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Brief paper

Finite-time output-feedback position and attitude tracking of a rigid body[☆]

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ABSTRACT

The position and attitude tracking of a rigid body without velocity measurements is addressed. Dual quaternions are used to describe the coupled rotational and translational motions of the rigid body, yielding compact forms of the kinematics and dynamics suitable for control law synthesis. An output-feedback pose (position and attitude) tracking controller is then designed by integrating techniques from passivity and homogeneity. More precisely, a passivity-enabling auxiliary system is proposed to provide necessary damping instead of velocity feedback and a homogeneous method is used to ensure finite-time convergence. The proposed controller guarantees uniform almost global finite-time stability of the closed-loop system and produces a well-defined vector field on the attitude configuration manifold, thus avoiding the unwinding phenomenon. Moreover, it can be split to obtain velocity-free controllers with finite-time convergence for the cases of translation-only or rotation-only control. Numerical examples verify the effectiveness of the proposed method.

1. Introduction

The position and attitude control of a rigid body in 3-D space remains a relevant problem due to its broad applications to important mechanical systems such as spacecraft, unmanned aerial vehicles, autonomous underwater vehicles, robotic manipulators, etc. For missions such as space-based interferometry, rendezvous and docking, cooperative aerial towing, etc. are problems which fit more naturally into simultaneous rather than sequential position and attitude control. Concurrent relative position and attitude control with high precision is thus a key enabling technology. The problem is challenging since the six-degree-of-freedom (six-DOF) kinematics and dynamics of rigid-body motion are both nonlinear and couple translation and rotation. If the position and attitude controllers are designed separately but concatenated for pose control, the stability of the overall six-DOF system may not be directly implied by the individually stable translation and rotation systems and must be further addressed.

State-feedback pose controllers have been developed by Filipe and Tsiotras (2015), Gui and Vukovich (2016a), Kristiansen, Nicklasson, and Gravdahl (2008), and Wang, Liang, Sun, Zhang, and Liu (2012) assuming availability of (translational and angular) velocity measurements from sensors. However, in reality velocity information can be either unreliable or unavailable due to various limitations. To eliminate the requirement for velocity feedback, a high-pass filter was utilized by Wong, Pan, and Kapila (2005) while a low-pass filter was constructed by Filipe and Tsiotras (2013) from a 3-D version for attitude-only control (Lizarralde & Wen, 1996). Both methods yielded asymptotic six-DOF tracking laws without a velocity observer. Unlike asymptotic stability, finite-time stability (FTS; Bhat & Bernstein, 2000a) implies convergence in a finite time, thus faster convergence rates and better robustness to disturbances. Note that the concept of FTS here is different from that of Amato, Ariola, and Cosentino (2006), which only focuses on the boundedness of states within a finite time interval.

To design finite-time output-feedback laws, a typical approach is to combine finite-time state-feedback laws with finite-time observers (Hong, Yang, Bushnell, & Wang, 2000) or differentiators (Levant, 2003), while avoiding finite-time escape. It is, however, fairly difficult to construct global finite-time observers for general nonlinear systems and most existing results are for certain single-input-single-output systems (Hong, 2002; Hong, Huang, & Xu, 2001; Li & Qian, 2006; Orlov, Aoustin, & Chevallereau, 2011).

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Recently, finite-time output-feedback controllers for attitude control were proposed by Du and Li (2013), Zou (2014), and Hu, Jiang, and Friswell (2014), all based on velocity observers. These designs require feedback domination of the entire system nonlinearity, including the Coriolis forces, and rely on an Euler–Lagrange formulation of the attitude dynamics in local coordinates. In addition, the resulting controllers yield semi-global FTS, which intrinsically relies on high-gain injection to enlarge the domain of attraction. These approaches can be extended to translation control but will lead to drawbacks similar to attitude-only controllers. A finite-time observer was developed by Sanyal, Izadi, and Bohn (2014) to estimate the states for rigid-body motion, but it still required velocity measurements. An observer-free dynamic output-feedback law was proposed by Su and Zheng (2015) for a double integrator system but it only ensures local FTS if homogeneous perturbations are present, similarly to Hong, Xu, and Huang (2002). These studies are mainly restricted to stabilization of some special time-invariant systems.

The above methods, however, cannot be directly applied to develop global finite-time output-feedback tracking laws for a rigid body due to the nonlinear, coupled, nonautonomous system dynamics. In addition, the attitude configuration SO(3) is a compact manifold without boundary which does not allow continuous global stabilization laws (Bhat & Bernstein, 2000b). If inappropriately designed, some quaternion-based controllers can induce the undesirable unwinding problem (Schlanbusch, Loria, & Nicklasson, 2012).

