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Brief paper

Optimal dual adaptive agile mobile wireless power control☆

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ABSTRACT

Mobile wireless channels change persistently and rapidly, so power control needs to adapt similarly in order to save battery power and mitigate interference. Training signals in every data packet are used to achieve this in current systems, albeit in a suboptimal fashion from a power consumption perspective. This problem is used as the basis for posing and solving in detail an optimal dual adaptive control problem and then deriving heuristic controllers from this optimal solution. *Dual* refers to the joint requirements of the control signal to probe the system for parameter estimation and to regulate the total energy use; these are conflicting requirements which reveal the complexity of optimal stochastic control in general. The information state is defined, computed and explicitly incorporated into the optimization. Performance and computational load comparisons are made between: the optimal control, the certainty equivalence control, and a simplified heuristic approach. The contribution of the paper is the explicit solution of a classically hard optimal stochastic control problem to expose the role of the information state.

1. Introduction

Mobility in wireless communications causes rapid channel variation which, in turn, forces the persistent adaptation of transmission power levels on a per-packet basis. Without this adaptation, battery life is seriously compromised and interference occurs between users. In the PCS1900 standard, rapid (re)acquisition of the correct power (and equalizer) is aided by a training signal present as a *mid-amble* in the center of every packet sent from the mobile station (MS) to the base station (BS) or vice versa. OFDM based systems use pilot signals in a similar fashion. From a control systems perspective, this is an example of an adaptive control system functioning without human intervention many billion of times per hour. It is highly non-stationary and this paper seeks to explore aspects of optimal adaptive control evident from this context.

In existing applied control laws the BS estimates the received signal-to-noise ratio (SNR) and sends a one-bit power control signal to the MS to increase or decrease current power by 2 dB. We consider the situation in which the MS has knowledge of the SNR estimate and seeks to adjust its transmission power of the training signal in an optimal adaptive fashion. This brings in duality as introduced by Fel'dbaum (Fel'dbaum, 1960, 1961, 1965), since higher power facilitates accurate SNR estimation while compromising energy usage. The contributions of the paper are the study of duality in a functioning adaptive control system and the exploration of alternative approaches associated with optimality.

An Optimal Dual Adaptive Control (ODAC) is derived using the *information state* (Bayesian filter) recursion coupled with the Stochastic Dynamic Programming Equation (SDPE) as in Kumar and Varaiya (1986). We believe that this is the first formulation of a meaningful practical persistent adaptation problem in such full detail. The ODAC learning is active and the fade parameter uncertainty is managed inherently in the control. We contrast this with suboptimal controllers by comparing: ODAC; Certainty Equivalence (CE) control; probing-enhanced ODAC; and a close-to-optimal heuristic. The comparison is in terms of performance versus computational complexity, since ODAC is effectively intractable . . . but optimal.

Most other works on dualized adaptive control (DAC) are suboptimal and borrow from the inclusion of probing into the control signal without optimality. For a thorough survey of suboptimal dual adaptive control methods, the reader is referred to Filatov and Unbehauen (2004), Wittenmark (1995). There are a number of successful industrial applications of dualized adaptive controllers, Allison, Ciarniello, Tessier, and Dumont (1995), Bugeja and Fabri (2009), Ismail and Dumont (2003) and Wittenmark and Elevitch (1985). From the perspective of this paper, current

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cellular mobile wireless also falls into the category of effective but suboptimal dualized adaptive power control. Our aim in this paper is to explore optimal dual adaptive control in this context.

2. Problem formulation

We treat the problem of MS optimal adaptive transmission power control over a memoryless (i.e. flat) fading additive Gaussian white noise (AGWN) channel with perfect feedback from the BS. We presume the fade is fixed over the packet time but changes packet to packet. The dynamics of this system are given by

$$\begin{aligned} x_k &= f u_{k-1} \qquad x_0 = f, \\ y_k &= x_k + w_k. \end{aligned} \tag{1}$$

for $k = 1, 2, \dots, N$, where:

f is the unknown channel fade, which we presume to be constant over one packet transmission time.

u_{k-1} is the transmitted signal chosen by the MS for the current (kth of N) packet training symbol. Given by $u_{k-1} = p_{k-1}a_{k-1}$, where

- a_{k-1} is a binary (BPSK) training sequence, $a_k \in \{-1, 1\}$, known to both MS and BS.
- p_{k-1} is the square root of the power of the training signal, known solely to the MS.

y_k is the received signal at the BS, and

w_k is additive Gaussian white noise of known variance σ_w^2.

We impose the following assumptions.

Assumption 1. (1.A) The channel fade can take one of I distinct values, $\{f[1], f[2], \dots, f[I]\}$.

(1.B) The target signal-to-noise ratio (SNR) at the BS for message transmission is $\gamma^\star = 6.79$ dB, which results in a usable bit-error rate (BER) of 10^{-3} for BPSK (Lee & Messerschmitt, 1990).

(1.C) There are I distinct transmission signal powers corresponding to each possible fade value and $\gamma^\star$.

$$u^\star[i]^2 = p^\star[i]^2 = \frac{\sigma_w^2\gamma^\star}{f[i]^2}. \tag{2}$$

We seek to minimize the training and message power by posing the following problem.

Stochastic Optimal Control Problem

Minimize the performance index

$$J = E\left[\sum_{k=0}^{N-1} c_k(x_k, u_k) + c_N(\pi_N)\right], \tag{3}$$

over all admissible causal feedback control policies,

$$u_k = g_k(Z^k) \in \{u^\star[1], \dots, u^\star[I]\},$$

with:

- *initial probability mass function (pmf) π_0 of the fade, f, and posterior pmf of the fade at symbol time N, π_N,*
- *history*

$$Z^k = (u_0, y_1, u_1, y_2, \dots, u_{k-1}, y_k), \tag{4}$$

- *stage cost for $k = 0, \dots, N-1$,*

$$c_k(x_k, u_k) = \left(u_k^{\star 2} - \frac{\sigma_w^2\gamma^\star}{f^2}\right)^2, \tag{5}$$

- *terminal cost with $i_N^\star = \operatorname{argmax}_i \pi_N$,*

$$c_N(\pi_N) = \left(u^\star[i_N^\star]^2 - \frac{\sigma_w^2\gamma^\star}{f[i_N^\star]^2}\right)^2, \tag{6}$$

The expectation in (3) is over the $\{w_k\}$ sequence and π_0.

This is a stochastic optimal control problem for choice of training signal powers. Since it involves an unknown parameter, f, it might also be labeled *adaptive*. The solution (to be presented) of this problem involves the *information state* and Stochastic Dynamic Programming (SDP) (Bar-Shalom, 1981; Kumar & Varaiya, 1986). It achieves the optimal balance between probing and regulation to minimize J in (3). It also is extraordinarily computationally demanding. Fel'dbaum posed a problem isomorphic to this but was unable to compute the solution with the tools available in the 1960s. This formulation has perfect feedback from BS to MS, which jointly comprises the controller. In the full problem discussed in Section 6 one needs to: separate BS and MS, posit another noisy fading channel connected them, and have each solve a joint optimization problem. This is a degree of difficulty beyond the statement above.

Definition 1. The **information state** is the conditional probability vector of the state x_k at the MS given the available history $\pi_k(Z^k)$.

Lemma 1. *For the AGWN channel with constant fade described by (1), the information state updates according to*

$$\pi_{k+1}(Z^{k+1}) := T_k(\pi_k(Z^k), y_{k+1}, u_k), \tag{7}$$

$$= \frac{1}{\pi_k(Z^k)D(y_{k+1})\underline{1}}\pi_k(Z^k)D(y_{k+1}), \tag{8}$$

with: $\underline{1} = \begin{bmatrix}1 & 1 & \dots & 1\end{bmatrix}^T$, and diagonal matrix,

$$D(y_{k+1})[i] = \frac{1}{\sqrt{2\pi}\sigma_w}\exp\left[-\frac{(y_{k+1} - f[i]u_k)^2}{\sigma_w^2}\right]. \tag{9}$$

This is Bayes' Rule

$$\begin{aligned} \Pr(f = f[i]|Z^{k+1}) &= \frac{\Pr(f = f[i], y_{k+1}|Z^k)}{\Pr(y_{k+1}|Z^k)}, \\ &= \frac{\Pr(y_{k+1}|Z^k, f = f[i])\Pr(f = f[i]|Z^k)}{\Pr(y_{k+1}|Z^k)}. \end{aligned}$$

The transformation T_k in (7) will appear in the solution via stochastic dynamic programming.

2.1. Duality

Fig. 1 shows the quadratic stage cost function (5) versus control value $u[i]$, where these values are separated by 2 dBm, as is used in practical mobile wireless systems such as PCS-1900. The graph is centered on the correct value $u^\star[i]$.

Two features are immediately apparent.

(i) The power penalty for incorrect selection of power grows quadratically with distance from $u^\star$.

(ii) In the practical dBm scale, the penalty for choosing too high a power is significantly greater than that for choosing dBm-equivalent too low a power.

Thus there is a penalty for incorrect power choice and this penalty is diminished for cautious lower power selections.

By contrast, it is directly proven from (9) that

$$\mathrm{E}\left[D(y_{k+1})[i]\right] = \frac{1}{2\sqrt{\pi}\sigma_w}\exp\left(-\frac{1}{4\sigma_w^2}(f - f[i])^2u_k^2\right). \tag{10}$$