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Brief paper

Convex liftings-based robust control design☆

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ABSTRACT

This paper presents a new approach for control design of constrained linear systems affected by bounded additive disturbances and polytopic uncertainties. This method hinges on so-called *convex liftings* which emulate control Lyapunov function by providing a constructive framework for optimization based control implementation. It will be shown that this method can guarantee the recursive feasibility and robust stability. Finally, a numerical example will be presented to illustrate this method.

1. Introduction

Originated in the seminal work (Lyapunov, 1907), Lyapunov stability stands as a fundamental concept in control theory (Loría & Panteley, 2006). In stability analysis, a Lyapunov function is usually of use to prove closed-loop stability, see Kalman and Bertram (1960), Brayton and Tong (1979) and Molchanov and Pyatnitskiy (1989). On the other hand, in control design, control Lyapunov functions are usually employed to design stabilizing/robust controllers, see among others (Khalil, 2002; Zubov & Boron, 1964). Accordingly, whenever such control Lyapunov functions are used in optimization based strategies, these should be chosen such that the recursive feasibility and closed-loop stability are all fulfilled. Different classes of control Lyapunov functions have been proposed in control theory (Michel, Nam, & Vittal, 1984; Polanski, 1995). In the context of linear quadratic control, infinite/finite quadratic cost functions usually serve as control Lyapunov functions, as shown in Anderson and Moore (2007), Chmielewski and Manousiouthakis (1996) and Sznaier and Damborg (1987). In particular, in linear model predictive control (MPC), such a control Lyapunov function has been used to design robust controllers to cope with polytopic uncertainties, leading to a linear matrix inequality problem, see Kothare, Balakrishnan, and Morari (1996). Polyhedral control Lyapunov functions have also been exploited in several studies, e.g., Bitsoris (1988b), Bitsoris and Vassilaki (1995), Blanchini (1994, 1995), Gutman and Cwikel (1987), Lazar (2010) and Vassilaki, Hennet, and Bitsoris (1988), since they lead to simple design procedures, i.e., composed of linear constraints. Convex piecewise affine control Lyapunov function for piecewise affine systems has also been considered in Baotic, Christophersen, and Morari (2006) and solved using dynamic programming, which may be impractical if disturbances and uncertainties are considered.

It is worth emphasizing that the robust control design proposed in Kothare et al. (1996) requires at each sampling time solving a linear matrix inequality (LMI) problem, the online evaluation thus becomes computationally demanding. Some improvements of this method are presented in Cuzzola, Geromel, and Morari (2002) and Wan and Kothare (2003). An effort to simplify this complexity has been proposed in Kouvaritakis, Rossiter, and Schuurmans (2000). However, this method can only guarantee the positive invariance of the initially ellipsoidal feasible set instead of asymptotic stability of the origin. Also, although the number of LMIs is decreased, however, solving online an LMI problem is still expensive in comparison to strict real-time requirements. Some extensions of the latter method have been proposed to reduce complexity, e.g., Khan and Rossiter (2012).

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Note however that making use of *degree of freedom* n_c is nothing other than solving a finite horizon MPC problem. Also, in the context of MPC, the optimal cost function usually serves as a Lyapunov function, therefore minimizing a nominal cost function as in this reference is meaningless, and robust stability is thus guaranteed by the constraint set. Further, the pre-imposition on the structure of controllers leads to conservativeness and possible loss of recursive feasibility. An alternative robust MPC scheme has been presented in Mayne, Seron, and Raković (2005) to take bounded additive disturbances into account. However, polytopic uncertainties considerably increase its computational complexity with respect to the prediction horizon. As an extension of this method, parameterized tube MPC has recently been proposed in Rakovic, Kouvaritakis, Cannon, Panos, and Findeisen (2012) to cope with bounded additive disturbances. Although implicit controller is computed based on its decomposed elements, the number of decision variables is of order $\mathcal{O}(q^N)$, with q to be the number of vertices of the given disturbance set and N to be the prediction horizon. As a consequence, accounting for polytopic uncertainty makes the online computation much more demanding, as the number of decision is of order $\mathcal{O}(q^N p^N)$, with p to be the number of vertices of the given polytopic uncertainty set. Further, dealing with tube cost function in this case becomes more complicated.

