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Distributed sampled-data control of nonholonomic multi-robot systems with proximity networks[☆]

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ABSTRACT

This paper considers the distributed sampled-data control problem of a group of mobile robots connected via distance-induced proximity networks. A dwell time is assumed in order to avoid chattering in the neighbor relations that may be caused by abrupt changes of positions when updating information from neighbors. Distributed sampled-data control laws are designed based on nearest neighbor rules, which in conjunction with continuous-time dynamics results in hybrid closed-loop systems. For uniform and independent initial states, a sufficient condition is provided to guarantee synchronization for the system without leaders. In order to steer all robots to move with the desired orientation and speed, we then introduce a number of leaders into the system, and quantitatively establish the proportion of leaders needed to track either constant or time-varying signals. All these conditions depend only on the neighborhood radius, the maximum initial moving speed and the dwell time, without assuming *a prior* properties of the neighbor graphs as are used in most of the existing literature.

1. Introduction

Cooperative control of multi-robot/agent systems (MRS/MAS) has generated wide interest in researchers in control and robotics communities. Compared with a single robot, multiple robots can cooperatively accomplish complicated tasks with the advantages of high efficiency and robustness to the link failures. Over the last decade, MRS have wide applications in implementing a large number of tasks ranging from coverage, deployment, rescue, to surveillance and reconnaissance. Among these tasks, a basic one is to reach synchronization, i.e., all robots reach the same state, which actually has close connection with many important engineering applications, such as rendezvous problem (Cortés, Martínez, & Bullo, 2006; Smith, Broucke, & Francis, 2007), agreement problem (Pease, Shostak, & Lamport, 1980), distributed optimization (Lobel & Ozdagar, 2011) and formation control (Cao, Yu, & Anderson, 2011).

Recently, the synchronization problem of MAS has been extensively studied in the literature where the neighbor relations are typically modeled as graphs or networks. For example, Jadbabaie, Lin, and Morse (2003) and Ren and Beard (2005), respectively, studied the first-order discrete-time MAS with undirected graphs and directed graphs. Olfati-Saber and Murray (2004) studied the MAS with first-order continuous-time dynamics. The nonholonomic unicycle MRS are investigated by Moshtagh, Michael, Jadbabaie, and Daniilidis (2009) and Montijano, Thunberg, Hu, and Sagüès (2013). MAS with nonlinear dynamics, time delays, and measurement noises are also considered (Moreau, 2005; Shi & Johansson, 2013; Wang & Liu, 2009; Xiao & Wang, 2008). In almost all existing results, the neighbor graphs are required to satisfy certain connectivity assumptions for synchronization. How to verify or guarantee such conditions has been a challenging issue. In order

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to maintain connectivity of dynamical communication graphs, potential function methods are commonly used when designing the distributed control laws (Ajorlou & Aghdam, 2013; Dimarogonas & Kyriakopoulos, 2007; Ji & Egerstedt, 2007).

For a real world MRS, it is more practical that the dynamics of the system are modeled in a continuous-time manner whereas the control laws are designed based on the sampled-data information. The sampled-data technique is of interest in many situations, such as unreliable information channels, limited bandwidth, transport delay. The synchronization of MAS with sampled-data control laws has been studied (Liu, Li, Xie, fu & Zhang, 2013; Xia & Chen, 2012), where the neighbor graphs are also required to satisfy certain connectivity assumptions. It is clear that the potential function techniques are not applicable for the analysis of MAS with continuous-time dynamics and sampled-data control, because connectivity of the networks might be lost between sampling instants. How to analyze the synchronization behavior of such kind of systems becomes more challenging. In this paper, we first present a distributed sampled-data algorithm for a group of nonholonomic unicycle robots with continuous-time dynamics, and provide a comprehensive analysis for the synchronization of the closed-loop hybrid system. In our model, each robot has limited sensing and communication range, and the neighbor relations are described by proximity networks. A dwell time is assumed when updating information from neighbors, implying that the control signals are kept constant between the sampled instants and only updated at discrete-time instants. With such sampled-data information, our design of distributed control laws based on nearest-neighbor rules will clearly result in a hybrid closed-loop system, which is different from the case of discrete-time MAS studied by Liu and Guo (2009) and Tang and Guo (2007), and is also different from the previous results given by Liu, Wang and Hu (2014) where the control law for the rotational speed is designed using the continuous-time information.

