Automatica 74 (2016) 222–229

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Energy shaping for position and speed control of a wheeled inverted pendulum in reduced space[☆]

Sergio Delgado[a], Paul Kotyczka[b,1]

[a] *Institute of Automatic Control (Prof. Dr.-Ing. B. Lohmann), Technische Universität München, Boltzmannstr. 15, 85748 Garching, Germany*

[b] *Laboratoire d'Automatique et de Génie des Procédés (LAGEP), Université Claude Bernard Lyon 1, 43 Boulevard du 11 Novembre 1918, 69622 Villeurbanne Cedex, France*

ARTICLE INFO

Article history:
Received 5 March 2015
Received in revised form
28 June 2016
Accepted 18 July 2016

Keywords:
Passivity-based control
Underactuated mechanical systems
Nonholonomic systems

ABSTRACT

The paper deals with the energy-based stabilization and speed control of a wheeled inverted pendulum, which is an underactuated, unstable mechanical system subject to nonholonomic constraints. We use the method of Controlled Lagrangians for the stabilization of an equilibrium characterized by the length of the driven path, the orientation, and the pitch angle. The approach is systematic and very intuitive, for it is physically motivated. Based on the stabilization results, we design a speed control law. After the presentation of the model under nonholonomic constraints in Lagrangian representation, we provide an elegant solution to the matching equations for kinetic and potential energy shaping for the considered system. Simulations show the applicability of the method, and the comparison with a linear controller emphasizes its performance.

1. Introduction

The *wheeled inverted pendulum* (WIP) – and its well-known commercial version, the *Segway* (Segway, 2016) – has gained interest for human assistance or transportation in the past several years due to its high maneuverability and simple construction (Li, Yang, & Fan, 2013). A WIP – shown from the side in Fig. 1 (left) – consists of a vertical body with two coaxial driven wheels mounted on the body. The actuation of both wheels in the same direction generates a forward (or backward) motion; opposite wheel velocities lead to a turning motion around the vertical axis. Mobile robotic systems based on the WIP, like the intelligent two wheeled road vehicle *B2* presented in Baloh and Parent (2003), or the novel and more car-like *Segway PUMA* and *Chevrolet En-V*, are being developed to be used as new personal urban transportation systems (General Motors, 2010; PUMA, 2016). Some institutes have also developed their own WIPs for research purposes, e.g., *JOE* (Grasser, D'Arrigo, Colombi, & Rufer, 2002) and *InPeRo* (Nasrallah, Michalska, & Angeles, 2007), to give only two examples. These systems can be further used as service robots like *KOBOKER* (Lee & Jung, 2011) or moving information platforms like the Ballbot *mObi* (mObi, 2016).

The stabilization and tracking control for the WIP is challenging: The system is underactuated, the upward position of the body represents an unstable equilibrium that needs to be stabilized by feedback, and, in addition, the system motion is restricted by nonholonomic (non-integrable) constraints (Bloch, 2003) and is, thus, not smoothly stabilizable at a point, as proven by Brockett (Brockett, 1983). Many researchers around the world have put great effort into designing stabilization and tracking control laws for the WIP, particularly using linearized models (Grasser et al., 2002; Li et al., 2013; Muralidharan & Mahindrakar, 2014). During the last decade, however, a strong focus is set on the nonlinear model and nonlinear control laws (Kausar, Stol, & Patel, 2012; Muralidharan, Ravichandran, & Mahindrakar, 2009; Nasrallah et al., 2007; Pathak, Franch, & Agrawal, 2005). For a very complete overview of the existing work on modeling and control of WIPs until 2012, the reader is referred to Chan, Stol, and Halkyard (2013).

Existing methods often do not exploit the mechanical structure of the system, feature a cumbersome design procedure, or lack robustness due to a partial feedback linearization. A solution can

☆ The material in this paper was partially presented at the 5th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non Linear Control (LHMNLC 2015), July 4–7, 2015, Lyon, France. This paper was recommended for publication in revised form by Associate Editor Jun-ichi Imura under the direction of Editor Toshiharu Sugie.

E-mail addresses: s.delgado@tum.de (S. Delgado), kotyczka@tum.de (P. Kotyczka).

