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Brief paper

Semi-global stabilization by an output feedback law from a hybrid state controller☆

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ABSTRACT

This article suggests a design method of a hybrid output feedback for SISO continuous systems. We focus on continuous systems for which there exists a hybrid state feedback law. A local hybrid stabilizability and a (global) complete uniform observability are assumed to achieve the stabilization of an equilibrium with a hybrid output feedback law. This is an existence result. Moreover, assuming the existence of a robust Lyapunov function instead of a stabilizability assumption allows to design explicitly this hybrid output feedback law. This last result is illustrated for linear systems with reset saturated controls.

1. Introduction

In recent years, many techniques for designing a stabilizing control law for nonlinear dynamical systems have been developed. It is now possible to achieve stabilization of equilibria for a large class of models. However, due to Brockett's necessary condition for stabilizability, it is well known that some systems cannot be stabilized by a continuous controller. Some of these systems can however be stabilized with a hybrid state feedback law, i.e. a discrete/continuous controller (see e.g. Prieur and Trélat (2006), where the Brockett integrator is stabilized with a quasi optimal hybrid control). Moreover, the use of hybrid control laws may be interesting to address performance issues (see e.g. Prieur, 2001). This explains the great interest of the control community in the synthesis of hybrid control laws (see Fichera, Prieur, Tarbouriech, & Zaccarian, 2013; Goebel, Sanfelice, & Teel, 2012; Hespanha, Liberzon, & Teel, 2008; Hetel, Daafouz, Tarbouriech, & Prieur, 2013; Yuan & Wu, 2014).

The output feedback stabilization problem has also attracted the attention of numerous researchers. Indeed, employing a state feedback law in most of the cases is impossible, since the sensors can only access partial measurements of the state. Output feedback laws may be designed from a separation principle. More precisely, two tools are designed separately: a stabilizing state feedback law and an asymptotic state observer. However, if this approach is fruitful for linear systems, the separation principle does not hold in general for nonlinear systems. For instance, there exist stabilizable and observable systems for which the global asymptotic stabilization by output feedback is impossible (Mazenc, Praly, & Dayawansa, 1994). Nevertheless, from weak stabilizability and observability assumptions, some semi-global results may be obtained (see e.g. Teel & Praly, 1994 or Isidori, 1995, Pages 125–172). However, in this case the observer and the state feedback have to be jointly designed (not separately) (see also Andrieu and Praly (2009) for some global results).

The aim of this paper is to address the stabilization by hybrid output feedback law. In Teel (2010), a local separation principle is stated. However, the construction of the observer is not explicit. Here, from a hybrid state feedback controller and an observability property, an algorithm is provided to build hybrid output feedback laws which stabilize semi-globally the equilibrium plant. If moreover a robust Lyapunov function is known, the feedback law design becomes explicit.

This article is organized as follows. Section 2 introduces the problem together with a hybrid stabilizability and an observability

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assumption. The main result is given in Section 3.1. An equivalent stabilizability assumption in terms of Lyapunov function is considered in Section 3.2. This allows to give a more explicit theorem. Section 4 explains how to prove the first theorem from the second theorem. In Section 5 technical lemmas are stated in order to construct the suggested output feedback law. An illustrative example is given in Section 6. Finally, Section 7 collects some concluding remarks.

Note that this paper is an extension of the conference paper Marx, Andrieu, and Prieur (2014). It includes a missing part (the observer design), proofs and a new illustration.

Notation: Given $\lambda \in \mathbb{N}$, $\mathbb{R}_{\geq\lambda} = [\lambda, +\infty)$. Given $n \in \mathbb{N}$, $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix, i.e. $I_n = \mathtt{diag}(1, \dots, 1)$. $\star$ states for symmetric terms. Given $n \in \mathbb{N}$, $\mathcal{L}^\infty_{loc}(\mathbb{R}, \mathbb{R}^n)$ denotes the set of measurable locally bounded functions $u : \mathbb{R} \to \mathbb{R}^n$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class-$\mathcal{K}$ (for short $\alpha \in \mathcal{K}$) if it is continuous, zero at zero, and strictly increasing. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class-$\mathcal{KL}$ (for short $\beta \in \mathcal{KL}$) if it satisfies (i) for each $t \geq 0$, $\beta(., t)$ is nondecreasing and $\lim_{t\searrow 0} \beta(s, t) = 0$, and (ii) for each $s \geq 0$, $\beta(s, .)$ is nonincreasing and $\lim_{t\to\infty} \beta(s, t) = 0$.

2. Problem statement

2.1. Hybrid state feedback law for a continuous time plant

The system under consideration is described by the following single-input single-output continuous dynamics:

$$\dot{x}_p = f_p(x_p) + g_p(x_p)u, \; y = h_p(x_p), \tag{1}$$

where $x_p \in \mathbb{R}^{n_p}$, $y \in \mathbb{R}$, $u \in \mathcal{U} \subset \mathbb{R}$. Note that $f_p : \mathbb{R}^{n_p} \to \mathbb{R}^{n_p}$ and $g_p : \mathbb{R}^{n_p} \to \mathbb{R}^{n_p}$, $h_p : \mathbb{R}^{n_p} \to \mathbb{R}$ are $n_p + 1$ times continuously differentiable.[1] $\mathcal{U}$ can be bounded (it yields a saturated control problem). Inspired by Prieur and Trélat (2006) and Sontag (1999), the origin, which is an equilibrium point for (1), is assumed to be stabilizable by a hybrid state feedback.