This paper proposes a method which only requires resolution of a linear programming problem at each sampling instant. Moreover, unlike the method in Blanchini (1994), which guarantees robust stability in the sense of Lyapunov (input-to-state stability), this paper proves a more flexible result by guaranteeing that the state converges to a given robust positively invariant set (minimal/maximal robust positively invariant set) as time tends to infinity. Note that such a constructed convex lifting is not a control Lyapunov function, which represents a relaxation and a supplementary degree of freedom with respect to the method in Blanchini (1994). Finally, to our best knowledge, convex liftings have never been used in control design and can be a valuable tool, offering additional flexibility for the existing constrained control methods.

2. Notation and definitions

Throughout this paper, $\mathbb{N}, \mathbb{N}_{>0}, \mathbb{R}, \mathbb{R}_+$ denote the set of nonnegative integers, the set of positive integers, the set of real numbers and the set of nonnegative numbers, respectively. For ease of presentation, with a given $N \in \mathbb{N}_{>0}$, by $\mathcal{I}_N$, we denote the index set: $\mathcal{I}_N := \{i \in \mathbb{N}_{>0} : i \leq N\}$. Also, we use $\mathcal{I}_N^2$ to denote the set defined as: $\mathcal{I}_N^2 = \mathcal{I}_N \times \mathcal{I}_N$.

A polyhedron is the intersection of finitely many closed halfspaces. A polytope is a bounded polyhedron. If P is an arbitrary polytope, then by $\mathcal{V}(P)$, we denote the set of its vertices. If $\mathcal{S}$ is an arbitrary set, then $\text{conv}(\mathcal{S})$ denotes the convex hull of $\mathcal{S}$. Also, we use $\dim(\mathcal{S})$ to denote the dimension of its affine hull. Moreover, if $\mathcal{S}$ is a full-dimensional set, then we use $\text{int}(S)$ to denote the interior of $\mathcal{S}$. Given a set $\mathcal{S} \subset \mathbb{R}^d$ and a matrix $A \in \mathbb{R}^{d\times d}$, then $A\mathcal{S}$ is defined as follows: $A\mathcal{S} := \{As : s \in \mathcal{S}\}$. Also, for any vector $x \in \mathbb{R}^d$, $\rho_{\mathcal{S}}(x)$ is defined as follows: $\rho_{\mathcal{S}}(x) := \min_{y\in\mathcal{S}} \sqrt{(y-x)^T(y-x)}$. Given two sets $\mathcal{S}_1, \mathcal{S}_2 \subset \mathbb{R}^d$, their Minkowski sum is denoted by $\mathcal{S}_1 \oplus \mathcal{S}_2$ and is defined by: $\mathcal{S}_1 \oplus \mathcal{S}_2 := \left\{y_1 + y_2 \in \mathbb{R}^d : y_1 \in \mathcal{S}_1, y_2 \in \mathcal{S}_2\right\}$. Also, $\mathcal{S}_1 \setminus \mathcal{S}_2$ is defined as follows: $\mathcal{S}_1 \setminus \mathcal{S}_2 := \left\{x \in \mathbb{R}^d : x \in \mathcal{S}_1,\ x \notin \mathcal{S}_2\right\}$.

