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Brief paper

Dynamic partial state feedback control of cascade systems with time-delay ☆

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ABSTRACT

This paper investigates the problem of global stabilization by partial state feedback for a class of cascade nonlinear systems with time-delay. Under suitable ISS conditions imposed on zero-dynamics, a delay-free, dynamic partial state feedback compensator is presented for achieving global state regulation. The controller is constructed by employing a dynamic gain based design method, together with the ideas of changing supply rates and adding an integrator. With appropriate choices of Lyapunov–Krasovskii functionals, it is shown that all the states of the time-delay cascade system can be regulated to the origin while maintaining boundedness of the closed-loop system. Two examples are given to illustrate the effectiveness of the proposed dynamic partial state feedback control scheme.

1. Introduction

Consider a cascade system with time-delay of the form

$$\begin{aligned}
\dot{z}_i &= f_{0i}(z_1,\ldots,z_i,z_1(t-d),\ldots,z_i(t-d),\\
&\qquad x_1,\ldots,x_i,x_1(t-d),\ldots,x_i(t-d)),\\
\dot{x}_i &= x_{i+1}+g_i(z_1,\ldots,z_i,z_1(t-d),\ldots,z_i(t-d),\\
&\qquad x_1,\ldots,x_i,x_1(t-d),\ldots,x_i(t-d)),\\
z_i(s) &= \zeta_i(s), \qquad x(s)=\mu(s), \quad s\in[-d,0],\\
&\qquad i=1,\ldots,r, \qquad\qquad (1)
\end{aligned}$$

where $z_i \in \mathbb{R}^{n_i}(n_i = 0, 1, 2, \ldots)$ and $x = [x_1, \ldots, x_r]^T \in \mathbb{R}^r (r \geq 1)$ are the system states, $u := x_{r+1} \in \mathbb{R}$ is the control input, and the constant $d \geq 0$ is an unknown time-delay of the system. For $i = 1, \ldots, r, f_{0i} : \mathbb{R}^{2n_1+\cdots+2n_i+2i} \to \mathbb{R}^{n_i}$ and $g_i : \mathbb{R}^{2n_1+\cdots+2n_i+2i} \to \mathbb{R}$ are C^1 mappings with $f_{0i}(0) = 0$ and $g_i(0) = 0$, and $\zeta_i(s) \in \mathbb{R}^{n_i}$ and $\mu(s) \in \mathbb{R}^r$ are continuous functions defined on $[-d, 0]$. Notably, $\dim z = \dim [z_1, \ldots, z_r]^T = n_1 + n_2 + \cdots + n_r$ with $n_i \geq 0$, which can be either less or bigger than, or equal to $\dim x = r$. Throughout this paper, it is assumed that only the partial state of the cascade system (1), namely, $x = [x_1, \ldots, x_r]^T$, is measurable and available for feedback design.

Without the time-delay, partial state feedback control of the cascade nonlinear system (1) has been studied in the literature; see, for instance, Chen (2009), Chen and Huang (2004), Jiang and Mareels (1997), Lin and Gong (2003), Lin and Pongvuthithum (2002) and the references therein. In the case when only z_1 appears in the cascade system (1), the problem of global stabilization was considered in Isidori (1999), Jiang and Mareels (1997), Lin and Gong (2003) and Lin and Pongvuthithum (2002) by using a nonlinear small-gain theorem, and the idea of changing supply rate combined with the backstepping design. In Chen (2009) and Chen and Huang (2004), global stabilization of the more general cascade system such as (1) without time-delay was shown to be possible by using a Lyapunov direct method. It was shown in Chen and Huang (2004) that the states $(z_1, \ldots, z_r)$, which model the dynamic uncertainty, represent the internal model of the cascade system when studying the robust output regulation of lower-triangular systems. The work (Chen, 2009) also provided an explicit construction of a Lyapunov function in superposition form for the cascade nonlinear system.