Assumption 1 (*Persistent Flow Stabilizability*)**.** There exists a hybrid controller defined by $(\mathcal{F}_c, \mathcal{J}_c, f_c, g_c, \theta_c)$, where $\mathcal{F}_c$ and $\mathcal{J}_c$ are closed sets, $\mathcal{F}_c \cup \mathcal{J}_c = \mathbb{R}^{n_p+n_c}$, $g_c : \mathbb{R}^{n_p+n_c} \to \mathbb{R}^{n_c}$, $f_c : \mathbb{R}^{n_p+n_c} \to \mathbb{R}^{n_c}$ and $\theta_c : \mathbb{R}^{n_p+n_c} \to \mathcal{U}$ are continuous functions and a positive value λ in (0, 1) such that the set $\{0\} \times [0, 1]$ in $\mathbb{R}^{n_p+n_c} \times \mathbb{R}$ is asymptotically stable for the system:

$$\begin{cases} \dot{x}_p = f_p(x_p) + g_p(x_p)\theta_c(x_p, x_c) \\ \dot{x}_c = f_c(x_p, x_c) \\ \dot{\sigma} = 1 - \sigma \end{cases} \quad (x_p, x_c, \sigma) \in \mathcal{F}_c \times \mathbb{R}_{\geq 0} \tag{2a}$$

$$\begin{cases} x_p^+ = x_p \\ x_c^+ = g_c(x_p, x_c) \\ \sigma^+ = 0 \end{cases} \quad (x_p, x_c, \sigma) \in \mathcal{J}_c \times \mathbb{R}_{\geq\lambda} \tag{2b}$$

with basin of attraction $\mathcal{B} \times \mathbb{R}_{\geq 0}$, where $\mathcal{B}$ is an open subset of $\mathbb{R}^{n_p+n_c}$.

The sets $\mathcal{F}_c \times \mathbb{R}_{\geq 0}$ and $\mathcal{J}_c \times \mathbb{R}_{\geq\lambda}$ are called respectively the flow and jump sets associated to the continuous and discrete dynamics. The notion of solutions and of asymptotic stability discussed all along the paper are borrowed from Goebel et al. (2012).

[1] These mappings are sufficiently smooth so that the mapping ϕ defined in (3) is C^1 and so that the function B defined in (12) is locally Lipschitz.

Remark 1. An important feature of the hybrid state feedback control law is that its dynamics include a timer σ. It implies that there exists a dwell time between two consecutive jumps and consequently it prevents the existence of Zeno solutions. In the case in which this property is not satisfied for the state feedback, a timer can be added as presented in Cai, Teel, and Goebel (2008, Part V, C.). Such a technique is called a *temporal regularization*. However, in this case, only semi-global practical stability is obtained. ○

The problem under consideration in this paper is to design a stabilizing output feedback law based on this hybrid state feedback. The design presented in this paper requires an observability property for system (1) as described in the following section.

2.2. Observability notions

Following Gauthier, Hammouri, and Othman (1992), define the C^1 mapping $\phi : \mathbb{R}^{n_p} \to \mathbb{R}^{n_p}$ as follows

$$\phi(x_p) = \begin{bmatrix} h_p(x_p) & L_{f_p}h_p(x_p) & \dots & L_{f_p}^{n_p-1}h_p(x_p) \end{bmatrix}^\top, \tag{3}$$

where $L_{f_p}^i h_p(x)$ denotes the ith Lie derivative of h_p along f_p. The observability assumption employed all along the paper can be now stated.

Assumption 2 (*(Global) Complete Uniform Observability (Gauthier et al., 1992)*)**.** System (1) is completely uniformly observable, that is

(i) The mapping $\phi : \mathbb{R}^{n_p} \to \phi(\mathbb{R}^{n_p}) = \mathbb{R}^{n_p}$ is a diffeomorphism;
(ii) System (1) is observable for any input $u(t)$, i.e. on any finite time interval $[0, T]$, for any measurable bounded input $u(t)$ defined on $[0, T]$, the initial state is uniquely determined on the basis of the output $y(t)$ and the input $u(t)$.

Remark 2. In Marx et al. (2014), from a weaker observability assumption, i.e. an observability property holding for just one control, a finite-time convergent observer and a hybrid state feedback controller have been used to design an output feedback law. Such a strategy does not need a persistent flow stabilizability assumption. However only a weak stability property is obtained for the closed-loop system. ○

3. Semi-global output feedback result

3.1. First main result

Inspired by the approach of Teel and Praly (1994), from Assumptions 1 and 2, a semi-global output feedback result may be obtained.

Theorem 1 (*Semi-global Asymptotic Stability*)**.** *Assume Assumptions 1 and 2 hold. Assume moreover that g_c satisfies that, for all (x_p, x_c) in $\mathcal{B} \cap \mathcal{J}_c$, the set*

$$\{(x_p, g_c(w, x_c)), w \in \mathbb{R}^{n_p}\} \tag{4}$$

is a compact subset of $\mathcal{B}$, then the origin of system (1) is semi-globally asymptotically stabilizable by a hybrid output feedback. In other words, for all compact sets Γ contained in $\mathcal{B}^p := \{x_p \in \mathbb{R}^{n_p}, (x_p, 0) \in \mathcal{B}\}$, there exist a C^1 function $\Psi_p : \mathbb{R}^{n_p} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and a positive real number c_x such that the set $\{0\} \times [0, 1]$ in $\mathbb{R}^{2n_p+n_c} \times [0, 1]$ is asymptotically stable for the system

$$\begin{cases} \dot{x}_p = f_p(x_p) + g_p(x_p)u \\ \dot{\hat{x}}_p = \Psi_p(\hat{x}_p, y, u) \\ \dot{x}_c = f_c(\tilde{x}_p, x_c) \\ \dot{\sigma} = 1 - \sigma \\ y = h_p(x_p), \; u = \theta_c(\tilde{x}_p, x_c) \end{cases} \tag{5a}$$