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Brief paper

# Set-membership filtering for time-varying systems with mixed time-delays under Round-Robin and Weighted Try-Once-Discard protocols ☆

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ABSTRACT

This paper is concerned with the set-membership filtering problem for a class of time-varying systems with mixed time-delays and communication protocols. Two kinds of well-known protocols (Round-Robin protocol and Weighted Try-Once-Discard protocol) are considered, with which the data transmission between the sensor nodes and the filter is implemented via a shared communication network that allows only one sensor node to send its measurement data at each transmission instant in order to prevent the data from collisions. The transmission order of sensor nodes is orchestrated by the underlying protocol of the network. The aim of the problem addressed is to design a set-membership filter capable of confining the state estimate of the system to certain ellipsoidal region subject to the bounded non-Gaussian noises. Sufficient condition is first derived for the existence of the desired filter at each time step in terms of a recursive algorithm. Then, two optimization problems are solved by optimizing the constraint ellipsoid of the estimation error subject to the underlying protocol. Simulation results demonstrate the effectiveness of the proposed filter design scheme.



## 1. Introduction

The past decades have witnessed a surge of research interest on networked systems due primarily to their extensive applications in various fields including environmental monitoring, industrial automation, smart grids and distributed/mobile communications. The key feature of networked systems is that the connections of system components are implemented via shared communication networks. Networked systems possess many advantages such as low cost, simple installation, reduced system wiring and high reliability. Compared with the traditional systems with point-to-point communication scheme, the utilization of communication network has led to rich yet complex network-induced behaviors (e.g. communication delays, packet dropouts and signal quantization) and these behaviors have attracted considerable research attention, see e.g. Ding, Wang, Shen, and Wei (2015), Liu, Xia, Chen, and Hu (2007), Xiong and Lam (2007, 2009) and Zhang, Shi, Chen, and Huang (2005).

Filtering problem is a fundamental research issue in signal processing and control communities (Basin, Loukianov, & Hernandez-Gonzalez, 2013; Caballero-Aguila, Hermoso-Carazo, & Linares-Perez, 2015; Hou, Dong, Wang, Ren, & Alsaadi, 2016; Lam, Zhang, Chen, & Xu, 2015; Yang, Dong, Wang, Ren, & Alsaadi, 2016; Yu, Dong, Wang, Ren, & Alsaadi, 2016; Zhang et al., 2016). For linear (or nonlinear) systems with Gaussian noises, the Kalman filtering (or extended Kalman filter) method is always recognized as a reliable filtering approach. Nevertheless, the Kalman filter may lead to unsatisfactory performance when the disturbances are non-Gaussian noises. Another well-known filtering method is the $\mathcal{H}_\infty$ filtering approach whose aim is to guarantee a given disturbance attenuation level on the estimation error subject to energy-bounded noises. For conventional $\mathcal{H}_\infty$ filtering method, there does

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not seem to be any provision to ensure the boundedness of the estimation error. In this regard, the so-called set-membership filtering method does well as it generates a satisfactory state estimate guaranteeing that the estimation error is confined to a bounded region in the state-space subject to the unknown-but-bounded noises. The origination of the set-membership filtering problem dated back to the 1960s and such a problem has gained recurring research interest in the past decade (Guo & Wang, 2007; Wei, Liu, Song, & Liu, 2015; Yang & Li, 2009a,b).

In most existing literature concerning the filtering problems of networked systems, it has been assumed that all the sensor nodes could simultaneously get access to the network to transmit signals. This assumption, however, is generally unrealistic for networked systems since real-world networks unavoidably suffer from limited bandwidth which is likely to give rise to data collisions in case of simultaneous multiple accesses. As such, the communication protocols are needed to orchestrate the transmission order of sensor nodes (Bauer, Donkers, van de Wouw, & Heemels, 2013; Xu, Su, Pan, Wu, & Xu, 2013; Zhang, Yu, & Feng, 2011). The widely utilized protocols in industry include, but are not limited to, the Round-Robin (RR) protocol (Ugrinovskii & Fridman, 2014), the Weighted Try-Once-Discard (WTOD) protocol (Donkers, Heemels, van de Wouw, & Hetel, 2011; Walsh, Ye, & Bushnell, 2002) and stochastic communication protocol (Long & Yang, 2014; Tabbara & Nešić, 2008). Compared with those traditional schemes without protocol scheduling, the utilization of communication protocol would bring in certain fundamental challenges (or protocol-induced effects) to the dynamics analysis issues. As such, it is necessary to examine how the inclusion of the communication protocol impacts on the control and filtering problems of networked systems. So far, some preliminary results have been reported on the analysis issue of networked systems subject to communication protocols.

