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Brief paper

Nodes selection strategy in cooperative tracking problem[☆]

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ABSTRACT

In this paper, a new optimization problem is addressed for node selection that has application potentials in input/output switches for sensors in control system design and leader determination in social networks. The purpose of the addressed problem is to develop a strategy for selecting a subset of nodes as controlled nodes in order to minimize certain objective function consisting of the convergence speed and the energy of control action, over a finite time-horizon. For networks with fixed controlled nodes, an upper bound of the objective function is obtained which is shown to be convex and independent of the time-horizon. For networks with switched controlled nodes, a greedy algorithm is proposed to reduce the computation complexity resulting from the length of the time-horizon, where the nodes selection is carried out over divided small time-intervals. The cost gap is also analyzed between the strategy of optimizing over the whole time-horizon and the strategy of optimizing over the small intervals. Finally, the proposed nodes selection strategy is validated through simulations and two regions are found in which the number of optimal controlled nodes is determined.

1. Introduction

Consensus-based dynamical networks have proven to be an effective yet flexible framework for modeling the multi-agent information sharing problems involving cooperative tasks with examples including wireless sensor navigation, spacecraft formation control, mobile robot rendezvous and unmanned aerial vehicle flocking. The last decade has witnessed surged research interest on the consensus problem and a rich body of results has been reported in the literature, see e.g. Cao, Morse, and Anderson (2008), Fax and Murray (2004), He, Cheng, Shi, Chen, and Sun (2014), Jadbabaie, Lin, and Morse (2003), Ren and Beard (2005), Xiao, Boyd, and Kim (2007), Xiao and Wang (2008), Yang, Wang, and Shi (2013) and Zuo (2015). In recent years, particular research efforts have been devoted to the controllability properties of the underlying interaction network in order to provide a systematic way for characterizing/designing useful interaction models, see Commault and Dion (2013), Egerstedt, Martini, Cao, and Camlibel (2012), Jafari, Ajorlou, and Aghdam (2011), Ji, Wang, Lin, and Wang (2009), Parlangeli and Notarstefano (2012), Porfiri and Bernardo (2008), Rahmani, Ji, Mesbahi, and Egerstedt (2009) and Yoon and Tsumura (2011) for some representative results. For example, Rahmani et al. (2009) have shown how the symmetry structure of the network, characterized in terms of its automorphism group, is directly related to the controllability of the corresponding multi-agent system. In Egerstedt et al. (2012), Egerstedt et al. have discussed the relationship between the network structure and the controllability properties in single-leader consensus networks, and have also summarized some recent problems/results appearing in the past five years.

In parallel with the controllability problem of multi-agent systems, the nodes selection problem has been a research focus over the past few years due to the fact that a suitable selection of the nodes would have a major impact on the effectiveness of adopting control technologies in real-world applications. For example, in the area of control system design, it is vitally important to select appropriate input/output sensors so as to maintain the desired control performances. In social networks, needless to say, selecting

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the right leaders is the key to ensure efficient spreading of the information to the whole network. The nodes selection problem has been recently studied for leader–follower systems and many results have appeared, see Clark, Alomair, Bushnell, and Poovendran (2014), Clark, Bushnell, and Poovendran (2014), Kawashima and Egerstedt (2012), Lin, Fardad, and Jovanovic (2014), Patterson and Bamieh (2010) and Porfiri and Florilli (2009). Specifically, Clark, Alomair et al. (2014) have studied leader selection in order to minimize convergence errors experienced by the follower agents, and Lin et al. (2014) have looked into the leaders selection problem by minimizing the mean-square deviation from consensus in stochastically forced networks.

In this paper, we consider a class of cooperative tracking problem (Chen, Chen, & Yuan, 2009; Li, Wang, & Chen, 2004; Zhou, Lu, & Lü, 2008) with aim to select a subset of nodes (referred to as the controlled nodes) to be injected into the control inputs so as to drive the remaining nodes to reach the desired consensus. Our attention is focused on how to select the controlled nodes in order to optimize some network performances. It is worth mentioning that the controlled nodes in the proposed system are affected by their adjacent nodes while the states of the leader nodes in the leader–follower system are fixed at the beginning. In most of the existing literature, the network performance index has been either the convergence speed or the network robustness against noises. Different from the existing literature, we propose a new network performance index described by a quadratic function that takes *both* the convergence speed *and* the energy of the control actions. Such a new index is motivated by the classical *LQR* problem that minimizes a cost function accounting for the trade-off between the undesired deviation and the energy incurred by the control action. In addition, we set a constraint on the number of the controlled nodes in order to take the resource limitation into consideration. Accordingly, we formulate the nodes selection problem as an optimization one that seeks a set of binary values indicating whether a node is selected to be the controlled node. In the case of selecting fixed nodes, we aim to select a set of nodes by minimizing the network performance index with an inequality constraint of the number of selected nodes. In the case of selecting switched nodes, we aim to select a set of nodes at each time step by minimizing the network performance index with the nodes number constraint at each time step and the constraint of using frequency of each node.

