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An improved summation inequality to discrete-time systems with time-varying delay☆

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ABSTRACT

The summation inequality plays an important role in developing delay-dependent criteria for discrete-time systems with time-varying delay. This note proposes an improved summation inequality to estimate the summation terms appearing in the forward difference of Lyapunov–Krasovskii functional. Compared with the inequality recently developed by the Wirtinger-based summation inequality and the reciprocally convex lemma, the proposed one reduces the estimation gap while requires the same number of decision variables. A relaxed stability criterion of a linear discrete-time system with a time-varying delay is established by using such novel inequality. Two numerical examples are given to demonstrate the advantages of the proposed method.

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1. Introduction

In the last few years, the stability analysis of discrete-time systems with time-varying delays has become a hot topic in the field of control theory (Feng, Lam, & Yang, 2015; Gao & Chen, 2007; He, Wu, Liu, & She, 2008; Huang & Feng, 2010; Kim, 2015; Kwon, Park, Park, Lee, & Cha, 2012, 2013; Meng, Lam, Du, & Gao, 2010; Nam, Pathirana, & Trinh, 2015; Nam, Trinh, & Pathirana, 2015; Peng, 2012; Seuret, Gouaisbaut, & Fridman, 2015; Shao & Han, 2011; Xu, Lam, Zhang, & Zou, 2014; Zhang & Han, 2015; Zhang, Peng, & Zheng, 2016; Zhang, Xu, & Zou, 2008). An important objective of stability analysis is to find the admissible delay region such that time-delay systems remain stable for the time-varying delay within this region (Zhang, He, Jiang, Wu, & Zeng, 2016). Delay-dependent stability criteria developed in the framework of the Lyapunov–Krasovskii functional (LKF) and the linear matrix inequality (LMI) are the most effective criteria to determine such admissible region. The following double summation term is frequently applied during the construction of LKF to obtain delay-dependent criterion (Seuret et al., 2015):

$$V_r(k) = \sum_{i=-h_2}^{-h_1-1} \sum_{j=k+i}^{k-1} \eta^T(j) R \eta(j) \tag{1}$$

where h_1 and h_2 are respectively the lower and the upper bounds of a time-varying delay (i.e., $h_1 \le d(k) \le h_2$), $R \ge 0$, and $\eta(k) = x(k+1) - x(k)$ with $x(k)$ being the system state. Then the following term will appear in the forward difference of $V_r(k)$:

$$\mathcal{S}(k) := \sum_{i=k-d(k)}^{k-h_1-1} \eta^T(i) R \eta(i) + \sum_{i=k-h_2}^{k-d(k)-1} \eta^T(i) R \eta(i) \tag{2}$$

During the development of stability criteria, a challenging problem is how to estimate the lower bound of the above summation term (Zhang & Han, 2015). Obtaining tighter bound of summation term (i.e., reducing the estimation gap) plays a key role in reducing the conservatism. In the early literature, the free-weighting matrix (FWM) approach (He et al., 2008) and the Jensen-based inequality (JBI) (Shao & Han, 2011) were two important methods for this issue. By relaxing the JBI, Wirtinger-based inequalities (WBIs) were simultaneously reported in Nam, Pathirana, and Trinh (2015) and Seuret et al. (2015) and later in Zhang and Han (2015). Very recently, an auxiliary function based inequality (AFBI) (Nam, Trinh, & Pathirana, 2015) and a free-matrix-based summation inequality (FMBI) (Chen, Lu, & Xu, 2016) inspired by the

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research of Zeng, He, Wu, and She (2015, 2016) were developed by further improving the WBI.

Those inequality-based estimation methods include two key steps to estimate $\mathcal{S}(k)$: (1) applying the JBI/WBI/AFBI to estimate two summation terms in $\mathcal{S}(k)$, respectively; and (2) using the reciprocally convex lemma (RCL) (Park, Ko, & Jeong, 2011) to handle the $d(k)$ appearing in the denominator. The recently developed techniques (the WBIs, the AFBI, and the FMIB) focus on the first step. To the best of the authors' knowledge, there is no research that discusses the tighter estimation of $\mathcal{S}(k)$ considering two steps together. This note aims to fill this research gap.