Among various communication protocols, the RR and WTOD protocols are widely employed by communication and signal processing communities. The RR protocol is also known as the time-division multiple access (TDMA) protocol or Token Ring protocol. Under the scheduling of the RR protocol, the transmission instants of all the sensor nodes are predetermined according to a fixed *circular* order. Obviously, the RR protocol is a periodic protocol. On the other hand, the WTOD protocol belongs to the class of quadratic protocols. Different from the "periodic assignment" behavior of the RR protocol, the WTOD protocol assigns the transmission instants to certain sensor nodes according to a given quadratic selection principle. For the purpose of characterizing the scheduling behaviors caused by protocols, the switched-linear-system framework has been introduced in Donkers et al. (2011) for discrete-time networked systems in which both the RR and the WTOD scheduling behaviors have been considered. Based on the switched-linear-system framework, the co-design problem of both the stabilizing controller and scheduling protocol has been investigated in Song, Yu, and Zhang (2009) for a class of networked control systems with multiple distributed transmission delays.

On the other hand, it is well known that time-delays exist widely in practice and may cause undesirable dynamic behaviors including oscillation and instability. In recent years, a rich body of literature has appeared on the control and filtering problems of networked systems with time-delays, see e.g. Hu, Wang, Gao, and Stergioulas (2012) and the references therein. Unfortunately, when it comes to the set-membership filtering problems with time-delays under communication protocols, the corresponding results have been really scattered due mainly to the difficulties in handling the coupling issues between the set membership and the scheduling protocols. To this end, a seemingly interesting research problem that is of clear engineering insight is to investigate the set-membership filtering problem for the time-varying networked systems with a communication protocol (the RR protocol or WTOD protocol), and this has motivated our present research.

Summarizing the above discussions, in this paper, we aim to deal with the set-membership filtering problem for the time-varying networked system with simultaneous presence of mixed time-delays and communication protocol scheduling. More specifically, the objective of this paper is to design a set-membership filter for the networked systems with mixed time-delays subject to the RR protocol and WTOD protocol, respectively. *The main contributions of this paper are highlighted as follows:* (1) *the set-membership filtering problem is, for the first time, investigated for time-varying systems with the protocol scheduling;* (2) *the influences from both the RR protocol and the WTOD protocol on the filter performance are considered; and* (3) *the filter gain matrix is obtained by solving a set of recursive matrix inequalities that are solvable via standard software package.*

**Notations**: The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the $n$ dimensional Euclidean space and set of all $n \times m$ real matrices. $\mathbb{N}\,(\mathbb{N}^+, \mathbb{N}^-)$ denotes the set of integers (nonnegative integers, negative integers). The notation $X \geq Y$ $(X > Y)$, where $X$ and $Y$ are real symmetric matrices, means that $X - Y$ is positive semi-definite (positive definite). Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ will, respectively, denote the expectation of the stochastic variable $x$ and expectation of $x$ conditional on $y$. 0 represents the zero matrix of compatible dimensions. The $n$-dimensional identity matrix is denoted as $I_n$ or simply $I$, if no confusion is caused. The shorthand diag$\{\cdots\}$ stands for a block-diagonal matrix. $\|A\|$ refers to the norm of a matrix $A$ defined by $\|A\| = \sqrt{\text{trace}(A^TA)}$. $M^T$ represents the transpose of $M$. In symmetric block matrices, "$*$″ is used as an ellipsis for terms induced by symmetry. The Kronecker delta function $\delta(a)$ is a binary function that equals 1 if $a = 0$ and equals 0 otherwise. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

## 2. Problem formulation and preliminaries

In this section, we first introduce some preliminaries related to the Round-Robin (RR) protocol and the Weighted Try-Once-Discard (WTOD) protocol, and then describe the problem setup.

### 2.1. Communication protocols

Consider a networked system with $N$ nodes labeled as $\{1, 2, \ldots, N\}$. In this system, all the nodes transmit their data via a shared communication network in which *only one node* is allowed to get access to the network at each transmission instant for the purpose of preventing the data from collisions. Let $\xi(k) \in \{1, 2, \ldots, N\}$ be the selected node obtaining access to the communication network at time instant $k$. The value of $\xi(k)$ is determined by the communication protocol of the network.

(1) *The Round-Robin (RR) protocol*:

Under the scheduling of the RR protocol, the value of $\xi(k)$ satisfies $\xi(k + N) = \xi(k)$ for all $k \in \mathbb{N}^+$ and $\xi(k) = k$ for $k \in \{1, 2, \ldots, N\}$. In other words, $\xi(k)$ can be calculated as:

$$\xi(k) = \text{mod}(k - 1, N) + 1. \tag{1}$$

In such a protocol, the number of nodes $N$ can be regarded as the period of the RR protocol. During each period of the RR protocol, each node has access to the communication network exactly once.

(2) *The Weighted Try-Once-Discard (WTOD) protocol*:

WTOD protocol is a dynamical protocol in which the value of $\xi(k)$ is determined by the following selection principle:

$$\xi(k) \triangleq \arg\max_{1\leq i\leq N} \left(y_i(k) - y_i^*(k)\right)^T Q_i \left(y_i(k) - y_i^*(k)\right) \tag{2}$$