Time-delay is frequently encountered in various engineering systems and often causes instability. For time-delay systems, many

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results have been obtained and reported in the literature (Bekiaris-Liberis & Krstic, 2013; Bresch-Pietri & Krstic, 2014; Fridman & Shaked, 2002; Gu, Kharitonov, & Chen, 2003; Krstic, 2010; Pepe, 2014; Richard, 2003). In Krstic (2010), it was pointed out that many systems such as feedback linearizable or strict-feedback systems with input delay are not globally stabilizable because they are not forward complete. With this observation, the work (Krstic, 2010) focused on the problem of input delay compensation for forward complete and strict-feedforward systems. It has led subsequent developments; see, for instance, the papers Bekiaris-Liberis and Krstic (2013); Bresch-Pietri and Krstic (2014) and references therein. By comparison, little effort has been made for the control of the cascade nonlinear system (1) with time-delay. Even in the case when the nonlinear system (1) contains no dynamic uncertainty z_i, the stabilization problem is still a difficult one as shown in Jankovic (2001), Karafyllis and Jiang (2010), Mazenc, Mondie, and Niculescu (2003), Zhang and Lin (2014). Following the work Chen (2009), we study in this paper the problem of how to control the time-delay nonlinear cascade system (1) by delay-independent, partial state feedback. As indicated in Chen and Huang (2004), even without delay, there is a connection between the solvability of the global output regulation problem and the global stabilizability of cascade nonlinear systems by partial state feedback. Therefore, the study of the cascade system (1) with time-delay is likely to provide a new insight in understanding the global output regulation of time-delay nonlinear systems and may pave a way to solve the global output regulation problem.

The main contribution of this paper is to prove that the global state regulation problem of the time-delay cascade system (1) is solvable by a *delay-free, dynamic partial state* feedback compensator if appropriate ISS conditions imposed on zero-dynamics are fulfilled. Specifically, we apply the idea from Zhang and Lin (2014), Zhang, Lin, and Lin (2016) to design a delay-independent, partial state feedback controller with dynamic gains that are updated by Riccati-like equations. The constructed dynamic partial state feedback compensator can globally regulate all the states of the time-delay cascade nonlinear system (1) to the origin while maintaining the boundedness of the resulted closed-loop system. The novelty lies in the development of a dynamic partial state feedback control strategy based on the backstepping design and changing supply rates, capable of counteracting the time-delay nonlinearities of the cascade system (1). The delay-free, dynamic partial state compensator is then designed in a recursive manner. Another new ingredient is the construction of Lyapunov–Krasovskii functionals that relies on dynamic gains, which play a crucial role in proving the global stability as well as the global state regulation of the cascade nonlinear system with time-delay.

Notations: Throughout this paper, we let v_d denote the time-delay term $v(t-d)$, for example, $z_{id} = z_i(t-d)$ and $x_{id} = x_i(t-d)$. Define $\bar{v}_i = [v_1, \ldots, v_i]^T \in \mathbb{R}^i$ for $i = 1, \ldots, r$. Hence, $\bar{x}_i = [x_1, \ldots, x_i]^T$, $\bar{x}_{id} = [x_{1d}, \ldots, x_{id}]^T$ and $\bar{l}_i = [l_1, \ldots, l_i]^T$.

2. Preliminaries

In this section, we collect some technical lemmas to be used in the sequel. The lemmas listed below play an important role in the design of a dynamic partial state feedback compensator for the system (1).

Lemma 2.1 (*Lin & Qian, 2002*). *Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ be a continuous function. Then, there are smooth scalar functions $a(x) \geq 0$, $b(y) \geq 0$, $c(x) \geq 1$ and $d(y) \geq 1$, such that*

$$|f(x,y)| \leq a(x) + b(y) \quad \text{and} \quad |f(x,y)| \leq c(x)d(y). \tag{2}$$

