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Brief paper

Two equivalent sets: Application to singular systems ☆

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ABSTRACT

In this paper, we consider the problem of state feedback control design for continuous singular systems by applying equivalent sets technique. A new formulation of dissipativity condition is proposed. Based on this condition, the desired state feedback controller is designed such that the closed-loop system is admissible and dissipative. For singular Markovian systems, necessary and sufficient conditions are proposed for the system to be admissible and for the state feedback control design. For time-delay singular Markovian systems, a new bounded real lemma is proposed and the corresponding H_∞ control problem is studied. Numerical examples are given to illustrate the effectiveness of the theoretic results developed.

1. Introduction

Singular systems which are formulated as a set of coupled differential and algebraic equations (Xu & Lam, 2006) can be employed to describe a variety of practical systems such as electrical circuits (Dai, 1989) and economic systems (Luenberger & Arbel, 1977). Therefore, many control and filtering problems for singular systems have attracted extensively attention (Masubuchi, Kamitane, Ohara, & Suda, 1997; Xu & Yang, 1999). Dissipativity introduced in Willems (1972) has played an important role in system and control theory and gives strong links among physics, system theory and control engineering (Lozano, Maschke, Brogliato, & Egeland, 2000). The theory of dissipativity generalizes the H_∞ performance and passivity and considers both gain and phase properties. Therefore, it can provide an appropriate framework for less conservative robust controller design and many results have been reported in literature. Necessary and sufficient dissipativity conditions for continuous-time systems are proposed in Xie, Xie, and De Souza (1998). The results have been extended to continuous-time singular systems in Masubuchi (2006). However, the necessary and sufficient conditions in Masubuchi (2006) contain an equality constraint and a semi-definite matrix inequality, that is, $E^TX = X^TE \geq 0$, which may cause trouble in checking the condition numerically. Therefore, a natural question is how to obtain necessary and sufficient dissipativity and dissipative control conditions without these constraints.

On the other hand, parameters in practical systems may experience random abrupt changes either because of failures, repair, or environmental changes of modification operating points. Markovian jump systems have been widely employed to model these dynamic systems (Mariton, 1990; Shi, Boukas, & Agarwal, 1999). The necessary and sufficient stability conditions of Markovian jump systems are given in terms of linear matrix inequalities (LMIs) in de Farias, Geromel, do Val, and Costa (2000). Based on this work, many important stabilization results are obtained in Wu, Shi, Shu, Su, and Lu (2016), Zhang, Leng, and Colaneri (2016). For singular Markovian jump systems, a necessary and sufficient stability condition is proposed in terms of LMIs in Xu and Lam (2006) which provides fundamental results for further studying other control and filtering problems. However, there exists equality constraint which makes the numerical computation fragile and synthesis of state feedback controller more difficult. To remove the equality constraint, an excellent work is given in the form of strict LMIs in Xia, Boukas, Shi, and Zhang (2009). But the state feedback control result in Xia et al. (2009) is only a necessary condition. Hence, a meaningful research problem is raised naturally, that is, how to establish a necessary and sufficient

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stability condition for singular Markovian jump systems which would make the state feedback control design easier.

In another research front line, a great deal of attention has been devoted to the study of time-delay singular Markovian jump systems (Wang, Wang, Xue, & Lu, 2013; Wang & Xu, 2015). More details about the development of time-delay singular Markovian jump systems can refer to Wang, Zhang, and Yan (2015). H_∞ performance whose analysis result is known as bounded real lemma is one of the most popular and important performance specifications. A bounded real lemma in terms of LMIs for time-delay singular Markovian jump systems is proposed in Wu, Su, and Chu (2009) for the first time. However, the equality constraint $E^TP_i = P_i^TE$ is involved which makes the numerical computation fragile. A new bounded real lemma in terms of strict LMIs is established in Wang et al. (2013). However, when the time-delay is time-varying, the controller design method will not work. The result in Wu, Su, and Shi (2012) is not only without equality constraint but also can be used for time-varying delay case. It should be pointed out that there are some restrictions on matrix E and free-weighting matrix W. Moreover, when the state feedback control is studied, the matrix $\mathscr{P}_iE$ is enlarged as $\mathscr{P}_iE+\sigma\mathscr{L}$ and the inverse of $\mathscr{P}_iE+\sigma\mathscr{L}$ is used. This will introduce conservatism and the inverse of $\mathscr{P}_iE+\sigma\mathscr{L}$ may not exist when matrix E is in a general form. Based on the motivations given earlier, a natural research problem is how to establish a less conservative H_∞ control condition in terms of strict LMIs without any constraint on matrix E for time-delay singular Markovian jump system which also can be used for time-varying delay case.

In this paper, a new equivalent sets approach is proposed to investigate the problems of admissibilization and dissipative control of singular systems. By employing an equivalent parametrization of the constrained sets, a novel necessary and sufficient dissipativity condition of singular systems is presented without the equality constraint. Based on the criterion, a necessary and sufficient condition for the existence of a state feedback controller is established to render the closed-loop system to be admissible and dissipative. For singular Markovian systems, necessary and sufficient conditions of admissibility analysis and state feedback control are obtained in terms of LMIs. For singular Markovian systems with time-delay, a new bounded real lemma and an H_∞ control method are provided. Numerical examples are given to demonstrate the effectiveness of the obtained results.

Notation: The notation used throughout the paper is standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices; $P>0$ (≥ 0) means that P is real symmetric and positive definite (semi-definite); I and 0 denote the identity matrix and zero matrix, respectively, with compatible dimensions; $\star$ stands for the symmetric terms in a symmetric matrix and sym(A) is defined as $A+A^T$; the notation diag(A,B) denotes the block diagonal matrix $\begin{bmatrix} A & 0 \\ 0 & B\end{bmatrix}$. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

2. Preliminaries

In this section, some useful lemmas and the equivalent sets are given which will be used to develop our main results in sequel.