This paper approaches simultaneous position and attitude tracking of a rigid body with neither translational nor angular velocity feedback. The six-DOF dynamics are formulated via dual quaternions, an extension of quaternions. They provide a compact, efficient, global description of rigid-body motion with only eight numbers, and facilitate the unified synthesis of pose control laws. The control design features two steps, namely, injecting damping via a dual-quaternion filter, which can be viewed as an extension of a previous quaternion filter (Abdessameud & Tayebi, 2009; Tayebi, 2008), and enabling a negative homogeneous degree by a properly constructed nonsmooth feedback. Both design procedures take advantage of the nonlinearity of system dynamics. As shown by rigorous analysis, the resultant output-feedback controller not only ensures uniform almost global FTS (UAGFTS) of the closed-loop systems but also avoids the unwinding problem. It can also be split to obtain finite-time output-feedback control laws for translation-only or rotation-only control. In particular, the rotational control law yields *a priori* bounded control torques. Illustrative examples show the effectiveness of the proposed method.

2. Preliminaries and problem formulation

2.1. Notations, quaternions, and dual quaternions

Throughout this paper, $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix and $\mathbb{I}_n = \{1, \ldots, n\}$. For $\lambda > 0$ and a weight vector $\boldsymbol{w} = [w_1, \ldots, w_n]^T \in \mathbb{R}^n$ with $w_i > 0$, $i \in \mathbb{I}_n$, a dilation operator $\Delta_\lambda^{\boldsymbol{w}}$ is defined by $\Delta_\lambda^{\boldsymbol{w}}\boldsymbol{x} = [\lambda^{w_1}x_1, \ldots, \lambda^{w_n}x_n]^T$ for $\boldsymbol{x} \in \mathbb{R}^n$. To deal with time-dependent functions and systems, $\Delta_\lambda^{\boldsymbol{w}}$ is extended as $\Delta_\lambda^{\boldsymbol{w}}(\boldsymbol{x}, t) = (\Delta_\lambda^{\boldsymbol{w}}\boldsymbol{x}, t)$ (Pomet & Samson, 1994). Given $x \in \mathbb{R}$ and $\boldsymbol{x} \in \mathbb{R}^n$, denote by $|x|$ the absolute value, $\|\boldsymbol{x}\|$ the Euclidean norm, and $\mathrm{sgn}^\alpha(\boldsymbol{x}) = [\mathrm{sgn}^\alpha(x_1), \ldots, \mathrm{sgn}^\alpha(x_n)]^T$, where $\alpha \geq 0$, $\mathrm{sgn}^\alpha(x) = \mathrm{sgn}(x)\,|x|^\alpha$ and $\mathrm{sgn}(\cdot)$ is the standard sign function. Note that $\mathrm{d}\,|x|^{1+\alpha}/\mathrm{d}t = (1+\alpha)\mathrm{sgn}^\alpha(x)\dot{x}$ and $\mathrm{d}\,\|\boldsymbol{x}\|^{1+\alpha}/\mathrm{d}t = (1+\alpha)\,\|\boldsymbol{x}\|^{\alpha-1}\boldsymbol{x}^T\dot{\boldsymbol{x}}$. In addition, $y = \mathcal{O}(x)$ means $|y| \leq c\,|x|$ for sufficiently small $|x|$ and some constant $c > 0$. For $\forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3$, $\boldsymbol{x}^\times$ is the skew-symmetric matrix satisfying $\boldsymbol{x}^\times\boldsymbol{y} = \boldsymbol{x} \times \boldsymbol{y}$, where $\times$ is the cross product on $\mathbb{R}^3$.

Let $\mathbb{Q} = \{\boldsymbol{q} = (q_0, \bar{\boldsymbol{q}}) : q_0 \in \mathbb{R}, \bar{\boldsymbol{q}} \in \mathbb{R}^3\}$ denote the set of quaternions. Let ε represent a dual unit satisfying $\varepsilon^2 = 0$ but $\varepsilon \neq 0$. A dual number is then defined as

