Automatica 74 (2016) 99–106

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Hierarchical Model Predictive Control of independent systems with joint constraints☆

Bruno Picasso, Xinglong Zhang, Riccardo Scattolini

Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, via Ponzio 34/5, 20133 Milan, Italy

ARTICLE INFO

Article history:
Received 14 November 2015
Received in revised form
30 May 2016
Accepted 20 June 2016

Keywords:
Hierarchical control
Model predictive control
Model reduction

ABSTRACT

This paper describes a hierarchical scheme for the control of independent stable systems subject to joint constraints. At the higher layer of the control structure reduced order dynamic models are used to minimize an economic cost function by adopting a long sampling time, while at the lower layer independent shrinking horizon MPC controllers working at a faster rate are designed for the original models to guarantee stability and convergence. A novel model reduction procedure is developed and simulation results are reported to witness the potentialities of the approach.

1. Introduction

Hierarchical control structures made by regulators working at different time scales are widely used in the process industry to cope with many significant problems, see the review Scattolini (2009) and the papers listed there. For instance, in the case of singularly perturbed systems, i.e. systems characterized by separable slow and fast dynamics, low level "fast" feedback controllers are designed to stabilize the fast dynamics, while at the higher level of the control structure a regulator working at lower frequency is in charge of stabilizing the slow dynamics and satisfy performance requirement, see e.g. Brdys, Grochowski, Gminski, Konarczak, and DrewaR (2008) and Van Henten and Bontsema (2009) for a couple of industrial examples. An in-depth theoretical analysis of multilayer structures for singularly perturbed systems made by Model Predictive Controllers (MPC) has been recently reported in Chen, Heidarinejad, Liu, and Christofides (2012); Chen, Heidarinejad, Liu, Munoz de la Pena, and Christofides (2011).

In a different setting, two-layer architectures are used for economic optimization. At the higher level, usually named Real Time Optimization (RTO) layer, the optimal working conditions are periodically recomputed to maximize profits and/or minimize costs, see e.g. Seborg, Mellichamp, Edgar, and Doyle (2010). In RTO, static models are often used in view of the implicit assumption that, in the periods between successive optimizations, the system reaches its steady-state conditions. Given the reference values of the plant variables computed with RTO, at the lower layer MPC is applied in view of its stabilizing properties and its ability to explicitly deal with constraints on the state, input, and output variables, see Rawlings and Mayne (2009). Many methods have been recently developed to merge the two layers, see e.g. Adetola and Guay (2010), Kadam and Marquardt (2007) and Würth, Hannemann, and Marquardt (2009), or to derive stabilizing MPC algorithms minimizing an economic cost according to the so called economic MPC (or EMPC), see e.g. Amrit, Rawlings, and Angeli (2011), Chen, Heidarinejad, Liu, and Christofides (2012), Diehl, Amrit, and Rawlings (2011) and Grüne (2013). However, as it has been well recognized in Ellis and Christofides (2014), in EMPC a sufficiently large prediction horizon must often be used to consider the long-term performance of the system, so that the resulting optimization problem can be difficult to solve in real-time.

In this setting, a significant problem concerns the design of hierarchical control systems for the coordination and control of independent subsystems which must cooperate to achieve prescribed performance. As a first example, consider the problem of controlling micro-grids made by independent components, such as batteries, gas-turbines, photovoltaic panels, wind generators, and loads. For these systems a high level MPC, working at a slow timescale, typically fifteen minutes, and relying on simplified models of the system components computes the nominal operating conditions guaranteeing that the overall energy balance of the grid is satisfied and optimized according to an economic performance index. At low level MPC controllers act at higher frequency, typically one

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor Ian R. Petersen.

E-mail addresses: picasso.bruno@gmail.com (B. Picasso), xinglong.zhang@polimi.it (X. Zhang), riccardo.scattolini@polimi.it (R. Scattolini).

http://dx.doi.org/10.1016/j.automatica.2016.07.030
0005-1098/

minute, and adjust the micro-grid operation to reduce the effect of disturbances or unmodeled dynamics, Parisio, Rikos, and Glielmo (2014) and Raimondi Cominesi, Farina, Giulioni, Picasso, and Scattolini (2015). Conceptually similar problems arise in many different engineering fields, such as in the control of the temperature in a building when the available thermal power generators must be coordinated according to an economic criterion, see for instance the problem considered in Morosan, Bourdais, Dumur, and Buisson (2011), or in industrial applications with many generation units, see for example the problem considered in Martì, Sarabia, and de Prada (2014) where two oxygen generators must feed three consumer units.

Motivated by these examples, in this paper we develop a hierarchical control structure for the coordination of independent linear dynamic systems with input and joint output constraints. At the higher layer, a "long" sampling time compatible with the prediction horizon required for economic optimization is adopted and reduced order dynamic models of the system's components are used to state and solve an EMPC algorithm guaranteeing feasibility and convergence. The outcomes of this layer are the components of the control variables to be held constant over the long sampling periods. At the lower layer, decentralized MPC controllers, one for each subsystem, are implemented in the "short" time scale and according to a shrinking horizon strategy to compensate for the model inaccuracies at the high level and to guarantee the overall stability, convergence, and the fulfillment of the joint constraints. In order to derive the main results, a novel model reduction procedure is proposed.

