t)\,dt\\ &\le e^{2\alpha b}\frac{4(b-a)^2}{\pi^2}\int_a^b \dot{f}^T(t)W\dot{f}(t)\,dt\\ &\le e^{2|\alpha|(b-a)}\frac{4(b-a)^2}{\pi^2}\int_a^b e^{2\alpha t}\dot{f}^T(t)W\dot{f}(t)\,dt,\end{aligned} \tag{1}$$

where the second inequality follows from Liu et al. (2010, Lemma 3.1). If $\alpha < 0$, the proof is similar but $e^{2\alpha b}$ should be replaced by $e^{2\alpha a}$ after the first and second inequalities in (1). ■

If $\alpha = 0$, Lemma 1 coincides with Liu et al. (2010, Lemma 3.1) that was used in Liu and Fridman (2012b) to construct a Lyapunov functional for stability analysis of a sampled-data system. Here we use the extended Wirtinger inequality of Lemma 1 for Lyapunov-based *exponential* stability analysis (see V_W term in (A.1)).

2. NCSs with two networks

Consider a linear system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + w_1(t),\\ y(t) &= Cx(t) + w_2(t),\end{aligned} \quad t \ge 0 \tag{2}$$

with the state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^l$, exogenous disturbance $w_1 \in \mathbb{R}^n$, measurement noise $w_2 \in \mathbb{R}^l$, and constant matrices A, B, and C. We assume that (A, B) is stabilizable and (A, C) is detectable meaning that there exist constant matrices $K \in \mathbb{R}^{m\times n}$ and $L \in \mathbb{R}^{n\times l}$ such that $A + BK$ and $A + LC$ are Hurwitz. Let $\{s_k\}$ be sampling instants such that

$$0 = s_1 < s_2 < \cdots, \qquad \lim_{k\to\infty} s_k = \infty, \qquad s_{k+1} - s_k \le h.$$

In this section, we assume that at each sampling time s_k ($k \in \mathbb{N}$ throughout the paper) the output $y(s_k)$ is transmitted to a controller, which generates a control signal and transmits it to actuators, where it is applied through zero-order hold (see Fig. 1). The controller and actuators are event-driven with updating times (see Fig. 2)

$$\xi_k = s_k + r_0 + \eta_k, \qquad t_k = \xi_k + r_1 + \mu_k,$$

where r_0 and r_1 are known constant transport delays, η_k and μ_k are time-varying delays such that

$$0 \le \eta_k \le \eta_M, \quad 0 \le \mu_k \le \mu_M, \quad \xi_k \le \xi_{k+1}, \quad t_k \le t_{k+1}. \tag{3}$$

Note that the sequences $\{\xi_k\}$ and $\{t_k\}$ should be increasing, but we do not require $\eta_k + \mu_k$ to be less than a sampling interval. We assume that the sensors' and controller's clocks are synchronized (Zhang et al., 2001) and together with $y(s_k)$ the time stamp s_k is transmitted so that $\eta_k = \xi_k - s_k - r_0$ can be calculated by the controller. The delay uncertainty μ_k is unknown.

To reduce the workload of a controller-to-actuators network, we incorporate the event-triggering mechanism (Tabuada, 2007). The idea is to send only those control signals $u(\xi_k)$ whose relative change is greater than some threshold. Namely, let the *nominal control* (without event-triggering) be

$$u(t) = \begin{cases}0, & t < \xi_1,\\ u(\xi_k), & t \in [\xi_k, \xi_{k+1}),\end{cases}$$

where $u(\xi_k)$ will be constructed later. Then the *applied control* signal $\bar{u}(t)$ is 0 for $t < \xi_1$ and

$$\bar{u}(t) = \begin{cases}\bar{u}(\xi_{k-1}), & t \in [\xi_k, \xi_{k+1}), \quad (5)\text{ is true},\\ u(\xi_k), & t \in [\xi_k, \xi_{k+1}), \quad (5)\text{ is not true},\end{cases} \tag{4}$$

where the event-triggering rule is given by

$$[\bar{u}(\xi_{k-1}) - u(\xi_k)]^T\,\Omega\,[\bar{u}(\xi_{k-1}) - u(\xi_k)] \le \sigma u^T(\xi_k)\Omega u(\xi_k) \tag{5}$$

with event-triggering parameters $0 \le \Omega \in \mathbb{R}^{m\times m}$, $\sigma \in [0, 1)$, and initial value $\bar{u}(\xi_0) = 0$. Then the system (2) transforms into

$$\begin{aligned}\dot{x}(t) &= Ax(t) + w_1(t), & t &\in [0, t_1),\\ \dot{x}(t) &= Ax(t) + B\bar{u}(\xi_k) + w_1(t), & t &\in [t_k, t_{k+1}),\\ y(t) &= Cx(s_k) + w_2(s_k), & t &\in [s_k, s_{k+1}).\end{aligned} \tag{6}$$

The purpose of this section is to construct a predictor-based controller that stabilizes (6). First, we construct the following observer for $x(t)$:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t - r_1) - L[y(t) - \hat{y}(t)], \quad t \ge 0,\\ \hat{y}(t) &= C\hat{x}(s_k), \quad t \in [s_k, s_{k+1})\end{aligned} \tag{7}$$