3. Problem settings

In this paper, we consider a discrete-time linear system:

$$x_{k+1} = A(k)x_k + B(k)u_k + w_k, \tag{1}$$

where x_k, u_k, w_k denote the state, control variable and additive disturbance at time k. The state-space matrices $[A(k)\, B(k)]$ are time-varying and assumed to belong to an *uncertainty matrix polytope* denoted by Ψ and defined below:

$$[A(k)\, B(k)] \in \Psi := \text{conv}\left\{[A_1\, B_1], \ldots, [A_L\, B_L]\right\}. \tag{2}$$

The state, control variables and disturbances are subject to constraints:

$$x_k \in \mathbb{X} \subset \mathbb{R}^{d_x}, \qquad u_k \in \mathbb{U} \subset \mathbb{R}^{d_u}, \qquad w_k \in \mathbb{W} \subset \mathbb{R}^{d_x}, \tag{3}$$

where $d_x, d_u \in \mathbb{N}_{>0}$, and $\mathbb{X}, \mathbb{U}, \mathbb{W}$ are polytopes containing the origin in their interior.

The objective is to find robust control laws which can cope with bounded additive disturbances and polytopic model uncertainties such that the closed loop is robustly stable. It is clear that if w_k is unknown, one cannot expect to guarantee asymptotic stability of the origin. In this case, asymptotic stability is replaced with an ultimate boundedness concept (Khalil, 2002; Kofman, Haimovich, & Seron, 2007) or input to state stability (Jiang & Wang, 2001).

4. Robust control design based on convex liftings

4.1. Robust positively invariant sets

Positively invariant sets have been studied over several decades. Due to their relevance in control theory, they turn out to be useful in many control related studies, e.g., Bitsoris (1988a,b), Bitsoris and Vassilaki (1995), Blanchini and Miani (2007) and Kerrigan (2001). The definition of a robust positively invariant set for system (1) is recalled below.

Definition 4.1. Given an admissible control law $u_k = Kx_k \in \mathbb{U}$, a set $\Omega \subseteq \mathbb{X}$ is called *robust positively invariant* with respect to (1) if

$$(A(k) + B(k)K)\Omega \oplus \mathbb{W} \subseteq \Omega, \quad \forall [A(k)\ B(k)] \in \Psi,$$

where Ψ is defined in (2).

To compute such a robust positively invariant set Ω, it is important to choose an appropriate unconstrained control law to cope with given bounded additive disturbances and polytopic uncertainties. More clearly, this control law should satisfy that there exists a Lyapunov function $V(x) : \mathbb{R}^{d_x} \to \mathbb{R}_+$ such that

$$V((A(k) + B(k)K)x_k) - V(x_k) < 0, \quad \forall\ [A(k)\ B(k)] \in \Psi.$$

The computation of such a gain K was studied in, e.g., Daafouz and Bernussou (2001) and Kothare et al. (1996). A simpler formulation is presented below:

$$\begin{aligned} &\min_{Z,Y} -\text{logdet}(Z) \\ &\text{subject to} \\ &Z = Z^T > 0 \\ &\begin{bmatrix} Z & (A_iZ + B_iY)^T \\ A_iZ + B_iY & Z \end{bmatrix} > 0, \quad \forall\, i \in \mathcal{I}_L. \end{aligned}$$

Then, gain K is determined by $K = YZ^{-1}$. It is already known that the above formulation is an LMI problem and is solvable by using semidefinite programming. The interested reader can find details in Boyd, El Ghaoui, Feron, and Balakrishnan (1994).

With respect to the state feedback $u_k = Kx_k$, the computation of a robust positively invariant set Ω for system (1) has been put forward in Nguyen (2014), as a simple extension of the idea presented in Gilbert and Tan (1991). Note also that prominent studies on the computation of the maximal and minimal positively invariant sets for a linear, discrete-time invariant system affected by bounded additive disturbances can be found in Kolmanovsky and Gilbert (1998) and Rakovic, Kerrigan, Kouramas, and Mayne (2005). Still, in the case system (1) is not affected by additive disturbances, then the minimal robust positively invariant set coincides with the origin due to its asymptotic stability, i.e., $\Omega = \{0\}$. Without loss of generality, we are hereafter interested in the case $\Omega \subseteq \mathbb{X} \subset \mathbb{R}^{d_x}$ represents a full-dimensional set.