Lemma 1 (*Xia et al., 2009*). *For matrix $E\in\mathbb{R}^{n\times n}$ with* rank(E) $=r\leq n$, *denote E_L and E_R are full column rank with $E=E_LE_R^T$,* rank(E_L) = rank(E_R) $=r$ *and let $P=P^T$ such that $E_L^TPE_L>0$, and Q is nonsingular. U with full row rank and Λ with full column rank are the left and right null matrices of matrix E, respectively, that is $UE=0$ and $E\Lambda=0$. Then, $PE+U^TQ\Lambda^T$ is nonsingular and its inverse is expressed as $(PE+U^TQ\Lambda^T)^{-1}=\bar{P}E^T+\Lambda\bar{Q}U$, where $\bar{P}=\bar{P}^T$ and $\bar{Q}$ is nonsingular such that $E_R^T\bar{P}E_R=(E_L^TPE_L)^{-1}$, $\bar{Q}=(\Lambda^T\Lambda)^{-1}Q^{-1}(UU^T)^{-1}$.*

Lemma 2. *The following sets are equivalent:*

$$\mathbb{A}=\left\{X\in\mathbb{R}^{n\times n}: E^TX=X^TE\geq 0,\ X \text{is nonsingular}\right\},$$

$$\mathbb{B}=\left\{X=PE+U^T\Phi\Lambda^T: P=P^T\in\mathbb{R}^{n\times n},\ E_L^TPE_L>0,\right.$$
$$\left.\Phi\in\mathbb{R}^{(n-r)\times(n-r)}\text{is nonsingular}\right\},$$

where E_L, E_R, $U^T\in\mathbb{R}^{n\times(n-r)}$ and $\Lambda\in\mathbb{R}^{n\times(n-r)}$ are defined in Lemma 1, respectively.

Proof (*Sufficiency*). Let $X=PE+U^T\Phi\Lambda^T$, we have $E^TX=X^TE=E_RE_L^TPE_LE_R^T\geq 0$ and X is nonsingular based on Lemma 1.

(Necessity) Without loss of generality, denote $X=\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22}\end{bmatrix}$, $E=\begin{bmatrix} I_r & 0 \\ 0 & 0\end{bmatrix}$, where $X_{11}\in\mathbb{R}^{r\times r}$ and $X_{22}\in\mathbb{R}^{(n-r)\times(n-r)}$, then we have $E_L=E_R=\begin{bmatrix} I_r \\ 0\end{bmatrix}$ and $U^T=\Lambda=\begin{bmatrix} 0 \\ I_{n-r}\end{bmatrix}$. By using $E^TX=X^TE$, we have $X_{12}=0$, $X_{11}\geq 0$. Due to rank(E^TX) = rank$(X_{11})=r$, we arrive at $X_{11}>0$. Recalling X is nonsingular, it yields that X_{22} is nonsingular. Then, we can find a matrix $P=P^T=\begin{bmatrix} X_{11} & X_{21}^T \\ X_{21} & I_{n-r}\end{bmatrix}$ and $\Phi=X_{22}$ such that $E_L^TPE_L=X_{11}>0$ and $X=PE+U^T\Phi\Lambda^T$. □

Remark 3. It should be noted that Lemma 2 is different from that in Feng and Lam (2013) because the set X_1 in Feng and Lam (2013) and the set $\mathbb{A}$ in this paper are different. On the other hand, the matrices E_L and E_R satisfying $E=E_L^TE_R$ are not used in Feng and Lam (2013). The system considered in Feng and Lam (2013) is discrete-time singular system and the synthesis results in Feng and Lam (2013) are just sufficient conditions while the system considered in this paper is continuous-time singular systems and the synthesis results in terms of strict LMIs are necessary and sufficient conditions for delay free singular systems.

3. Dissipative control of singular systems

In this section, the problems of dissipativity analysis and dissipative control of singular systems are addressed by employing the two equivalent sets defined in Lemma 2.

Consider a class of linear continuous singular systems described by

$$\begin{cases} E\dot{x}(t)=Ax(t)+B_ww(t), & x(0)=x_0 \\ z(t)=Cx(t)+D_ww(t) \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector; x_0 is the initial condition; $w(t)\in\mathbb{R}^p$ represents the exogenous input which includes disturbances to be rejected, and $z(t)\in\mathbb{R}^q$ is the controlled output; A, B_w, C and D_w are constant matrices with appropriate dimensions. In contrast with standard linear systems with $E=I$, the matrix $E\in\mathbb{R}^{n\times n}$ has $0<\text{rank}(E)=r<n$. First, we give some definitions and lemmas on unforced system (1):

Definition 4 (*Dai, 1989*).

(1) The singular system in (1) is said to be regular if $\det(sE-A)$ is not identically zero.
(2) The singular system in (1) is said to be impulse-free if $\deg\{\det(sE-A)\}=\text{rank}(E)$.
(3) The singular system in (1) is said to be asymptotically stable, if all the finite roots of $\det(sE-A)=0$ have negative real parts.
(4) The singular system in (1) or the pair (E,A) is said to be admissible if the system is regular, impulse-free and asymptotically stable.

For a supply rate $s(w,z)=\begin{bmatrix} w \\ z\end{bmatrix}^TS\begin{bmatrix} w \\ z\end{bmatrix}$ with $S\in\mathbb{R}^{(p+q)\times(p+q)}$, the definition of dissipativity is given as follows: