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Brief paper

A proportional-integral extremum-seeking controller design technique☆

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ABSTRACT

This paper proposes an alternative extremum seeking control design technique for the solution of real-time optimization control problems. The technique considers a proportional-integral approach that minimizes the impact of a time-scale separation on the transient performance of the extremum-seeking controller. It is assumed that the equations describing the dynamics of the nonlinear system and the cost function to be minimized are unknown and that the objective function is measured. The dynamics are assumed to be of relative degree one with respect to the objective function. Two simulation examples are presented to demonstrate the effectiveness of the proposed technique.

1. Introduction

Extremum-seeking control (ESC) is an established control technology that is used to solve various classes of real-time optimization problems (Tan, Moase, Manzie, Nesic, & Mareels, 2010). It is an extremely appealing approach in practice since it does not require any specific knowledge of the process dynamics and the cost function. However, the application of ESC is generally limited to the solution of steady-state optimization problem in which the optimization procedure is handled at a much slower time-scale than the unknown process dynamics. As demonstrated in the seminal work of Krstic and co-workers (see Krstic & Wang, 2000), there are extremely valid technical justifications for the need for a time-scale separation in ESC. With the help of averaging analysis and singular perturbation methods, the analysis leads to a very general convergence results with minimal assumptions on the process dynamics and cost function. The stability analysis relies on two components: (1) an averaging analysis of the persistently perturbed ESC loop and (2) a time-scale separation of ESC closed-loop dynamics between the fast transients of the system dynamics and the slow quasi steady-state extremum-seeking task. Unfortunately, the need for a time-scale separation inevitably leads to slow transient performance.

Many researchers have considered various approaches to overcome the limitations of ESC. In Krstic (2000), performance improvements of ESC were considered using a dual-mode ESC and a phase compensation strategy. The non-local properties on ESC was studied in Tan, Nesic, and Mareels (2006). This work extends the work in Krstic and Wang (2000) by considering the case where the fast dynamics can be assumed to be uniformly global asymptotically stable along the equilibrium manifold. In Adetola and Guay (2007) and Guay, Dochain, and Perrier (2004), an alternative ESC algorithm is considered where an adaptive control and estimation approach is used. The key aspect of this approach is that the equilibrium map is parameterized and the parameters are estimated with the help of a tailored adaptive estimation technique. The results in Nesic, Mohammadi, and Manzie (2010) unify the approaches based on singular perturbation and parameter estimation by considering the case where the objective function is parameterized in a known fashion. Moase and Manzie (2012) propose a fast extremum seeking approach for a class of Wiener–Hammerstein processes. Recent work reported in Ghaffari, Krstic, and Nesic (2012) and Moase, Manzie, and Brear (2010) have proposed a Newton-based extremum-seeking technique that provides an estimate of the inverse of the Hessian of the cost function. This technique can effectively alleviate the convergence problems associated with the increase of the gain of the Newton update. A time-varying ESC design technique was proposed in Guay and Dochain (2015) that utilizes an alternative mechanism

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for the estimation of the gradient. The approach is shown to provide significant improvement in transient performance. Other alternative techniques such as proposed by Zhang and Ordóñez (2009) make use of sampled gradient measurements to improve the convergence properties of ESC techniques that implement numerical optimization techniques. A sliding-mode approach is presented in Fu and Özgüner (2011). The main feature of existing approaches is the use of some variation of a gradient descent algorithm that requires the integration of an estimate of the gradient of the unknown cost function subject to some form of filter and a dither signal.

Although the limitations associated with the tuning of ESC are generally well understood, the two time-scale nature of ESC systems remains problematic. Under the two time-scale assumption, the optimization operates at a quasi steady-state, or slow, time-scale such that the search for optimal operating conditions does not affect the process dynamics. To overcome the time-scale separation, one must incorporate some knowledge of the transient behavior of the process dynamics. In the case where a model is available, one can use adaptive extremum seeking technique as proposed in Guay and Zhang (2003) to stabilize a nonlinear system to the unknown optimum of a known but unmeasured cost function. Such techniques can solve the steady-state optimization ESC problem without the need for time-scale separation. In Scheinker and Krstic (2013), Lie bracket averaging techniques are considered to stabilize unknown dynamical systems using ESC. Although the objectives of the study presented in Scheinker and Krstic (2013) are different from those pursued in this study, the Lie bracket averaging approach proposed remains relevant as a potential mechanism to remove the time-scale separation requirement in ESC. At the current time, its application to ESC remains based on the assignment of a time-scale parameter.

