Automatica 77 (2017) 120–132

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Robust finite-time estimation of biased sinusoidal signals: A volterra operators approach[☆]

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ARTICLE INFO

Article history:
Received 7 July 2015
Received in revised form
5 September 2016
Accepted 14 October 2016

Keywords:
Finite-time estimation
Sinusoidal parameters estimation
Volterra operator
Kernels

ABSTRACT

A novel finite-time convergent estimation technique is proposed for identifying the amplitude, frequency and phase of a biased sinusoidal signal. Resorting to Volterra integral operators with suitably designed kernels, the measured signal is processed yielding a set of auxiliary signals in which the influence of the unknown initial conditions is removed. A second-order sliding mode-based adaptation law – fed by the aforementioned auxiliary signals – is designed for finite-time estimation of the frequency, amplitude, and phase. The worst case behavior of the proposed algorithm in presence of the bounded additive disturbances is fully characterized by Input-to-State Stability arguments. The effectiveness of the estimation technique is evaluated and compared with other existing tools via extensive numerical simulations.

1. Introduction

A wide variety of techniques are proposed in the literature to address the frequency estimation problem including extended Kalman filters (see e.g. Hajimolahoseini, Taban, and Soltanian-Zadeh (2012) and the reference cited there in), phase locked loop (PLL) tools (see e.g. Karimi-Ghartemani and Ziarani (2004) and Pin (2010)), adaptive notch filters (ANF) (see e.g. Hsu, Ortega, and Damm (1999) and Mojiri, Yazdani, and Bakhshai (2010)) and techniques based on the internal model principle (Brown & Zhang, 2004).

The presence of a bias perturbation affecting the sinusoidal signal has recently generated considerable research efforts. An extension of the so-called EPLL approach presented in Karimi-Ghartemani and Ziarani (2004) has been recently proposed in Karimi-Ghartemani, Ali Khajehoddin, Jain, Bakhshai, and Mojiri (2012) to address sinusoidal signals affected by a bias term by including an additional integrator in the EPLL algorithm. However, only local stability properties can be proved or, when averaging analysis is used, global results are available but they are valid only for small adaptation gains (see Pin, 2010).

ANF techniques represent an effective alternative to handle the possible bias in the measured signal by introducing an augmented integral loop that makes the modified ANF a frequency-locked-loop (FLL) system (see, Fedele, Picardi, & Sgro, 2009; Karimi-Ghartemani et al., 2012). Resorting to a bank of such FLL filters, multi-sinusoidal estimation problems have also been dealt with in Fedele and Ferrise (2014). A variant of the FLL, namely second-order generalized integrator-based orthogonal signal generator (OSG-SOGI) is also exploited in Fedele and Ferrise (2012) and Fedele, Ferrise, and Muraca (2011) to cope with a single sinusoidal signal affected by a bias term.

Another important family of frequency estimation methods is based on the state-variable filtering techniques (see e.g., Pin, Parisini, & Bodson, 2011; Pyrkin, Bobtsov, Efimov, & Zolghadri, 2011). Specifically, in Bobtsov, Efimov, Pyrkin, and Zolghadri (2012), a fourth-order frequency estimator – characterized by a switched adaptation gain – is proposed: this algorithm provides extra attenuation of high-frequency noise in steady state. Recently, an algorithm able to cope with a large class of structured perturbations parametrized in the family of time-polynomial functions has been proposed in Pin, Chen, Parisini, and Bodson (2014). The robustness of the method against bounded unstructured perturbations is characterized by an Input-to-State-Stability (ISS) analysis. Furthermore, in Chen, Pin, Ng, Hui, and Parisini (2015) and Chen,

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Andrea Garulli under the direction of Editor Torsten Söderström.

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http://dx.doi.org/10.1016/j.automatica.2016.10.031
0005-1098/

Pin, and Parisini (2014), a parallel pre-filtering scheme is presented that extends the single pre-filter used in Pin et al. (2014) and Pin et al. (2011) with a simplified adaptation law.