1 The second author is on leave of absence from Institute of Automatic Control (TUM) with a European Commission Marie-Skłodowska-Curie Fellowship (Project reference 655204, EasyEBC).

http://dx.doi.org/10.1016/j.automatica.2016.07.045
0005-1098/

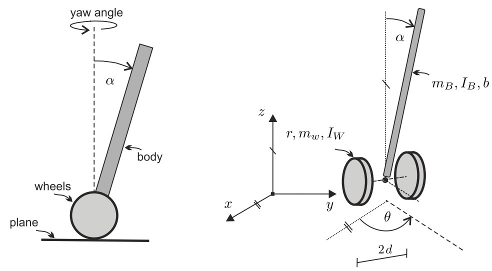

Fig. 1. The wheeled inverted pendulum.

be provided by energy shaping methods. These control techniques, like the method of Controlled Lagrangians, or Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC), have been successfully used for the stabilization of underactuated mechanical systems (Chang, Bloch, Leonard, Marsden, & Woolsey, 2002; Ortega, Spong, Gómez-Estern, & Blankenstein, 2002) as well as the speed control for electromechanical systems (Ortega, Loria, Nicklasson, & Sira-Ramirez, 1998). These methods are attractive, since they shape the energy of the system but preserve its physical structure and, thus, appear *natural*. The idea of shaping the energy has been also extended to the stabilization of nonholonomic mechanical systems (Blankenstein, 2002; Maschke & van der Schaft, 1994). However, for the *asymptotic* stabilization of a desired *configuration*, a discontinuous or time-varying control law is required (Astolfi, 1996; Brockett, 1983). This can be achieved via energy shaping by assigning non-smooth potential functions (Fujimoto, Sakai, & Sugie, 2012).

In this paper, which generalizes and completes the results of the conference paper (Delgado & Kotyczka, 2015), we present a feasible and elegant solution of the energy shaping problem for position and speed control of the WIP. Instead of considering the six-dimensional manifold $\tilde{\mathcal{Q}}$, which represents the configuration space of the WIP, we restrict our analysis to a lower dimensional space $\mathcal{Q}$, on which the system evolves *unconstrained*. Based on the WIP's nonlinear model, we design a passivity-based stabilizing and speed controller (for constant speed references) in the reduced space $\mathcal{Q}$. Since the closed-loop mechanical-type energy is used as Lyapunov function, the framework is remarkably intuitive, for it is physically motivated. The controller is thereafter parametrized applying local linear dynamics assignment (LLDA), a method used to fix design parameters in nonlinear passivity-based control by making use of the linearized model (Kotyczka, 2013). Note that the simplicity of controller design, in turn, requires appropriate planning of the trajectories in the reduced space $\mathcal{Q}$. The applicability and performance of the developed controllers is shown with a series of simulations. To sum it up, the novelty of the paper is the systematic and integrated design of a position and speed controller for the wheeled inverted pendulum system in a single, energy-based framework. An emphasis is put on the structural advantages of the approach.

Convention: For compactness, we will use the notation $\mathrm{s}(\alpha) = \sin\alpha$, and $\mathrm{c}(\alpha) = \cos\alpha$. When obvious from the context, arguments are omitted for simplicity.

2. Modeling

In a *simple* mechanical system with k nonholonomic constraints, the n-dimensional manifold $\tilde{\mathcal{Q}}$ is the configuration space, its tangent bundle $T\tilde{\mathcal{Q}}$ is the velocity phase space and a smooth non-integrable distribution $\mathcal{D} \subset T\tilde{\mathcal{Q}}$ characterizes the constraints. The Lagrangian L is a map $L : T\tilde{\mathcal{Q}} \to \mathbb{R}$ and is defined as the kinetic energy minus the potential energy $L = K - V$. A curve $q(t)$ is said to satisfy the constraints if $\dot{q}(t) \in \mathcal{D}_q$, for all $q \in \tilde{\mathcal{Q}}$ and all times t. The constraint distribution $\mathcal{D}$ is assumed to be regular, i.e., of constant rank $n - k$.

