Automatica 74 (2016) 71–79

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stability of a class of delayed port-Hamiltonian systems with application to microgrids with distributed rotational and electronic generation[☆]

Johannes Schiffer[a,1], Emilia Fridman[b], Romeo Ortega[c], Jörg Raisch[d,e]

[a] School of Electronic and Electrical Engineering, University of Leeds, Leeds LS2 9JT, UK
[b] Tel Aviv University, Tel Aviv 69978, Israel
[c] Laboratoire des Signaux et Systémes, École Supérieure d'Electricité (SUPELEC), Gif-sur-Yvette 91192, France
[d] Technische Universität Berlin, Einsteinufer 11, 10587 Berlin, Germany
[e] Max-Planck-Institut für Dynamik komplexer technischer Systeme, Sandtorstr. 1, 39106 Magdeburg, Germany

ARTICLE INFO

Article history:
Received 25 August 2015
Received in revised form
19 April 2016
Accepted 9 July 2016

Keywords:
Microgrid control
Microgrid stability
Smart grid applications
Droop control
port-Hamiltonian systems
Time delay systems
Lyapunov–Krasovskii functionals

ABSTRACT

Motivated by the problem of stability in droop-controlled microgrids with delays, we consider a class of port-Hamiltonian systems with delayed interconnection matrices. For this class of systems, delay-dependent stability conditions are derived via the Lyapunov–Krasovskii method. The theoretical results are applied to an exemplary microgrid with distributed rotational and electronic generation and illustrated via a simulation example. The stability analysis is complemented by providing an estimate of the region of attraction of a microgrid with delays.

1. Introduction

1.1. Motivation

Time delays are a highly relevant phenomenon in many engineering applications. They appear, e.g., in networked control, sampled-data and biological systems (Fridman, 2014a). In particular, time delays may substantially deteriorate the performance of a system, e.g., with regard to stability properties of its equilibria. Therefore, it is of paramount importance in a large variety of applications to explicitly consider time delays in the system design and analysis process.

In this paper, we derive conditions for stability of a class of port-Hamiltonian (pH) systems with delays. PH systems theory provides a systematic framework for modeling and analysis of network models of a large range of physical systems and processes (van der Schaft, 2000; van der Schaft & Jeltsema, 2014). In particular, the geometric structure of a pH model underscores the importance of the energy function, the interconnection pattern and the dissipation of a system. With regard to stability analysis, the main advantage of a pH representation is that the Hamiltonian usually is a natural candidate Lyapunov function (van der Schaft, 2000). Unfortunately, in the presence of delays this does not apply in general. Yet, it seems natural to seek to construct alternative Lyapunov function candidates by using the Hamiltonian as a point of departure aiming to exploit the structural properties of pH systems.

The present work is further motivated by the problem of the effect of time delays on microgrid (μG) operation. The μG is an emerging concept for an efficient integration of renewable

☆ This work was partially supported by Israel Science Foundation (Grant No. 1128/14). The material in this paper was partially presented at the 54th IEEE Conference on Decision and Control, December 15–18, 2015, Osaka, Japan. This paper was recommended for publication in revised form by Associate Editor Shun-ichi Azuma under the direction of Editor Toshiharu Sugie.

E-mail addresses: j.schiffer@leeds.ac.uk (J. Schiffer), emilia@eng.tau.ac.il (E. Fridman), ortega@lss.supelec.fr (R. Ortega), raisch@control.tu-berlin.de (J. Raisch).

[1] Fax: +44 113 3432032.

http://dx.doi.org/10.1016/j.automatica.2016.07.022
0005-1098/

distributed generation (DG) units (Guerrero, Loh, Chandorkar, & Lee, 2013; Hatziargyriou, Asano, Iravani, & Marnay, 2007). A μG is a locally controllable subset of a larger electrical network and is composed of several DG units, storage devices and loads (Guerrero et al., 2013). A particular characteristic of a μG is that it can be operated either in grid-connected or in islanded mode, i.e., disconnected from a larger power system.

Typically, a large share of the power units in a μG are renewable and storage units connected to the network via DC/AC inverters. On the contrary, most conventional generation units are interfaced to the grid via synchronous generators (SGs). As inverters possess significantly different physical properties from SGs, many challenging problems arise in future power grids (Guerrero et al., 2013; Hatziargyriou et al., 2007). Amongst these, system stability is one of the most relevant and critical (Guerrero et al., 2013).

So far, most stability analysis of μGs has focused on purely inverter-based μGs (Münz & Metzger, 2014; Schiffer, Ortega, Astolfi, Raisch, & Sezi, 2014a; Simpson-Porco, Dörfler, & Bullo, 2013). Yet, from a practical point of view, most present and near-future applications concern μGs with a mixed generation pool consisting of SG- and inverter-interfaced units. Following Schiffer, Goldin, Raisch, and Sezi (2013), we refer to such a system as a *μ G with distributed rotational and electronic generation (MDREG).* The predominant type of conventional units in MDREGs are diesel gensets (Krishnamurthy, Jahns, & Lasseter, 2008) and, hence, we focus on these in the present work.

The most commonly employed control scheme to operate MDREGs is droop control. This is a decentralized proportional control scheme, the main objectives of which are stability and power sharing. Droop control is the standard basic control scheme for SG-based networks (Kundur, 1994) and has also been adapted to inverter-interfaced units (Guerrero et al., 2013). As shown, e.g., in Schiffer et al. (2013), droop control ensures a compatible joint operation of SG- and inverter-interfaced DG units.

