Automatica 77 (2017) 268–278

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

On robust input design for nonlinear dynamical models ☆

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ARTICLE INFO

Article history:
Received 5 October 2015
Received in revised form
31 May 2016
Accepted 7 November 2016

Keywords:
System identification
Input design
Particle filter
Nonlinear systems

ABSTRACT

We present a method for robust input design for nonlinear state-space models. The method optimizes a scalar cost function of the Fisher information matrix over a set of marginal distributions of stationary processes. By using elements from graph theory we characterize such a set. Since the true system is unknown, the resulting optimization problem takes the uncertainty on the true value of the parameters into account. In addition, the required estimates of the information matrix are computed using particle methods, and the resulting problem is convex in the decision variables. Numerical examples illustrate the proposed technique by identifying models using the expectation–maximization algorithm.

1. Introduction

Input design is concerned with generating an excitation signal that maximizes the information retrieved from an experiment, quantified in terms of a cost function related to the intended model application. Some of the initial contributions are discussed in Cox (1958) and Goodwin and Payne (1977). Since then, many contributions to the subject have been presented; see e.g. Fedorov (1972), Gevers (2005), Hildebrand and Gevers (2003), Whittle (1973) and the references therein.

In the case of dynamical systems, the existing results on input design are mostly focused on linear models. The assumption of a linear model structure can reduce the complexity of the problem, leading to formulations that are convex in the decision variables (Ljung, 1999). In this case, the convexity of the problem is achieved by designing the power spectrum of the input signal. Several approaches to input design for linear models have been proposed in the literature involving, e.g., linear matrix inequalities (LMI) (Jansson & Hjalmarsson, 2005; Lindqvist & Hjalmarsson, 2000), Markov chains (Brighenti, Wahlberg, & Rojas, 2009), and time domain techniques (Suzuki & Sugie, 2007). With the exception of the methods in Jansson and Hjalmarsson (2005) and Lindqvist and Hjalmarsson (2000) that rely on convexification of the problem, the previous formulations are non-convex, which illustrates the difficulty of solving the input design problem.

In recent years, there has been an interest to extend the input design methods to nonlinear (NL) model structures. The main issue is that the frequency domain methods cannot be applied, which restricts the applicability of convex formulations (Jansson & Hjalmarsson, 2005; Lindqvist & Hjalmarsson, 2000). The first approaches to the problem considered NL finite impulse response (FIR) models (Hjalmarsson & Mårtensson, 2007; Larsson, Hjalmarsson, & Rojas, 2010). In Hjalmarsson and Mårtensson (2007) the input design problem is analyzed using the knowledge from linear systems, while in Larsson et al. (2010) the input design problem is solved over a set of marginal distributions of stationary processes.

An extension of the input design problem to structured NL models is presented in Vincent, Novara, Hsu, and Poolla (2009, 2010), where the model is given by an interconnection of linear models and static nonlinearities. The class of NL model structures is also generalized in Forgione, Bombois, Van den Hof, and Hjalmarsson (2014), where the input signal is optimized over an alphabet with finite cardinality. A multilevel excitation is also considered in De Cock, Gevers, and Schoukens (2013) for identification of Wiener models. The restriction to a finite alphabet is relaxed in Gopaluni, Schön, and Wills (2011), where an ARX process is designed as input for the identification of NL state-space models (SSMs). A graph theoretical methodology to design

☆ This work was supported by the Swedish Research Council under contracts 621-2013-5524, 621-2011-5890 and 621-2009-4017, and by the European Research Council under the advanced grant LEARN, contract 267381. The material in this paper was partially presented at the 19th World Congress of the International Federation of Automatic Control (IFAC 2014), August 24–29, 2014, Cape Town, South Africa. This paper was recommended for publication in revised form by Associate Editor Alessandro Chiuso under the direction of Editor Torsten Söderström.

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http://dx.doi.org/10.1016/j.automatica.2016.11.030
0005-1098/

inputs for identification of NL output-error models is developed in Valenzuela, Rojas, and Hjalmarsson (2013, 2015), which is extended to NL-SSMs in Valenzuela, Dahlin, Rojas, and Schön (2014).