2.1. The Lagrange–d'Alembert equations

The widely used Lagrange–d'Alembert equations (Bloch, 2003; Delgado & Kotyczka, 2015; Pathak et al., 2005)

$$\frac{d}{dt}(\nabla_{\dot{q}} L) - \nabla_q L = A(q)\lambda + \sum F_{ext} \tag{1}$$

describe the dynamics of systems subject to k nonholonomic (Pfaffian) constraints of the form

$$A^T(q)\dot{q} = 0, \quad A \in \mathbb{R}^{n\times k}. \tag{2}$$

Assuming there are no external forces/torques other than the input torques $\tilde{\tau}$, the equations of motion (1) have the form

$$\tilde{M}(q)\ddot{q} + \tilde{C}(q,\dot{q})\dot{q} + \nabla_q V(q) = \tilde{\tau} + A(q)\lambda, \tag{3}$$

where $\tilde{M} = \tilde{M}^T > 0$ is the positive definite mass matrix, and the term $\tilde{C}\dot{q}$ represents the Coriolis and centripetal forces. The Lagrange multipliers $\lambda \in \mathbb{R}^k$ represent the constraint forces to satisfy (2). Due to these constraints, the admissible velocities at $q \in \tilde{\mathcal{Q}}$ must be of the form $\dot{q} = S(q)\nu$, where $S \in \mathbb{R}^{n\times n-k}$ is a full rank matrix satisfying $A^T S = 0$ for all $q \in \tilde{\mathcal{Q}}$, and $\nu \in \mathbb{R}^{n-k}$ are local coordinates of the constrained tangent space. The admissible velocities at q lie, therefore, in the subspace of $T_q\tilde{\mathcal{Q}}$ spanned by the columns of S, i.e., the space $\mathcal{D}_q$. Now, replace $\dot{q} = S\nu$ and $\ddot{q} = S\dot{\nu} + \dot{S}\nu$ in (3), and eliminate the constraints by pre-multiplying it by S^T:

$$S^T\tilde{M}S\dot{\nu} + S^T(\tilde{M}\dot{S} + \tilde{C}S)\nu + S^T\nabla_q V = S^T\tilde{\tau}. \tag{4}$$

The dynamical system represented by (4) can as well be written in the form

$$M\dot{\nu} + C\nu + S^T\nabla_q V = \tau + J\nu, \tag{5}$$

where $M = S^T\tilde{M}S$, and $\tau = S^T\tilde{\tau}$. Since the matrix C is solely defined by the *Christoffel symbols* of M, the matching of the systems (4) and (5) requires, in general, additional gyroscopic forces $J\nu$, where $J(q,\nu) = -J^T(q,\nu)$. In particular, $J\nu$ is equivalent to the term depending on the curvature of the Ehresmann connection in Bloch (2003) and Delgado, Gajbhiye, and Banavar (2015), or the Jacobi–Lie bracket term in the structure matrix of the Hamiltonian formulation (van der Schaft & Maschke, 1994).

2.2. The wheeled inverted pendulum (WIP)

Fig. 1 shows a simple scheme of the WIP. Let the configuration space be $\tilde{\mathcal{Q}} = \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$ and define local coordinates $q = (x, y, \theta, \alpha, \varphi_r, \varphi_l) \in \tilde{\mathcal{Q}}$. The position of the WIP on the horizontal plane is given by (x, y). The yawing and pitching angles are each identified by θ and α. The coordinates φ_r and φ_l represent the absolute rotations of the right and left wheel, respectively. The equations

$$A^T\dot{q} = \begin{bmatrix} -\mathrm{s}(\theta) & \mathrm{c}(\theta) & 0 & 0 & 0 & 0 \\ \mathrm{c}(\theta) & \mathrm{s}(\theta) & d & 0 & -r & 0 \\ \mathrm{c}(\theta) & \mathrm{s}(\theta) & -d & 0 & 0 & -r \end{bmatrix}\dot{q} = 0 \tag{6}$$

represent the rolling-without-slipping constraints of the wheels. The natural choice of admissible velocities $\nu = [v\ \dot{\alpha}\ \dot{\theta}]^T$, where v is the forward velocity of the WIP, results in

$$\dot{q} = S\nu = \begin{bmatrix} \mathrm{c}(\theta) & 0 & 0 \\ \mathrm{s}(\theta) & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1/r & 0 & d/r \\ 1/r & 0 & -d/r \end{bmatrix} \begin{bmatrix} v \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix}. \tag{7}$$