Lemma 2.2. *If $f(x, y)$ is a real-valued continuous function, there exist smooth scalar functions $g(x) \geq 0$ and $h(y) \geq 0$ satisfying*

$$f(x,y)(\|x\| + \|y\|) \leq g(x)\|x\| + h(y)\|y\|. \tag{3}$$

Proof. From Lemma 2.1 it follows that there exist smooth scalar functions $a(x) \geq 0$ and $b(y) \geq 0$, such that

$$f(x,y)(\|x\| + \|y\|) \leq (a(x) - a(0))\|y\| + (b(y) - b(0))\|x\| + (a(x) + b(0))\|x\| + (a(0) + b(y))\|y\|. \tag{4}$$

By smoothness, there are smooth functions $\bar{a}(x) \geq 0$ and $\bar{b}(y) \geq 0$ satisfying $a(x) - a(0) \leq \bar{a}(x)\|x\|$ and $b(y) - b(0) \leq \bar{b}(y)\|y\|$. This, together with the completion of square, leads to

$$(a(x) - a(0))\|y\| \leq \frac{1}{2}\bar{a}^2(x)\|x\|^2 + \frac{1}{2}\|y\|^2,$$
$$(b(y) - b(0))\|x\| \leq \frac{1}{2}\bar{b}^2(y)\|y\|^2 + \frac{1}{2}\|x\|^2. \tag{5}$$

Substituting (5) into (4) results in (3). □

Lemma 2.3. *Let $X = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ and $\alpha : \mathbb{R}^n \to \mathbb{R}$ be a nonnegative C^2 function with $\alpha(0) = 0$. Then, there exist smooth scalar functions $b_i(x_i) \geq 0$, $i = 1, \ldots, n$, such that*

$$a(X) \leq \sum_{i=1}^{n} x_i^2 b_i(x_i). \tag{6}$$

Proof. By the mean value theorem with an integration remainder,

$$\alpha(X) - \alpha(0) = \int_0^1 d\alpha(\theta X) = R(X)X = X^T R^T(X), \tag{7}$$

where $R(X) := \int_0^1 \frac{\partial \alpha}{\partial \beta}|_{\beta=\theta X} d\theta$ is a $1 \times n$ covector.

Note that $\alpha(X) \geq 0$ arrives its minimum $\alpha(0)$ at $X = 0$, thus $\frac{\partial \alpha}{\partial X}(0) = 0$, i.e., $R^T(0) = 0$. Using the same trick, the n-dimensional vector $R^T(X)$ can be decomposed as

$$R^T(X) - R^T(0) = \left(\int_0^1 \frac{\partial R^T}{\partial \beta}\bigg|_{\beta=\theta X} d\theta\right) X := H(X)X \tag{8}$$

where $H(X)$ is a symmetric, $n \times n$ Hessian matrix.

Using (7)–(8), we arrive at

$$\alpha(X) \leq \|H(X)\| \cdot \|X\|^2 \leq \beta(x_1, \ldots, x_n)(x_1^2 + \cdots + x_n^2),$$

which, combined with Lemmas 2.1 and 2.2, yields

$$\alpha(X) = \alpha(x_1, \ldots, x_n) \leq \sum_{i=1}^{n} x_i^2 b_i(x_i),$$

for appropriate smooth functions $b_i(x_i) \geq 0$, $i = 1, \ldots, n$. □

3. Global stabilization by dynamic partial state feedback

To control the cascade nonlinear system (1) with time-delay by means of partial state feedback (i.e., only the state x), we make the following assumption on $(z_1, \ldots, z_r)$-dynamics of (1).

Assumption 3.1. For $i = 1, \ldots, r$, there exists a C^1 Lyapunov function $V_{0i}(z_i)$, which is positive definite and proper, such that

$$\frac{\partial V_{0i}}{\partial z_i} f_{0i}(\bar{z}_i, \bar{z}_{id}, \bar{x}_i, \bar{x}_{id}) \leq -\|z_i\|^2 + \alpha_i(\bar{z}_{i-1}, \bar{z}_{(i-1)d}, \bar{x}_i, \bar{x}_{id}), \tag{9}$$

where $\alpha_i(\bar{z}_{i-1}, \bar{z}_{(i-1)d}, \bar{x}_i, \bar{x}_{id}) \geq 0$ is a C^2 function with $\alpha_i(0, 0, 0, 0) = 0$. □