The existing results on input design allow to optimize input signals when the system contains NL functions, but the restrictions on the system dynamics and/or the input structure are the main limitations of most of the previous contributions. Moreover, with the exception of multilevel excitation (Forgione et al., 2014; Larsson et al., 2010), and stationary processes (Brighenti et al., 2009; Valenzuela et al., 2014, 2013, 2015), most of the proposed methods cannot handle amplitude limitations on the input signal, which could arise due to physical and/or safety reasons.

The previously mentioned input design methods assume that a prior estimate of the model parameters is available for optimization. The requirement of such knowledge is a common issue in input design and different solutions to this difficulty have been proposed (Gerencsér, Hjalmarsson, & Mårtensson, 2009; Rojas, Aguero, Welsh, Goodwin, & Feuer, 2012; Rojas, Hjalmarsson, Gerencsér, & Mårtensson, 2011; Rojas, Welsh, Goodwin, & Feuer, 2007; Welsh & Rojas, 2009).

The main contribution of this article is to propose a robust input design method for the identification of NL-SSMs with input constraints, which extends the model structure considered in Valenzuela et al. (2013, 2015), and the nominal input design presented in Valenzuela et al. (2014). The optimal input signal is considered to be a realization of a stationary process, which maximizes a scalar function of the Fisher information matrix (FIM). To pose a tractable convex problem, we restrict the optimization to a set of marginal distributions of stationary processes with a finite alphabet. This set is a polytope and hence it can be described by a convex combination of its vertices. The vertices are cumulative distribution functions that can be found using de Bruijn graphs, as discussed in Valenzuela et al. (2014, 2013). Since the vertices of the set are known, we can draw an input realization and compute an estimate of the FIM for each vertex using particle methods (Del Moral, Doucet, & Jasra, 2006; Doucet & Johansen, 2011). The estimates of the information matrices are computed using the method introduced in Segal and Weinstein (1989), which only requires one realization of the input–output data, and thus reducing the computational effort when estimating the FIM compared to Valenzuela et al. (2014).

To make the input design robust against model uncertainty, the optimization problem considers a measure of the uncertainty of the parameters, which relaxes the requirements on the knowledge of the system assumed in Valenzuela et al. (2014, 2013, 2015). The method is illustrated through numerical examples, where the designed input is employed to identify a NL-SSM using the expectation–maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977; McLachlan & Krishnan, 2008; Schön, Wills, & Ninness, 2011).

The rest of this article is organized as follows. Section 2 states the problem and the main challenges when designing inputs for identification of NL-SSM. Section 3 describes the graph theoretical approach to input design. Section 4 discusses the estimation of the FIM using particle methods. A summary of the proposed robust input design method is presented in Section 5. The generation of the optimal input signal is addressed in Section 6. To illustrate the correctness and utility of the method, two numerical examples are discussed in Section 7. Finally, Section 8 concludes this work and presents future research directions.

Notation: Throughout this article, $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{R}^p$ denotes the set of p-dimensional vectors with real entries, $\mathbb{R}^{p\times r}$ is the set of $p \times r$ matrices with real entries, and $\mathbb{R}_+$ the set of positive real numbers. $\mathbf{P}$, $\mathbf{E}$, and $\text{Var}\{\cdot\}$ stand for a probability measure, the expected value, and the variance, respectively. Sometimes a subscript is added to $\mathbf{P}$ and $\mathbf{E}$ to clarify the stochastic process considered by these operators. Finally, for a finite set $\mathcal{A}$, $|\mathcal{A}|$ denotes its cardinality.

2. Problem formulation

Consider a NL-SSM described for all $t \geq 1$ by

$$x_t|x_{t-1} \sim f_\theta(x_t|x_{t-1}, u_{t-1}), \tag{1a}$$
$$y_t|x_t \sim g_\theta(y_t|x_t, u_t), \tag{1b}$$
$$x_0 \sim \mu_\theta(x_0), \tag{1c}$$

where f_θ, g_θ, and μ_θ denote probability density functions (pdf) parameterized by $\theta \in \Theta \subset \mathbb{R}^{n_\theta}$ (where Θ is an open set). Here, $u_t \in \mathbb{R}^{n_u}$ denotes the input signal, $x_t \in \mathbb{R}^{n_x}$ are the (unobserved/latent) internal states, and $y_t \in \mathbb{R}^{n_y}$ are the measured outputs.

