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Brief paper

# Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances☆

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ABSTRACT

Different from traditional centralized control, one major challenge of distributed consensus tracking control lies in the constraint that the desired reference trajectory is only accessible by part of the subsystems. Currently, most existing schemes require the availability of partial knowledge of the reference trajectories to all of the subsystems or information exchange of local control inputs. In this paper, we investigate distributed adaptive consensus tracking control without such requirements for nonlinear high-order multi-agent systems subjected to mismatched unknown parameters and uncertain external disturbances. By introducing compensating terms in a smooth function form of consensus errors and certain positive integrable functions in each step of virtual control design, a new backstepping based distributed adaptive control protocol is proposed. An extra estimator is designed in each subsystem to handle the parametric uncertainties involved in its neighbors' dynamics, which avoids information exchange of local neighborhood consensus errors among connected subsystems. It is shown that global uniform boundedness of all the closed-loop signals and asymptotically output consensus tracking can be achieved. Simulation results are provided to verify the effectiveness of our scheme.


## 1. Introduction

Consensus of multi-agent systems, due to its wide potential applications, has become a rapidly emerging topic in various research communities over the past decades. Distributed consensus control normally aims at achieving an agreement for the states or the outputs of network-connected systems, by designing controller for each subsystem based on only locally available information collected within its neighboring area. This control issue can be further classified into leaderless consensus control (Jadbabaie, Lin, & Morse, 2003; Moreau, 2005; Ren & Beard, 2005) and leader-following consensus control (Arcak, 2007; Bai, Arcak, & Wen, 2008, 2009; Hong, Hu, & Gao, 2006; Ren, 2007) according to whether the final consensus values are predetermined. Note that in most of currently available results on the latter issue, the desired references are set by the behaviors of specific leaders with similar dynamics to the followers and zero/known inputs. With the consideration of more general cases of desired trajectories, such issue is also referred to as consensus tracking control in some references such as Li, Liu, Ren, and Xie (2013), Wang, Huang, Wen, and Fan (2014), Yoo (2013) and Zhang and Lewis (2012).

As opposed to traditional tracking control of single system, the main challenge of distributed consensus tracking control of multi-agent systems lies in the constraint that the common time-varying reference trajectory is only known by part of the subsystems. Some effective distributed control protocols have been proposed in this area. In Bai et al. (2008, 2009) and Hong et al. (2006), partial knowledge of the reference trajectories are assumed available to all of the subsystems. Distributed observers are then designed in

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the subsystems which cannot fully access the reference trajectory to estimate its remaining uncertainties. In Ren (2007), to generate the local control law of each subsystem, the information of both states and control signals of its neighbors need be collected. Thus perfect consensus tracking can be achieved though the reference trajectory is totally unknown by some subsystems. However, such design of mutually dependent inputs may bring new challenges during implementation if they are generated without a prescribed priority (Wang, Wen, Huang, & Li, 2016). Alternative solutions to asymptotically consensus tracking are provided in Dong (2012) and Li et al. (2013), by introducing signum functions of local consensus errors in the proposed distributed control approaches.

It is worth pointing out that most of the aforementioned results are developed based on system models with relatively simple structures such as pure integrators or linear systems. Besides, intrinsic subsystem uncertainties are not considered although they are often unavoidable in modeling and control of practical systems. As we know, adaptive control is a promising tool to handle parametric and structural uncertainties in the systems. However, the results on distributed adaptive consensus control of uncertain multi-agent systems are still limited due to the difficulty in constructing adaptive laws with only local interactions (Mei, Ren, & Ma, 2011). In Das and Lewis (2010), first-order nonlinear multi-agent systems with unknown nonlinear dynamics and disturbances are considered. By incorporating neural network and robust control techniques, semi-global uniform ultimate boundedness of consensus errors is ensured if the local control gains are chosen to be sufficiently large. The results are extended to systems with higher-order multi-agent systems in El-Ferik, Qureshi, and Lewis (2014) and Zhang and Lewis (2012). In Yu and Xia (2012), it is assumed that the reference trajectory is linearly parameterized and the basis function vectors are known by all the subsystems. Then distributed adaptive control algorithm is presented for achieving perfect consensus tracking of first-order multi-agent systems with unknown parameters. The results are extended in Yu, Shen, and Xia (2013) to solve adaptive finite-time consensus problem of uncertain higher-order distributed agents. Nevertheless, the proposed distributed adaptive laws can only be implementable with extra information transmission of local neighborhood consensus errors among the linked subsystems. Based on similar assumptions on linearly parameterized reference trajectories, two new distributed adaptive control schemes are proposed recently (Hu & Zheng, 2014; Wang et al., 2014) and the convergence of consensus tracking errors to zero is established. In Hu and Zheng (2014), second-order leader–follower system with unknown dynamics and relative position measurements is treated. In Wang et al. (2014), output consensus tracking of nonlinear multi-agent systems in parametric strict feedback form (Krstic, Kanellakopoulos, & Kokotovic, 1995) is investigated. Similar problem to Wang et al. (2014) is handled in Yoo (2013) by adopting dynamic surface design approach and semi-globally uniformly ultimately bounded consensus tracking errors are finally obtained.