Estimation techniques based on adaptive observers represent a valid alternative to the aforementioned methods and several recent approaches are proposed in the literature. These observer-based methods have been extensively analyzed in terms of their stability properties and several papers show that global or semi-global stability can be achieved (see, for example Bobtsov, 2008; Carnevale & Astolfi, 2009, 2011; Chen, Pin, Ng et al., 2014; Chen, Pin, & Parisini, 2013; Hou, 2012; Marino & Tomei, 2002; Xia, 2002 and the references cited therein).

While the aforementioned algorithms provide in most cases only asymptotic stability guarantees, there exist a few other methods that are capable to achieve finite-time convergence,, which is a very desirable feature in several application contexts like, for example, micro-grids power systems that are affected by severe frequency fluctuations due to low inertia of generators. For instance, non-asymptotic methods based on algebraic derivatives are proposed in Trapero, Sira-Ramirez, and Battle (2007a,b); however these tools are affected by singularities due to the scalar division based algorithm (this issue has been tackled in Liu, Gibaru, and Perruquetti (2011) and Luviano-Juarez, Cortes-Romero, and Sira-Ramirez (2015) by using a recursive least squares algorithm, while preserving the deadbeat property). Furthermore, a modulating function-based approach is presented in Fedele and Coluccio (2010), which allows non-asymptotic frequency detection by suitably-designed modulating functions. The well-known Prony's method and its many variants represent another class of techniques specialized for estimating the frequency of complex sinusoidal functions (see, for example, Osborne and Smyth (1995) and the references cited therein). Recently, the Prony's problem is addressed by an algebraic method reported in Ushirobira, Perruquetti, Mboup, and Fliess (2013). Nevertheless, all the methods currently available lack a theoretical investigation of the finite-time convergence properties in the presence of measurement noise.

In this paper, we propose a robust parametric finite-time estimation methodology for biased sinusoidal signals by employing a class of kernel-based linear integral operators, which allow to establish a relationship independent of the unknown initial value of the state of the signal's generator, and in turn yield the adaptation algorithms to identify the sinusoidal parameters in a non-asymptotic way. This is a remarkable result since also in a very recent paper (Na, Yang, Wu, & Guo, 2015), the parameter adaptation scheme depends on a relationship which holds only asymptotically, due to the unknown initial error (asymptotic decay), and theoretically prevents the finite-time convergence of the estimates.

In the spirit of prior work presented by the authors on the sole frequency estimation problem (see Pin, Chen, & Parisini, 2015), this paper deals with a finite-time convergent estimation scheme in which the frequency, amplitude and phase (AFP) of a noisy sinusoidal signal are estimated in finite-time. As shown in the very recent paper (Pin, Assalone, Lovera, & Parisini, 2016) dealing with non-asymptotic continuous-time systems identification, Volterra operators induced by suitably defined bivariate kernels, turn out to be an enabling tool for finite-time estimation. In contrast with existing works, the behavior of the estimator in the presence of a bounded additive measurement disturbance is rigorously characterized by ISS arguments. To the best of the authors' knowledge, this is the first finite-time convergent sinusoidal estimator the behavior of which is analyzed also in the presence of unstructured and bounded measurement perturbations.

The paper is organized as follows: Section 2 introduces several useful notations and basic definitions and provides the problem formulation. In Section 3, the Volterra integral operators are characterized whereas in Section 4, the finite-time estimation technique is illustrated. In Section 5, the stability and robustness properties of the proposed estimation tool are dealt with. Extensive simulation results are provided in Section 6 and Section 7 draws some concluding remarks.

2. Problem statement and preliminaries

Consider a biased sinusoidal signal

$$y(t) = A_0 + A^* \sin(\vartheta(t)), \qquad \dot{\vartheta}(t) = \omega^*, \quad \vartheta_0 = \phi \tag{1}$$

where $A_0 \in \mathbb{R}_{>0}$, $A^* \in \mathbb{R}_{>0}$ and $\omega^* \in \mathbb{R}_{>0}$ are the unknown offset, amplitude and angular frequency, respectively, ϑ is the instantaneous angle, ϕ denotes the initial phase shift.

