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Stochastic boundary control design for extensible marine risers in three dimensional space☆

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ABSTRACT

This paper presents a new design of boundary controllers for global practical $\mathcal{K}_\infty$-exponential p-stabilization of vibration motions of extensible marine risers in three-dimensional (3D) space under both stochastic and deterministic sea loads. The control design and analysis of well-posedness and stability of the closed-loop system are carried out based on a new Lyapunov-type theorem, which is developed for studying well-posedness and p-stability of a class of stochastic evolution systems (SESs) in Hilbert space. Since this theorem eases difficulties in verification of the coercivity condition but requires conditions of a form similar to Lyapunov-type theorems for stochastic lumped-parameter systems, it has a potential application to other stochastic distributed-parameter systems.

1. Introduction

Reducing the riser vibration by means of boundary feedback control is effective since this control system is simple for implementation and does not introduce drag. In existing works on boundary control of marine risers, e.g., Do (2011), Do (2017), Do (in press), Do and Pan (2008, 2009), Fard and Sagatun (2001), Ge, He, How, and Choo (2010), He and Ge (2015), He, Ge, How, Choo, and Hong (2011); He, Ge, Voon, How, and Choo (2014); He, Sun, and Ge (2015); He, Zhang, and Ge (2013), Lu, Chen, Yao, and Wang (2013) and Nguyen, Do, and Pan (2013), the Lyapunov direct method is used. The common feature includes: (1) search for a proper Lyapunov functional, which consists of the system energy plus a perturbed term to utilize the structural damping; (2) design a boundary control to make time derivative of the Lyapunov functional negative definite; and (3) perform well-posedness and stability analysis. There are two types of the perturbed term, which result in two types of boundary controls. Let $\boldsymbol{\eta}(s,t)$ denote the vector of the riser displacements with the spatial variable s and time t; $\boldsymbol{\eta}_t := \frac{\partial \boldsymbol{\eta}}{\partial t}$; $\boldsymbol{\eta}_s := \frac{\partial \boldsymbol{\eta}}{\partial s}$; $\boldsymbol{\eta}^B := \boldsymbol{\eta}|_{s=L}$; and $\boldsymbol{\Gamma}$, $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ be positive definite matrices. The two types of the perturbed term V_p result in either "PD" or "DD" boundary control $\boldsymbol{u}_b$ in the following table.

"PD" boundary control	"DD" boundary control
$V_p = \int_0^L \boldsymbol{\eta}^T \boldsymbol{\Gamma} \boldsymbol{\eta}_t ds$	$V_p = \int_0^L \boldsymbol{\eta}_s^T \boldsymbol{\Gamma} \boldsymbol{\eta}_t s ds$
$\boldsymbol{u}_b = -\boldsymbol{K}_1 \boldsymbol{\eta}^B - \boldsymbol{K}_2 \boldsymbol{\eta}_t^B$	$\boldsymbol{u}_b = -\boldsymbol{K}_1 \boldsymbol{\eta}_s^B - \boldsymbol{K}_2 \boldsymbol{\eta}_t^B$

It is crucial to choose proper matrices $\boldsymbol{\Gamma}$ (small), $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ to make the Lyapunov functional proper and its derivative negative definite. The "DD"-control introduces some (small) damping but requires measurement of slope $\boldsymbol{\eta}_s^B$ instead of displacement $\boldsymbol{\eta}^B$ in "PD"-control. Apart from the above Lyapunov-based approach, a novel method based on the backstepping method (Krstic, Kanellakopoulos, & Kokotovic, 1995) was developed to design boundary controllers for flexible systems in Bohm, Krstic, Kuchler, and Sawodny (2014), Krstic and Smyshlyaev (2008a,b), Krstic, Siranosian, Balogh, and Guo (2007) and Krstic, Siranosian, and Smyshlyaev (2006). This approach can introduce large damping to the system but is difficult to apply to the riser system in this paper due to difficulties in finding proper gain kernels.

Although sea loads on marine risers are both deterministic and stochastic (Faltinsen, 1993), the above works on boundary control of the risers have considered only deterministic loads. This deteriorates the control performance or even results in an unstable closed-loop system, see Remark 4.2.2. The stochastic loads on risers

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require: (1) an amendment in the riser dynamics in Do (2017); (2) tools for control design and stability analysis for stochastic partial differential equations (SPDEs). Well-posedness and stability of stochastic beams under Lipschitz conditions were studied in Brzezniak, Maslowski, and Seidler (2005), Chow (2007), Chow and Menaldi (2014) and Zhang (2007). Note that it is difficult to apply well-posedness results developed for parabolic SPDEs in Da Prato and Zabczyk (1992), Gawarecki and Mandrekar (2011), Liu (2006), Liu and Mandrekar (1997) and Pardoux (1979), to stochastic risers since their motions are described by hyperbolic SPDEs due to difficult verification of the coercivity condition.

The above review motivates a consideration of reducing vibration of extensible marine risers under stochastic and deterministic sea loads in 3D by boundary control. The main contributions of this paper include three folds. First, a mathematical model describing motion of extensible marine risers in 3D under both deterministic and stochastic sea loads is derived in an appropriate form for boundary control design. In comparison with the models used in existing works (e.g., Do, 2011, Do, 2017, Do, in press, Do & Pan, 2008, 2009, Fard & Sagatun, 2001, Ge et al., 2010, He & Ge, 2015, He et al., 2011, 2014, 2015, 2013, Lu et al., 2013, Nguyen et al., 2013), the stochastic components of sea loads are included.