$$\hat{a} = a_r + \varepsilon a_d, \quad a_r,\ a_d \in \mathbb{R}, \tag{1}$$

where a_r and a_d are called the real and dual parts, respectively. Dual vectors and dual quaternions can be viewed as two generalizations of dual numbers. Dual numbers of the form $\hat{\boldsymbol{a}} = \boldsymbol{a}_r + \varepsilon\boldsymbol{a}_d$ are called dual vectors if $\boldsymbol{a}_r, \boldsymbol{a}_d \in \mathbb{R}^3$, and dual quaternions if $\boldsymbol{a}_r, \boldsymbol{a}_d \in \mathbb{Q}$. The sets of dual quaternions and dual vectors are denoted by $\mathbb{DQ}$ and $\mathbb{DQ}_V$, respectively. Let $\mathbf{1} = (1, \bar{\mathbf{0}})$ and $\mathbf{0} = (0, \bar{\mathbf{0}})$ with $\bar{\mathbf{0}} = [0, 0, 0]^T$ denote the identity and zero elements on $\mathbb{Q}$, respectively. Following this, define $\hat{\mathbf{1}} = \mathbf{1} + \varepsilon\mathbf{0}$ and $\hat{\mathbf{0}} = \mathbf{0} + \varepsilon\mathbf{0}$ to be the identity and zero element on $\mathbb{DQ}$ and $\hat{\bar{\mathbf{0}}} = \bar{\mathbf{0}} + \varepsilon\bar{\mathbf{0}}$ to be the zero element on $\mathbb{DQ}_V$. As a complement to the dual unit, the operator $\mathrm{d}/\mathrm{d}\varepsilon$ is introduced such that $\mathrm{d}\hat{\boldsymbol{a}}/\mathrm{d}\varepsilon = \boldsymbol{a}_d$ for $\forall \hat{\boldsymbol{a}} \in \mathbb{DQ}$ and when applied twice $\mathrm{d}^2\hat{\boldsymbol{a}}/\mathrm{d}\varepsilon^2 = \mathbf{0}$. In contrast, $\varepsilon\hat{\boldsymbol{a}} = \varepsilon\boldsymbol{a}_r$ and $\varepsilon(\varepsilon\hat{\boldsymbol{a}}) = \mathbf{0}$.

The multiplication on $\mathbb{Q}$ and $\mathbb{DQ}$ is defined by

$$\boldsymbol{q} \otimes \boldsymbol{q}' = (q_0q_0' - \bar{\boldsymbol{q}} \cdot \bar{\boldsymbol{q}}', q_0\bar{\boldsymbol{q}}' + q_0'\bar{\boldsymbol{q}} + \bar{\boldsymbol{q}} \times \bar{\boldsymbol{q}}'), \quad \boldsymbol{q}, \boldsymbol{q}' \in \mathbb{Q},$$

$$\hat{\boldsymbol{a}} \otimes \hat{\boldsymbol{b}} = \boldsymbol{a}_r \otimes \boldsymbol{b}_r + \varepsilon(\boldsymbol{a}_d \otimes \boldsymbol{b}_r + \boldsymbol{a}_r \otimes \boldsymbol{b}_d), \quad \hat{\boldsymbol{a}}, \hat{\boldsymbol{b}} \in \mathbb{DQ}.$$

Note that $\otimes$ is associative and distributive but not commutative. The conjugates of $\boldsymbol{a} \in \mathbb{Q}$ and $\hat{\boldsymbol{a}} \in \mathbb{DQ}$ are given by $\boldsymbol{a}^* = (a_0, -\bar{\boldsymbol{a}})$ and $\hat{\boldsymbol{a}}^* = \boldsymbol{a}_r^* + \varepsilon\boldsymbol{a}_d^*$. The conjugation operation satisfies $(\boldsymbol{a} \otimes \boldsymbol{b})^* = \boldsymbol{b}^* \otimes \boldsymbol{a}^*$ and $(\hat{\boldsymbol{a}} \otimes \hat{\boldsymbol{b}})^* = \hat{\boldsymbol{b}}^* \otimes \hat{\boldsymbol{a}}^*$ for $\forall \boldsymbol{a}, \boldsymbol{b} \in \mathbb{Q}$ and $\forall \hat{\boldsymbol{a}}, \hat{\boldsymbol{b}} \in \mathbb{DQ}$. In addition, the set of unit quaternions and unit dual quaternions are defined by $\mathbb{Q}_U = \{\boldsymbol{q} \in \mathbb{Q} : \boldsymbol{q}^* \otimes \boldsymbol{q} = \mathbf{1}\}$ and $\mathbb{DQ}_U = \{\hat{\boldsymbol{q}} \in \mathbb{DQ} : \hat{\boldsymbol{q}}^* \otimes \hat{\boldsymbol{q}} = \hat{\mathbf{1}}\}$. The swap of $\hat{\boldsymbol{a}} \in \mathbb{DQ}$ is defined by $\hat{\boldsymbol{a}}^s = \boldsymbol{a}_d + \varepsilon\boldsymbol{a}_r$. The following operations are also needed:

$$\hat{c} \odot \hat{\boldsymbol{a}} = (c_r + \varepsilon c_d) \odot (\boldsymbol{a}_r + \varepsilon\boldsymbol{a}_d) = c_r\boldsymbol{a}_r + \varepsilon c_d\boldsymbol{a}_d, \quad \hat{\boldsymbol{a}} \in \mathbb{DQ},$$

$$\hat{\boldsymbol{a}} \times \hat{\boldsymbol{b}} = \boldsymbol{a}_r \times \boldsymbol{b}_r + \varepsilon(\boldsymbol{a}_d \times \boldsymbol{b}_r + \boldsymbol{a}_r \times \boldsymbol{b}_d), \quad \hat{\boldsymbol{a}}, \hat{\boldsymbol{b}} \in \mathbb{DQ}_V,$$

$$\hat{\boldsymbol{a}} \circ \hat{\boldsymbol{b}} = \boldsymbol{a}_r \cdot \boldsymbol{b}_r + \boldsymbol{a}_d \cdot \boldsymbol{b}_d, \quad \hat{\boldsymbol{a}}, \hat{\boldsymbol{b}} \in \mathbb{DQ}_V,$$

where $\odot$ denotes the product of a dual number with a dual quaternion while $\circ$ is called the dual quaternion circle product. In addition, given $\hat{\boldsymbol{a}} \in \mathbb{DQ}$ and $\lambda \in \mathbb{R}$, let $\lambda\hat{\boldsymbol{a}} = \lambda\boldsymbol{a}_r + \varepsilon\lambda\boldsymbol{a}_d$. Given $\boldsymbol{A} \in \mathbb{R}^{3\times 3}$ and $\hat{\boldsymbol{a}} \in \mathbb{DQ}_V$, let $\boldsymbol{A}\hat{\boldsymbol{a}} = \boldsymbol{A}\boldsymbol{a}_r + \varepsilon\boldsymbol{A}\boldsymbol{a}_d$. Note that 3-D (dual) vectors can, in fact, be viewed as (dual) quaternions with vanishing scalar parts, which are practically treated as zero when operating with (dual) quaternions. For more concepts and properties about dual numbers and dual quaternions the reader is referred to Filipe and Tsiotras (2015), Gui and Vukovich (2016a) and references therein.

2.2. Definitions and lemmas

Definition 1 (*Pomet and Samson (1994)*)**.** Consider a time-varying system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t), \quad \boldsymbol{x} \in \mathbb{R}^n, \tag{2}$$

where $\boldsymbol{f}(\boldsymbol{x}, t) = [f_1(\boldsymbol{x}, t), \ldots, f_n(\boldsymbol{x}, t)]^T \in \mathbb{R}^n$ is continuous with respect to $\boldsymbol{x}$. The vector field $\boldsymbol{f}(\boldsymbol{x}, t)$ is said to be homogeneous of degree $k \in \mathbb{R}$ with respect to a dilation $\Delta_\lambda^{\boldsymbol{w}}$ if $f_i(\Delta_\lambda^{\boldsymbol{w}}(\boldsymbol{x}, t)) = \lambda^{w_i+k}f_i(\boldsymbol{x}, t)$ for $\forall i \in \mathbb{I}_n$, $\forall \boldsymbol{x} \in \mathbb{R}^n$, and $\forall \lambda > 0$.

Definition 2 (*Moulay and Perruquetti (2008)*)**.** Consider system (2) with $\boldsymbol{f}(\mathbf{0}_{n\times 1}, t) = \mathbf{0}_{n\times 1}$ and denote by U a neighborhood of $\boldsymbol{x} = \mathbf{0}_{n\times 1}$. Then, the origin is uniformly locally FTS (ULFTS) if it is (1) uniformly Lyapunov stable in U and (2) uniformly finite-time convergent in U, i.e., there exists a function $T : U \to \mathbb{R}_{\geq 0}$ such that for any $(\boldsymbol{x}_0, t_0) \in U \times \mathbb{R}_{\geq 0}$, the solution satisfies $\boldsymbol{x}(t, \boldsymbol{x}_0, t_0) \in U$ for $t \in [0, T(\boldsymbol{x}_0))$ and $\boldsymbol{x}(t, \boldsymbol{x}_0, t_0) = \mathbf{0}_{n\times 1}$ for $t \geq T(\boldsymbol{x}_0) + t_0$. If $U = \mathbb{R}^n$, the origin is uniformly globally FTS (UGFTS).