The paper is organized as follows: in Section 2 the original models of the subsystems are introduced together with their reduced order representation, and the MPC problems at the higher and lower levels are formulated. Section 3 describes the model reduction procedure, while in Section 4 the feasibility and convergence properties of the scheme are proven. An example is described in Section 5, while some conclusions and hints for future work are discussed in Section 6. For readability, the proofs of the main results are reported in the Appendix.

2. The two-layer control structure

The overall system under control is composed by M independent, discrete-time, linear dynamical systems described by

$$\Sigma_i : \begin{cases} x_i(h+1) = A_L^i x_i(h) + B_L^i u_i(h) \\ y_i(h) = C_L^i x_i(h), \end{cases} \quad i = 1, 2, \ldots, M, \tag{1}$$

where h is the time index of the base (fast) time scale, $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathscr{U}_i \subseteq \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{p_i}$. The following assumption holds.

Assumption 1. For any $i = 1, 2, \ldots, M$:

(i) the state x_i is measurable;
(ii) A_L^i is Schur stable;
(iii) B_L^i and C_L^i are full rank;
(iv) the pair (A_L^i, B_L^i) is reachable.

For the overall system $\Sigma = (\Sigma_1, \ldots, \Sigma_M)$ the goal is to design a controller such that, letting $y = [\, y_1' \; y_2' \; \cdots \; y_M' \,]' \in \mathbb{R}^p$ be the collective output, for a given $N \in \mathbb{N}$, $N > 1$, and for a given reference signal $y^o(k) \in \mathbb{R}^{\bar{p}}$, $k \in \mathbb{N}$ being the time index of the slow time scale, the following joint output constraint is satisfied:

$$\forall\, k \geq 1, \quad \mu\big(y(kN), y^o(k)\big) \geq 0, \tag{2}$$

for some specified function μ.

The stated control problem can be trivially solved by considering the overall system Σ as a whole and by resorting to well known (stabilizing) MPC for systems subject to constraints.

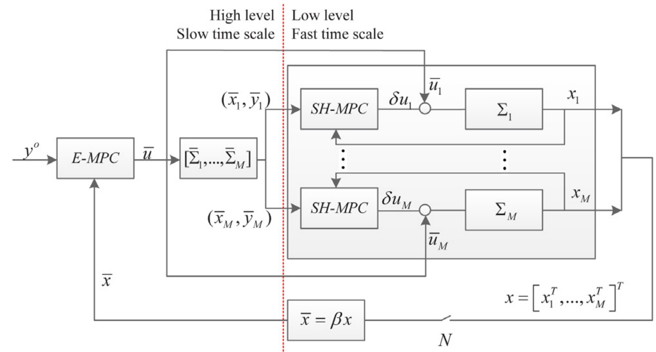

Fig. 1. Hierarchical control scheme. E-MPC = Economic MPC, SH-MPC = Shrinking Horizon MPC.

However, in so doing, the centralized solution of the resulting problem could be hampered by various factors, such as the high dimension of $\sum$ and/or the need to consider its long-term behavior, with the consequent need to use long prediction horizons, see Ellis and Christofides (2014). For this reason, the two-layer control scheme described in the following and depicted in Fig. 1 is proposed. The hierarchical structure is such that:

- at the higher layer an MPC controller, running at a slow time scale, i.e. every N time steps of the fast time scale, is designed for an overall low-order centralized model made by the ensemble of reduced order models of the Σ_i's. This controller must guarantee some fundamental properties, such as convergence in the long time scale and the fulfillment of the joint constraint (2), and can be designed according to economic criteria, see for example Amrit et al. (2011), Diehl et al. (2011) and Grüne (2013).
- at the lower layer a set of M decentralized MPC controllers, each one designed for the full model (1) of the corresponding subsystem Σ_i, are implemented in the fast time scale according to a shrinking horizon strategy. The low-level controllers are in charge of adjusting the nominal input computed by the high-level controller so as to compensate for unmodeled dynamics and to guarantee stability and performance.

Remark 1. A similar solution for integrating economic optimization and MPC by means of a hierarchical control scheme has been proposed in Ellis and Christofides (2014). However, in our approach the higher layer directly computes the nominal control action to be held constant over the long sampling time, and the lower layer can correct it at a higher frequency to compensate for model inaccuracy. On the contrary, in Ellis and Christofides (2014) the higher layer computes the reference signals for the systems (1) locally controlled at the lower layer. As an additional difference, in our scheme the higher layer relies on simplified models of the systems, so reducing the size of the control problem to be solved, while in Ellis and Christofides (2014) the full models (1) are used.

Remark 2. When the reference signal y^o is constant and the problem is feasible at the initial time instant $h = 0$, the recursive feasibility at the higher layer is guaranteed by the properties of the selected economic MPC algorithm, while at the lower layer it is proven in the following Section 4.1. In other cases, if at $h = 0$ the problem at the higher layer is infeasible due to the control constraints and to the joint output constraint (2), or the reference signal is time varying, a typical solution is to transform (2) into soft constraints by introducing slack variables, see for instance Maciejowski (2001); this solution is the one adopted in the following Section 5. Another possible solution consists in considering also $y^o(k)$ as an optimization variable and computing it as the feasible signal nearest to the ideal one, according to an approach already used in Betti, Farina, and Scattolini (2013) and Limon, Alvarado, Alamo, and Chamacho (2008).