Second, a Lyapunov-type theorem is proposed for study of well-posedness and stability (global p-stability, global (practical) $\mathcal{K}_\infty$-exponential p-stability) of a class of SESs. This theorem just requires: (1) continuity and local monotonicity conditions on the system functions; and (2) "usual" conditions on the Lyapunov function and its infinite generator. The usual conditions are of a form similar to those for stochastic lumped-parameter systems in Deng, Krstic, and Williams (2001), Khasminskii (1980) and Mao (2007). Thus, difficulties in verification of the coercivity condition as in Gawarecki and Mandrekar (2011) are relaxed.

Third, boundary controllers are designed to achieve global well-posedness and (practical) $\mathcal{K}_\infty$-exponential p-stability of the marine risers based on the proposed Lyapunov-type theorem. In comparison with existing works (Do, 2011, 2017, in press; Do & Pan, 2008, 2009; Fard & Sagatun, 2001; Ge et al., 2010; He & Ge, 2015; He et al., 2011, 2014, 2015, 2013; Lu et al., 2013; Nguyen et al., 2013), the control design in this paper shares a common feature of finding a proper Lyapunov function. However, the difficulty is to design boundary controls to make the infinite generator (instead of time derivative) of the Lyapunov function negative definite. The infinite generator contains Hessian terms due to stochastic components. Although the final form of the boundary controllers is "PD", the Hessian terms make the control design much harder than those in the existing works. Moreover, well-posedness and stability analysis need to use the Lyapunov-type theorem developed for SESs in this paper. Note that the works in Do (2017) for vibration control and Do (in press) for large defection control are developed for risers/beams under deterministic loads, and that well-posedness and stability analysis tools developed in Do (2017) and Do (in press) are for deterministic evolution systems, see also Remarks 2.1 and 3.1. The common feature among Do (2017) and Do (in press) and this paper is the treatment in Hilbert space.

Notations. We denote by the symbols $\wedge$ and $\vee$ the min and max operators, respectively. These operators are also applied to more than two arguments (e.g., $a \wedge b \wedge c := \min(a, b, c)$ and $a \vee b \vee c := \max(a, b, c)$; a, b, and c are scalars). The symbol $\mathbb{E}$ denotes the expected value. The symbol col denotes the column operator.

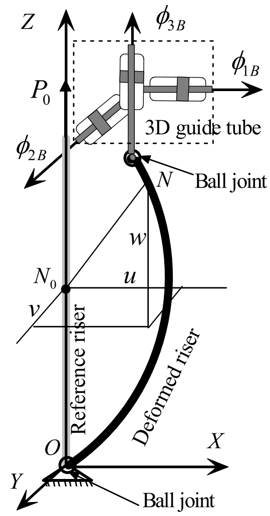

Fig. 1. Riser coordinates and boundary conditions.

2. Problem formulation

2.1. Stochastic model of marine risers in 3D

The riser's configuration is shown in Fig. 1. Let (u, v, w) denote (perturbed) displacements of the riser along the OX-, OY-, and OZ-axis from the point N_0 of the reference riser center line to the point N of the deformed riser center line at time t. To derive the equations of motion of the riser, we use the extended Hamiltonian principle (Craig & Kurdila, 2006):

$$\int_{t_1}^{t_2} \delta(\mathcal{T} - \mathcal{V} + \mathcal{W}_c)dt = 0, \tag{1}$$

where $\mathcal{T}$ is the kinetic energy, $\mathcal{V}$ is the potential energy, $\mathcal{W}_c$ denotes the virtual work by nonconservative forces and the virtual momentum transport at boundaries, and δ is variation taken during the time interval. Assume that the plane sections of the riser remain plane after deformation; the riser is locally stiff and its material is homogeneous, isotropic and linearly elastic; torsional moments are neglected. The kinetic energy $\mathcal{T}$ is

$$\mathcal{T} = \frac{m_o}{2}\int_0^L (u_t^2 + v_t^2 + w_t^2)dz + \frac{m_{1H}}{2}u_t^2(L,t) + \frac{m_{2H}}{2}v_t^2(L,t) + \frac{m_{3H}}{2}w_t^2(L,t), \tag{2}$$

where the arguments (z, t) are dropped for clarity, $m_o = \varrho A + m_a$ with ϱ the mass per unit length, A the cross section area of the riser, and m_a the added mass; L is the riser length at the initial configuration; the symbol $\bullet_t$ denotes $\frac{\partial \bullet}{\partial t}$; $m_{iH}, i = 1, 2, 3$ are the mass of the actuators; and $(u_t(L,t), v_t(L,t), w_t(L,t))$ denote the value of (u_t, v_t, w_t) at $z = L$. If an actuator allowing parallel displacements is used, then $m_{1H} = m_{2H} = m_{3H}$. If individual actuators are installed on a 3D guide tube mechanism, then $m_{1H} \neq m_{2H} \neq m_{3H}$, see Fig. 1. The potential energy $\mathcal{V}$ consisting of the strain energy and the energy due to the riser tension and bending moment is:

$$\mathcal{V} = \frac{EA}{2}\int_0^L \varepsilon^2 dz + \frac{P_0}{2}\int_0^L (u_z^2 + v_z^2)dz + \frac{EI}{2}\int_0^L (\kappa_1^2 + \kappa_2^2)dz, \tag{3}$$

where E is Young's modulus; I is the moment of inertia of the riser cross section; P_0 is the constant axial force; $\varepsilon(z,t)$ is the axial strain; κ_1 and κ_2 are bending curvatures in OXZ and OYZ planes;