ESC problems cannot be currently solved in the absence of time-scale separations if explicit process models are not available. This paper attempts to bridge this gap in the application of ESC. It develops and generalizes a proportional-integral ESC (PI-ESC) design technique first introduced in Guay and Dochain (2014). The PI-ESC operates using two modes. The integral control mode corresponds to the standard ESC task and is used to identify the steady-state optimum conditions. The proportional control mode is designed to ensure that the measured cost function is optimized instantaneously. The combined action of the two modes provides an additional proportional action tuning parameter that can be used to minimize and, potentially eliminate, the impact of time-scale separation on the transient performance of ESC systems. In this manuscript, we establish new convergence properties of the PI-ESC proposed in Guay and Dochain (2014). We also provide precise convergence conditions that can be extended to systems where the cost function is not necessarily of relative degree one. It is also shown that the PI-ESC can be used to stabilize a class of unstable control systems.

The paper is organized as follows. A brief description of the ESC problem is given in Section 2. The proposed proportional-integral ESC controller is described in Section 3. Two simulation examples are presented in Section 4 followed by brief conclusions in Section 5.

2. Problem description

We consider a class of nonlinear systems of the form:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, u is the vector of input variables taking values in $\mathcal{U} \subset \mathbb{R}^p$ and $y \in \mathbb{R}$ is the variable to be minimized. It is assumed that $f(x)$ and $g(x)$ are smooth vector valued functions of x and that $h(x)$ is a smooth function of x.

The objective is to steer the system to the equilibrium x^* (and corresponding u^*) that achieves the minimum value of $y(= h(x^*))$. Some additional assumptions are required concerning the cost function $h(x)$.

Assumption 1. The cost $h(x)$ is such that

(1) $\frac{\partial h(x^*)}{\partial x} = 0$
(2) $\frac{\partial^2 h(x)}{\partial x \partial x^T} > \beta I, \ \forall x \in \mathbb{R}^n$

where β is a strictly positive constant.

We denote the Lie derivatives of $h(x)$ with respect to $f(x)$ and $g(x)$ as $L_f h$ and $L_g h$, respectively. The Lie derivative is the directional derivative of the function $h(x)$ given by:

$$L_f h = \frac{\partial h}{\partial x} f, \qquad L_g h = \frac{\partial h}{\partial x} g.$$

In this study, we will consider a state-feedback control of the form $u = -k^* L_g h^T + \hat{u}$ where $\hat{u}$ is a constant vector and, k^*, a nonnegative constant.

The equilibrium (or steady-state) map is the n dimensional vector $x = \pi(\hat{u})$ that solves the following equation:

$$f(\pi(\hat{u})) - k^* g(\pi(\hat{u})) L_g h(\pi(\hat{u}))^T + g(\pi(\hat{u}))\hat{u} = 0.$$

The corresponding equilibrium cost function is given by:

$$y = h(\pi(\hat{u})) = \ell(\hat{u}). \tag{3}$$

At equilibrium, the problem is reduced to finding the minimizer u^* of $y = \ell(u^*)$. In the following, we let $\mathcal{D}(\hat{u})$ represent a neighborhood of the equilibrium $x = \pi(\hat{u})$. We also require the following properties for the dynamics:

Assumption 2. The dynamics (1) are such that the output $h(x)$ has strong relative degree one $\forall x \in \mathcal{D}(\hat{u})$ and $\forall \hat{u} \in \mathcal{U}$.

Under Assumption 2, one can find a diffeomorphism $[\xi, y] = \Theta(x)$ that transforms the control system dynamics into the well-known Byrnes–Isidori form given by:

$$\dot{\xi} = \phi(\xi, y)$$
$$\dot{y} = L_f h + L_g h u$$

where $\xi \in \mathbb{R}^{n-1}$, ϕ is a smooth vector valued function of ξ and $y = h(x)$. The next assumption associates a stability result to the control system using the proposed state-feedback controller.

Assumption 3. The normal form dynamics have the following property:

- the zero dynamics of the system are input to state stable from y to ξ with Lyapunov function $W(\xi)$,
- the function $W(\xi) + h(x)$ is such that:
  $$\beta_1 \|x - \pi(\hat{u})\|^2 \leq W(\xi) + h \leq \beta_2 \|x - \pi(\hat{u})\|^2$$
  for some positive constants β_1, β_2,
- there exists a nonnegative constant k^* such that:
  $$\frac{\partial W}{\partial \xi}\phi(\xi, y) + L_f h - k^* \left\|L_g h\right\|^2 + L_g h \hat{u} \leq -\alpha_3 \|x - \pi(\hat{u})\|^2 \tag{4}$$
  for some positive constant α_3 $\forall x \in \mathcal{D}(u)$ and $\forall \hat{u} \in \mathcal{U}$.

Finally, the following additional assumption concerning the steady-state cost function $\ell(u)$ is required.