As mentioned in the Introduction, our objective consists in estimating A^*, ω^* and ϑ of the sinusoidal signal (1) (i.e., AFP) within an arbitrarily small finite-time.

For the reader's convenience and for the sake of completeness, let us briefly recall some basic concepts of linear integral operators' algebra (see, for example Burton, 2005 and the references therein), which are needed to derive the main results presented in the subsequent sections.

In this paper, we use transformations acting on the Hilbert space $\mathcal{L}^2_{loc}(\mathbb{R}_{\geq 0})$ of locally square-integrable functions with domain $\mathbb{R}_{\geq 0}$ and range $\mathbb{R}$ (*i.e.*, $u(\cdot) \in \mathcal{L}^2_{loc}(\mathbb{R}_{\geq 0}) \Leftrightarrow (u(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}) \wedge (\int_B |u(t)|^2 dt < \infty, \forall \text{ compact } B \subset \mathbb{R}_{\geq 0})$. Given a function $u \in \mathcal{L}^2_{loc}(\mathbb{R}_{\geq 0})$, its image through the Volterra (linear, integral) operator $\mathcal{V}_K$ induced by a Hilbert–Schmidt $\mathcal{HS}$ *Kernel Function* $K(\cdot,\cdot) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is usually denoted by $[\mathcal{V}_K u](\cdot)$, and is defined by the inner product:

$$[\mathcal{V}_K u](t) \triangleq \int_0^t K(t,\tau) u(\tau) d\tau, \quad t \in \mathbb{R}_{\geq 0}. \tag{2}$$

Any explicit function of time $u(t) : t \to u(t) \in \mathbb{R}$, such that $u(\cdot) \in \mathcal{L}^2_{loc}(\mathbb{R}_{\geq 0})$ will be addressed in this paper as a signal. Then, given two scalars $a, b \in \mathbb{R}_{\geq 0}$, with $a < b$, let us denote by $u_{[a,b]}(\cdot)$ and $u_{(a,b]}(\cdot)$ the restriction of a signal $u(\cdot)$ to the closed interval $[a, b]$ and to the left open interval $(a, b]$, respectively. Moreover, let $\mathbf{u}(t) \in \mathbb{R}^n$, $\forall t \geq 0$ be an i-times differentiable vector of signals, we denote by $\mathbf{u}^{(1)}$ the vector of the ith order time-derivative signals. Then, we recall the following useful definition:

Definition 1 (*Weak (Generalized) Derivative*)**.** Let $u(\cdot) \in \mathcal{L}^1_{loc}(\mathbb{R}_{\geq 0})$. We say that $u^{(1)}(\cdot)$ is a *weak derivative* of $u(\cdot)$ if

$$\int_0^t u(\tau)\left(\frac{d}{d\tau}\phi(\tau)\right) d\tau = -\int_0^t u^{(1)}(\tau)\phi(\tau) d\tau, \quad \forall t \in \mathbb{R}_{\geq 0}$$

for all $\phi \in \mathbb{C}^\infty$, with $\phi(0) = \phi(t) = 0$. □

We remark that $u^{(1)}(\cdot)$ is unique up to a set of zero Lebesgue measure, *i.e.*, it is defined almost everywhere. If u is differentiable in the conventional sense, then its weak derivative is identical to its conventional derivative. Classical rules for the derivation of sum or products of functions also hold for the weak derivative. Given a kernel function $K(\cdot,\cdot)$ in two variables, its ith order weak derivative with respect to the second argument will be denoted as $K^{(i)}$, $i \in \mathbb{Z}_{\geq 0}$.

For obvious practical implementability reasons, it is convenient to devise a differential form for the operators. By applying the Leibniz differentiation rule to the Volterra integral, the transformed signal $[\mathcal{V}_k x](t)$, for $t \geq 0$, can be obtained as the