This note proposes an improved summation inequality by considering two terms of $\mathcal{S}(k)$ together. It is tighter than the one obtained by combining the WBI and the RCL but keeps the same number of decision variables. A new stability criterion for a linear discrete-time system with a time-varying delay is established by applying the proposed inequality. Finally, two numerical examples are given to illustrate the effectiveness of the proposed inequality and the corresponding criterion.

Throughout this note, the superscripts T and -1 mean the transpose and the inverse of a matrix, respectively; $\mathcal{R}^n$ denotes the n-dimensional Euclidean space; $\|\cdot\|$ refers to the Euclidean vector norm; $P>0$ (≥ 0) means that P is a symmetric positive-definite (semi-positive-definite) matrix; diag$\{\cdot\}$ denotes a block-diagonal matrix; $\mathrm{Sym}\{X\}=X+X^T$; and the symmetric term in a symmetric matrix is denoted by $*$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

Consider the following linear discrete-time system with a time-varying delay:

$$\begin{cases} x(k+1)=Ax(k)+A_dx(k-d(k)), & k\geq 0\\ x(k)=\phi(k), & k\in[-h_2,0]\end{cases} \tag{3}$$

where $x(k)\in\mathcal{R}^n$ and $\phi(k)$ are the system state and the initial condition, respectively; A and A_d are the system matrices; and $d(k)$ is a positive integer which is time-varying and satisfies

$$1\leq h_1\leq d(k)\leq h_2. \tag{4}$$

This note is concerned with the stability of system (3). As mentioned in Section 1, a challenging problem is how to estimate the summation term $\mathcal{S}(k)$. Therefore, the first aim of this note is to develop a more effective estimation method for this task. Then, this note will apply the proposed method to derive a new stability criterion for judging the influence of the time-varying delay on the stability of system.

Several WBIs with different forms were simultaneously reported in Nam, Pathirana, and Trinh (2015), Seuret et al. (2015) and later in Zhang and Han (2015). The ones to be applied in this note are recalled from Seuret et al. (2015).

Lemma 1 (*Wirtinger-based Inequality Seuret et al., 2015*). *For a given symmetric positive definite matrix R, integers $b>a$, any sequence of discrete-time variable x: $\mathcal{Z}[a,b]\to\mathcal{R}^n$, the following inequalities hold*

$$\sum_{i=a}^{b-1}\eta^T(i)R\eta(i)\geq\frac{1}{b-a}\begin{bmatrix}\vartheta_1\\ \vartheta_2\end{bmatrix}^T\begin{bmatrix}R & 0\\ 0 & 3\left(\dfrac{b-a+1}{b-a-1}\right)R\end{bmatrix}\begin{bmatrix}\vartheta_1\\ \vartheta_2\end{bmatrix} \tag{5}$$

$$\geq\frac{1}{b-a}\begin{bmatrix}\vartheta_1\\ \vartheta_2\end{bmatrix}^T\begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix}\begin{bmatrix}\vartheta_1\\ \vartheta_2\end{bmatrix} \tag{6}$$

where $\vartheta_1=x(b)-x(a)$, and $\vartheta_2=x(b)+x(a)-2\sum_{i=a}^{b}\frac{x(i)}{b-a+1}$.

The following RCL is usually applied to combine with the WBI in the literature.

Lemma 2 (*Reciprocally Convex Lemma (RCL) Park et al., 2011*). *For a given scalar α in the interval $(0,1)$, symmetric positive definite matrices U_1 and U_2, and any matrix X such that $\begin{bmatrix}U_1 & X\\ * & U_2\end{bmatrix}\geq 0$, the following inequality holds*

$$\begin{bmatrix}\dfrac{1}{\alpha}U_1 & 0\\ * & \dfrac{1}{1-\alpha}U_2\end{bmatrix}\geq\begin{bmatrix}U_1 & X\\ * & U_2\end{bmatrix}. \tag{7}$$

The estimation of the $\mathcal{S}(k)$ via the WBI and the RCL leads to the following lemma.