In MDREGs, time delays appear due to several reasons and also in several network components. First, the power-stroke and ignition delay of a diesel engine is represented by a time delay in standard models (Guzzella & Amstutz, 1998; Kuang, Wang, & Tan, 2000; Roy, Malik, & Hope, 1991). Second, in a practical setup, the droop control scheme is applied to an inverter, respectively an SG, via digital control. Digital control usually introduces additional effects such as clock drifts (Schiffer, Ortega, Hans, & Raisch, 2015b) and time delays (Kukrer, 1996; Maksimovic & Zane, 2007; Nussbaumer, Heldwein, Gong, Round, & Kolar, 2008), which may have a deteriorating impact on the system performance. According to Nussbaumer et al. (2008), the main reasons for the appearance of time delays are sampling of control variables and calculation time of the digital controller. In the case of inverters, the generation of the pulse-width-modulation (PWM) to determine the switching signals for the inverter induces an additional delay. We refer the reader to, e.g., Nussbaumer et al. (2008) for further details. Hence, time delays are a relevant phenomenon in MDREGs, which makes it important to investigate their influence on stability. This motivates the analysis below.

1.2. About the paper

The present paper focuses on the impact of time delays on stability of MDREGs. To that end, and following Schiffer et al. (2014a) and Schiffer, Fridman, and Ortega (2015a), we represent the MDREG as a pH system with delays. Motivated by this, we derive delay-dependent conditions for stability for a class of pH systems with delays, containing the MDREG model as a special case. The stability conditions are established by following Fridman (2014b), Fridman, Dambrine, and Yeganefar (2008) and Kao and Pasumarthy (2012) and constructing a *nonlinear and non-quadratic* Lyapunov–Krasovskii functional (LKF) from the Hamiltonian and its gradient. That the LKF can be nonlinear and non-quadratic follows from the fact that both the Hamiltonian and its gradient are, in general, nonlinear functions of the system states. Compared to that, standard LMI-based approaches (Fridman, 2014a,b) rely on LKFs, which are quadratic in the state variables. The latter is, in general, very restrictive.

The main contributions of the present paper are (i) to introduce a model of a droop-controlled MDREG which explicitly considers delays of the DG unit dynamics, (ii) to represent this MDREG model as a pH system with fast- and slowly-varying delays, (iii) to provide stability conditions for a class of pH systems with fast- and slowly-varying delays via the LK method, (iv) to provide an estimate of the region of attraction of an MDREG with delays and (v) to illustrate the usefulness of our conditions on an exemplary μG. Hence, the present paper extends our previous work (Schiffer et al., 2015a) in several regards: we take diesel engines into account, provide stability conditions for slowly- and fast-varying delays and derive an estimate of the region of attraction of an MDREG with delays.

1.3. Existing literature

Stability analysis of pH systems with delays has been the subject of previous research (Aoues, Lombardi, Eberard, & Di-Loreto, 2015; Aoues, Lombardi, Eberard, & Seuret, 2014; Kao & Pasumarthy, 2012; Pasumarthy & Kao, 2009; Yang & Wang, 2010). The main motivation of that work is a scenario in which several pH systems are interconnected via feedback paths which exhibit a delay. This setup yields a closed-loop system with skew-symmetric interconnections, which can be split into non-delayed skew-symmetric and delayed skew-symmetric parts. However, the model of an MDREG with delays derived in this work is not comprised in the class of pH systems studied in Aoues et al. (2014), Kao and Pasumarthy (2012), Pasumarthy and Kao (2009) and Yang and Wang (2010), since the delays do not appear skew-symmetrically. In that regard, the class of systems considered in the present work generalizes the class studied in Aoues et al. (2014), Kao and Pasumarthy (2012), Pasumarthy and Kao (2009) and Yang and Wang (2010), see Section 3. Unlike (Aoues et al., 2014; Kao & Pasumarthy, 2012; Pasumarthy & Kao, 2009), we also provide conditions for stability in the presence of fast-varying delays, which typically arise in the context of digital control (Fridman, 2014b; Liu & Fridman, 2012). In addition, we apply the derived approach to a practically relevant application, namely an MDREG. Compared to this, in Aoues et al. (2014), Kao and Pasumarthy (2012), Pasumarthy and Kao (2009) and Yang and Wang (2010) only academic examples were considered. The effect of time delays on μG stability has only been investigated in Efimov, Ortega, and Schiffer (2015) and Efimov, Schiffer, and Ortega (2016) for a two-inverter-scenario. In particular, none of the aforementioned analyses on μG stability (Münz & Metzger, 2014; Schiffer et al., 2013, 2014a; Simpson-Porco et al., 2013) take the effect of time delays into account.

The remainder of the paper is structured as follows. A model of an MDREG with delays is derived in Section 2. In Section 3, the considered class of pH systems with delays is introduced, for which delay-dependent conditions for stability are provided in Section 4. In Section 5, the results are applied to an exemplary MDREG for which we also provide an estimate of the region of attraction. Conclusions and topics of future work are given in Section 6.

Notation. We define the sets $\bar{n} = \{1, 2, \ldots, n\}$, $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} | x \geq 0\}$, $\mathbb{R}_{>0} = \{x \in \mathbb{R} | x > 0\}$, $\mathbb{R}_{<0} = \{x \in \mathbb{R} | x < 0\}$, $\mathbb{Z}_{\geq 0} = \{0, 1, 2, \ldots\}$. For a set $\mathcal{V}$, let $|\mathcal{V}|$ denote its cardinality. For a set of, possibly unordered, positive natural numbers $\mathcal{V} = \{l, k, \ldots, n\}$, the short-hand $i \sim \mathcal{V}$ denotes $i = l, k, \ldots, n$. Let $x = \mathrm{col}(x_i) \in \mathbb{R}^n$ denote a vector with entries x_i for $i \sim \bar{n}$, $\underline{0}_n$