It is widely recognized that the main challenge in selecting nodes stems from the Boolean constraints which give rise to a combinatorial optimization problem. In this paper, we introduce an iterative method to relax the Boolean constraints to a convex hull by replacing the l_0 norm with the l_1 norm. The main contributions of this paper are highlighted as follows. (1) In the case when the controlled nodes are fixed, an upper bound of the objective function is derived which is shown to be independent of the time-horizon, and the addressed nodes selection problem is converted into a convex optimization one that can be readily solved by some standard methods such as the interior point algorithm. (2) In the case when the controlled nodes switch, a greedy algorithm is exploited to solve the nodes selection problem over many small time-intervals, thereby significantly reducing the computation complexity for large-scale networks over lengthy time-horizon. The relationships among the network parameters are discussed and the cost gap of the objective function is also examined between the original optimization strategy and the transformed optimization strategy with the greedy algorithm. (3) By simulations, a tradeoff is found to exist between the convergence speed and the energy of control actions, where the number of optimally controlled nodes is determined under certain conditions.

Notations: $\mathbb{R}^n$ is the n-dimensional Euclidean space. When the matrix X is positive semi-definite (positive definite), it is denoted as $X \succeq 0$ ($X \succ 0$). $\mathrm{tr}(\cdot)$ is the trace of a matrix. I_n is the n-dimensional identity matrix. $\mathbf{1}_n$ is the vector with all components being 1. $\rho(\cdot)$ is the spectral radius of a matrix. $\mathrm{diag}(X)$ denotes the diagonal matrix with it diagonal blocks being X.

2. Problem formulation

Consider a network described as an undirected graph $G = (V, E)$ with $V = (1, 2, \ldots, n)$ being the set of n nodes and the edges $E \subset V \times V$ representing the communication links. Denote the set of neighbors of node i by $N_i = \{j : (i, j) \in E\}$. Each node can exchange information with its neighbors. The interconnection topology of the network is described by a weighted matrix $W = [w_{ij}]$, where $w_{ii} = 1 - \sum_{j \in N_i} w_{ij}$ and $0 < w_{ij} < 1$ if $(i, j) \in E$; otherwise, $w_{ij} = 0$. Here, we assume that the network G is connected, and W is thus an irreducible non-negative matrix.

In the network, each node updates its state as

$$x_i(k+1) = \sum_{j=1}^{n} w_{ij} \cdot x_j(k) + l \cdot \gamma_i(k) \cdot u_i(k) \tag{1}$$

where

$$u_i(k) = c - x_i(k). \tag{2}$$

Here, $x_i(k) \in \mathbb{R}^m$ is the state of the ith node at the time step k, $c \in \mathbb{R}^m$ is the desired state. Note that $0 < l < \min_i\{w_{ii}\}$, $1 \le i \le n$ is a constant gain. If the ith node is injected into the control input at time step k, then $\gamma_i(k) = 1$, which is referred to as the controlled node; otherwise, $\gamma_i(k) = 0$. In the case of selecting fixed nodes, the controlled nodes are fixed, *i.e.*, $\gamma_i(k)$ is constant for all $k = 1, 2, \ldots$. In the case of selecting switched nodes, the controlled node switches at each time step k. To simplify the addressed problem, we set $m = 1$. However, all the results can be extended to the case $m > 1$ by using the Kronecker product.

By collecting all the states of the nodes, we define $x(k) \triangleq [x_1(k), \ldots, x_n(k)]'$, $u(k) \triangleq [u_1(k), \ldots, u_n(k)]'$, $\Gamma(k) \triangleq \mathrm{diag}(\gamma_1(k), \ldots, \gamma_n(k))$. The single node dynamics in conjunction with the control inputs (2) can be represented as the following vector form

$$x(k+1) = \hat{W}(k)x(k) + cl \cdot \Gamma(k)\mathbf{1}_n, \tag{3}$$

where $\hat{W}(k) = W - l\Gamma(k)$.

In this paper, we are interested in designing an algorithm to determine $\gamma_i(k)$ so that the networked system (3) achieves the desired agreement with satisfactory performance. First, we examine whether all the nodes converge to the desired state c by adding the control inputs on a part of nodes.

It is worth mentioning that the reachability problem of the cooperative tracking protocol have been studied extensively. The following result follows from the existing results in Clark, Alomair et al. (2014), Lin et al. (2014) and Rahmani et al. (2009).

Proposition 1. *Consider an undirected network consisting of n nodes with dynamics (1). In the case of selecting fixed nodes, the states of all the nodes converge to the desired state if an arbitrary node is selected as the controlled node. In the case of selecting switched nodes, the states of all the nodes converge to the desired state if an arbitrary node is selected as the controlled node at each time step.*

Proof. Under the assumption that G is connected, W is nonnegative and irreducible. By Proposition 4.1 in Rahmani et al. (2009), it is easy to show that the dynamic system (1) achieves agreement asymptotically in both cases of fixed and switched nodes. ■

Remark 1. Note that the reachability of the cooperative tracking protocol (3) is independent of the number of the controlled nodes. Some existing works have shown that the number of the controlled nodes influences the convergence speed.