The objective in this article is to design an input signal $u_{1:n_{\text{seq}}} := (u_1, \ldots, u_{n_{\text{seq}}})$, as a realization of a stationary process, such that the NL-SSM (1) can be identified with maximum accuracy as defined by a scalar function of the FIM (Ljung, 1999). In the sequel, we assume that there exists at least one parameter $\theta_0 \in \Theta$ such that the model (1) exactly describes the pdfs of the system, i.e., there is no undermodeling (Ljung, 1999).

Given $u_{1:n_{\text{seq}}}$, the FIM is defined as

$$\mathcal{I}_F^{n_{\text{seq}}}(\theta_0) := \mathbf{E}\left\{\mathcal{S}(\theta_0)\mathcal{S}^\top(\theta_0) | \, u_{1:n_{\text{seq}}}\right\}, \tag{2}$$

where $\mathcal{S}(\theta_0)$ denotes the score function, i.e.,

$$\mathcal{S}(\theta_0) := \nabla_\theta \, \ell_\theta(y_{1:n_{\text{seq}}})\big|_{\theta=\theta_0}. \tag{3}$$

Here, $\ell_\theta(y_{1:n_{\text{seq}}})$ denotes the log-likelihood function

$$\ell_\theta(y_{1:n_{\text{seq}}}) := \log p_\theta(y_{1:n_{\text{seq}}}|u_{1:n_{\text{seq}}}). \tag{4}$$

We note that the expected value in (2) is with respect to the stochastic processes in (1). Since we consider $u_{1:n_{\text{seq}}}$ as a realization of a stationary process, here we are interested in the *per-sample* FIM, defined as

$$\begin{aligned}\mathcal{I}_F(\theta_0) &:= \frac{1}{n_{\text{seq}}}\mathbf{E}_u\left\{\mathcal{I}_F^{n_{\text{seq}}}(\theta_0)\right\} \\ &= \frac{1}{n_{\text{seq}}}\mathbf{E}\left\{\mathcal{S}(\theta_0)\mathcal{S}^\top(\theta_0)\right\},\end{aligned} \tag{5}$$

where the expected value in (5) is over both the stochastic processes in (1), and the stochastic vector $u_{1:n_{\text{seq}}}$.

We note that (5) depends on the cumulative distribution function (cdf) of $u_{1:n_{\text{seq}}}$, denoted by $P_u(u_{1:n_{\text{seq}}})$. Therefore, the input design problem is to find a cdf $P_u^{\text{opt}}(u_{1:n_{\text{seq}}})$ which maximizes a scalar function of (5), $\mathcal{H} : \mathbb{R}^{n_\theta\times n_\theta} \times \Theta \to \mathbb{R}$, where[1] $\mathcal{H}$ is a matrix concave function in its first argument (Boyd & Vandenberghe, 2004, pp. 108). Different choices of $\mathcal{H}$ have been proposed in the literature, see e.g. Rojas et al. (2007); some examples are $\mathcal{H}(A, \theta) = \log\det(A)$, and $\mathcal{H}(A, \theta) = -\text{tr}\{A^{-1}\}$ for $A \in \mathbb{R}^{n_\theta\times n_\theta}$ non-singular.

To simplify our problem, we will assume that u_t can only adopt a finite number c_{seq} of values. We denote this set of values as $\mathcal{C}$. With the previous assumption, we can define the following set:

$$\begin{aligned}\mathcal{P}_\mathcal{C} := \Bigg\{ p_u : \mathcal{C}^{n_{\text{seq}}} \to \mathbb{R} \,\Bigg|\, & p_u(\mathbf{x}) \geq 0, \ \forall \mathbf{x} \in \mathcal{C}^{n_{\text{seq}}}; \\ & \sum_{\mathbf{x}\in\mathcal{C}^{n_{\text{seq}}}} p_u(\mathbf{x}) = 1; \\ & \sum_{v\in\mathcal{C}} p_u(v, \mathbf{z}) = \sum_{v\in\mathcal{C}} p_u(\mathbf{z}, v), \forall \mathbf{z} \in \mathcal{C}^{n_{\text{seq}}-1} \Bigg\}.\end{aligned} \tag{6}$$

[1] We let $\mathcal{H}$ have an argument on Θ as the function can explicitly depend on the model parameter.