Lemma 3. *For a symmetric positive definite matrix R, any matrix X satisfying $\begin{bmatrix}\tilde R & X\\ * & \tilde R\end{bmatrix}\geq 0$ with $\tilde R=\mathrm{diag}\{R,3R\}$, the $\mathcal{S}(k)$ defined in (2) is estimated as*

$$\mathcal{S}(k)\geq\frac{1}{h_{21}}\zeta^T(k)\begin{bmatrix}E_1\\ E_2\end{bmatrix}^T\begin{bmatrix}\tilde R & X\\ * & \tilde R\end{bmatrix}\begin{bmatrix}E_1\\ E_2\end{bmatrix}\zeta(k) \tag{8}$$

where

$$h_{21}=h_2-h_1,\quad d=d(k) \tag{9}$$

$$\zeta(k)=\left[x^T(k),x^T(k-h_1),x^T(k-d),x^T(k-h_2),\right.$$
$$\left.v_1^T(k),v_2^T(k),v_3^T(k)\right]^T \tag{10}$$

$$v_1(k)=\sum_{i=k-h_1}^{k}\frac{x(i)}{h_1+1},\qquad v_2(k)=\sum_{i=k-d}^{k-h_1}\frac{x(i)}{d-h_1+1} \tag{11}$$

$$v_3(k)=\sum_{i=k-h_2}^{k-d}\frac{x(i)}{h_2-d+1} \tag{12}$$

$$E_1=\begin{bmatrix}e_2-e_3\\ e_2+e_3-2e_6\end{bmatrix},\qquad E_2=\begin{bmatrix}e_3-e_4\\ e_3+e_4-2e_7\end{bmatrix} \tag{13}$$

$$e_i=\left[0_{n\times(i-1)n},I_{n\times n},0_{n\times(7-i)n}\right],\quad i=1,2,\ldots,7. \tag{14}$$

Proof. Using WBI (6) and RCL (7) to estimate the $\mathcal{S}(k)$ yields

$$\begin{aligned}\mathcal{S}(k)&=\sum_{i=k-d}^{k-h_1-1}\eta^T(i)R\eta(i)+\sum_{i=k-h_2}^{k-d-1}\eta^T(i)R\eta(i)\\ &\geq\frac{1}{d-h_1}\zeta^T(k)E_1^T\tilde RE_1\zeta(k)+\frac{1}{h_2-d}\zeta^T(k)E_2^T\tilde RE_2\zeta(k)\\ &\geq\frac{1}{h_{21}}\zeta^T(k)\begin{bmatrix}E_1\\ E_2\end{bmatrix}^T\begin{bmatrix}\tilde R & X\\ * & \tilde R\end{bmatrix}\begin{bmatrix}E_1\\ E_2\end{bmatrix}\zeta(k).\end{aligned}$$

3. A relaxed summation inequality

This section develops an improved summation inequality for estimating $\mathcal{S}(k)$, shown in the following lemma.

Lemma 4. *For a symmetric positive definite matrix R, any matrix X, the $\mathcal{S}(k)$ defined in (2) is estimated as*

$$\mathcal{S}(k)\geq\frac{1}{h_{21}}\zeta^T(k)\begin{bmatrix}E_1\\ E_2\end{bmatrix}^T\left(\begin{bmatrix}\tilde R & X\\ * & \tilde R\end{bmatrix}+\begin{bmatrix}\dfrac{h_2-d}{h_{21}}T_1 & 0\\ 0 & \dfrac{d-h_1}{h_{21}}T_2\end{bmatrix}\right)$$
$$\times\begin{bmatrix}E_1\\ E_2\end{bmatrix}\zeta(k) \tag{15}$$

where $\tilde R=\mathrm{diag}\{R,3R\}$, $T_1=\tilde R-X\tilde R^{-1}X^T$ and $T_2=\tilde R-X^T\tilde R^{-1}X$.

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