For a multi-agent system, we may design a distributed algorithm to guarantee synchronization of the system, but the synchronization state is inherently determined by the initial states and model parameters. In many practical applications, we expect that the system achieves a desired synchronization state and we can treat that state as a reference signal. The agents that have access to the reference signal are referred to as leaders, and they can help steer the MRS to the desired state. Although a large number of theoretical analysis and results for the leader–follower model have been provided, further theoretical investigation is still needed due to some limitations in the existing theory: (i) similar to the leaderless case, the neighbor graphs are required to be connected or contain spanning trees to guarantee that the followers track the reference signal (Jadbabaie et al., 2003; Tove, Dimarogonas, Egerstedt & Hu, 2010), but there are few results to address how to verify such conditions. (ii) in order to guide all agents to accomplish complicated tasks, such as tracking time-varying signals and the containment control problem, a number of (not only one) leaders need to be introduced into the system (Cao, Ren & Egerstedt, 2012; Couzin, Krause, Franks & Levy, 2005; Dimarogonas, Tsiotras, & Kyriakopoulos, 2009). However, quantitative theoretical results for the number of leaders needed are still lacking. Hence, this paper considers also a multi-unicycle system with multiple leaders and presents some new quantitative results. The sampled-data information is used to design the control laws for the followers and leaders. We analyze the MRS with heterogeneous agents where the leaders and followers have different dynamics since the reference signal is only obtained by the leaders, and quantitatively provide the proportion of leaders needed to track the reference signals.

The main contributions of this paper are summarized into the following three aspects. (i) For the leaderless case, we establish a sufficient condition, imposed on the neighborhood radius, the dwell time and the maximum moving speed, to guarantee synchronization of the nonholonomic unicycles, which overcomes the difficulty of requiring *a prior* connectivity assumption on neighbor graphs used in most of the existing results. (ii) For the leader–follower model, we provide the proportion of leaders needed to guide all robots to track a reference signal which can be constant or slowly time-varying. These quantitative results illustrate that adding leaders is a feasible approach to guide MRS to accomplish some complicated tasks. (iii) For both the leaderless case and leader-following case, we provide comprehensive analysis for nonlinear hybrid closed-loop systems. Different from the work of Liu and Guo (2009) and Tang and Guo (2007), we need to estimate the synchronization rate of the continuous-time variables (i.e., speed and orientation). Here the speed and orientation are determined by the corresponding values at sampling time instants and they are updated according to the states of relevant neighbors, and the neighbors are defined via the positions of all robots. Hence, the positions, orientations and moving speeds of all robots are coupled. We deal with the coupled relationships by combining the dynamical trajectories of the robots at discrete-time instants with the analysis of continuous-time dynamics in sampling intervals.

The rest of this paper is organized as follows. In Section 2, we present the problem formulation for a leaderless model and provide the main result for synchronization. In Section 3, we first investigate the leader-following problem where the leaders have constant reference signal, and quantitatively provide the ratio of the number of leaders to the number of followers needed to track the signal. We then extend our result to the dynamical tracking where the leaders have time-varying reference signal, and present some simulations to illustrate our theoretical results. Concluding remarks are presented in Section 4.

For a vector $x \in \mathbb{R}^m$, x' denotes the transpose of x, and $\|x\|$ denotes the 2-norm, i.e., $\|x\| = (x'x)^{1/2}$. For a square matrix $A = (a_{ij})_{n\times n}$, $\|A\|$ denotes the 2-norm of A, i.e., $\|A\| = \sqrt{\lambda_{\max} AA'}$. For any two positive sequences $\{a_n, n \geq 1\}$ and $\{b_n, n \geq 1\}$, $a_n = O(b_n)$ means that there exists a positive constant C independent of n, such that $a_n \leq Cb_n$ for any $n \geq 1$; $a_n = o(b_n)$ (or $(a_n \ll b_n)$) means that $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$; $a_n = \Theta(b_n)$, if there exist two positive constants C_1 and C_2, such that $C_1 b_n \leq a_n \leq C_2 b_n$.

2. Leaderless synchronization

2.1. Problem formulation

Consider a group of n unicycle robots (or agents) moving in a plane. For a robot i ($i = 1, 2 \cdots, n$), the position of its center at time t ($t \geq 0$) is denoted by $X_i(t) = (x_i(t), y_i(t))' \in \mathbb{R}^2$. The orientation and moving speed of each robot are affected by the states of its local neighbors. A pair of two robots is said to be neighbors if their Euclidean distance is less than a pre-defined radius r_n. We use $\mathcal{N}_i(t)$ to denote the set of the robot i's neighbors at time t, i.e.,

$$\mathcal{N}_i(t) = \left\{ j : \Delta_{ij}(t) < r_n \right\}, \qquad (1)$$

where $\Delta_{ij}(t) = \|X_i(t) - X_j(t)\|$ is the Euclidean distance between robots i and j. The cardinality of the set $\mathcal{N}_i(t)$, i.e., the degree of the agent i, is denoted as $d_i(t)$. When the robots move in the plane, the neighbor relations dynamically change over time. We use graph $G_t = \{V, E_t\}$ to describe the relationship between neighbors at time t, where the vertex set $V = \{1, 2, \ldots, n\}$ is composed of all robots, and the edge set is defined as $E_t = \{(i, j) \in V \times V : \Delta_{ij}(t) < r_n\}$. The neighbor graphs are distance-induced, and also called geometric graphs or proximity networks.

Let $\theta_i(t)$ and $v_i(t)$ denote the moving orientation and translational speed of the ith robot at time t. The dynamics of the robots