In this paper, a new distributed adaptive backstepping control scheme is developed to achieve output consensus tracking for nonlinear multi-agent systems. The main contributions of this paper can be summarized as follows.

- The considered multi-agent system model is more general than those in most existing results on distributed consensus control including Arcak (2007), Bai et al. (2008, 2009), Das and Lewis (2010), Hong et al. (2006), Ren (2007) and Yu and Xia (2012) in the following terms. (i) The subsystems are nonlinear and allowed to have arbitrary relative degree and nonidentical dynamics; (ii) intrinsic mismatched unknown parameters and uncertain disturbances are simultaneously involved.
- In contrast to Bai et al. (2008, 2009), Hu and Zheng (2014), Wang et al. (2014) and Yu and Xia (2012), the assumptions of linearly parameterized reference signals and the corresponding basis function vectors being known by all subsystems are no longer needed. In this paper, it is assumed that the desired trajectory $y_r$ is known exactly for only part of the subsystems in the group, while it is only known that $|\dot{y}_r|$ is upper bounded by an arbitrarily unknown constant for the rest subsystems.
- It is worth noting that the non-differentiable signum function based distributed control approaches in Dong (2012) and Li et al. (2013) are not applicable to solve the output consensus tracking of the considered multi-agent system under the relaxed assumption on the reference signal. To address this issue, new compensating terms are introduced in the form of smooth functions with respect to consensus errors and certain positive integrable time-varying functions. Then the effects invoked by the uncertainties including bounded external disturbances and unknown $\dot{y}_r$ in part of subsystems can be counteracted completely. It is shown that global uniform boundedness of all the closed-loop signals and asymptotically consensus tracking for all subsystem outputs can be achieved in this paper, as opposed to El-Ferik et al. (2014), Yoo (2013) and Zhang and Lewis (2012).
- In each subsystem $i$, extra estimates for the external uncertainties in its neighbors' system dynamics are developed with only locally available information (i.e. $x_i$, $x_j$ for $j \in \mathcal{N}_i$). Additional information transmission of local control inputs or local neighborhood consensus errors among connected subsystems required in Ren (2007), Yu et al. (2013) and Yu and Xia (2012) can be avoided.

Simulation results on an application example with five one-link manipulators are provided to verify the effectiveness of the proposed distributed adaptive control scheme.

## 2. Problem formulation

### 2.1. System model

In this paper, we consider a group of $N$ nonlinear subsystems which can be transformable to the following parametric strict-feedback form.

$$\begin{aligned}
\dot{x}_{i,q} &= x_{i,q+1} + \varphi_{i,q}(x_{i,1}, \ldots, x_{i,q})^T \theta_i + d_{i,q}(t), \\
&\quad q = 1, \ldots, n-1 \\
\dot{x}_{i,n} &= b_i \beta_i(x_i) u_i + \varphi_{i,n}(x_i)^T \theta_i + d_{i,n}(t) \\
y_i &= x_{i,1}, \quad \text{for } i = 1, \ldots, N
\end{aligned} \tag{1}$$

where $x_i = [x_{i,1}, \ldots, x_{i,n}]^T \in \Re^n$, $u_i \in \Re$, $y_i \in \Re$ are the states, the control input and the output of the $i$th subsystem, respectively. $\theta_i \in \Re^{p_i}$ is a vector of unknown constants and the control coefficient $b_i \in \Re$ is an unknown non-zero constant. $\varphi_{i,j} : \Re^j \to \Re^{p_i}$ for $j = 1, \ldots, n$ and $\beta_i : \Re^n \to \Re^1$ are known smooth nonlinear functions. $d_{i,q}(t) \in \Re$ for $q = 1, \ldots, n$ denotes uncertain external disturbances.

### 2.2. Information transmission condition among the N subsystems

Suppose that the information transmission condition among the group of $N$ subsystems can be represented by an undirected graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ denotes the set of indexes corresponding to each subsystem, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges between two distinct subsystems. Since $\mathcal{G}$ is undirected, if the edge $(i, j) \in \mathcal{E}$, then $(j, i) \in \mathcal{E}$. This indicates that subsystems $i$ and $j$ are directly connected with each other, i.e. they can obtain state and structure information from each other. In this case, subsystem $j$ is called a neighbor of subsystem $i$, and vice versa. We denote the set of neighbors for subsystem $i$ as $\mathcal{N}_i \triangleq \{j \in \mathcal{V} :$