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On delay radii and bounds of MIMO systems☆

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ABSTRACT

The delay margin of a time-delay system constitutes the fundamental limit beyond which no single controller may exist to robustly stabilize an unstable delay plant for a range of delay values. For single-input single-output (SISO) systems with a linear time-invariant (LTI) controller, this margin is known to be finite for an unstable plant, and bounds on the delay margin are available. This paper extends the existing results to multi-input multi-output (MIMO) systems. We derive upper bounds on a generalized notion called delay radius. Our results show that for a delay whose direction is orthogonal to that of an unstable pole, no constraint is imposed by the pole on that delay, while if the delay direction is parallel to that of a nonminimum phase zero, its allowable range will be further restricted by the nonminimum phase zero.

1. Introduction

We consider the feedback system depicted in Fig. 1, where $P_\tau(s)$ is a family of plants that vary with an unknown delay τ, with $P_0(s)$ being the delay-free plant:

$$P_\tau(s) = e^{-\tau s}P_0(s), \quad \tau \geq 0. \tag{1}$$

The *delay margin* problem is to determine

$$\tau^* = \inf\{r : \text{ There exists no } K(s) \text{ to stabilize } P_\tau(s),\ \forall \tau \in [0,\ r]\}. \tag{2}$$

In other words, we seek to find the largest delay range within which $P_\tau(s)$ can be robustly stabilized by a *fixed, single* finite-dimensional LTI controller $K(s)$. This problem has been under scrutiny for some time. In Michiels and Niculescu (2007) (pp. 154), the delay margin was determined for the first-order system achievable by static feedback, and in Silva, Datta, and Bhattacharyya (2002) for the first-order system when PID controllers are used. More generally, for SISO systems with a single unstable pole, the exact margin was found in Middleton and Miller (2007), which in turn serves as an upper bound for general LTI systems with an arbitrary number of unstable poles. These bounds consequently provide a limit beyond which no single LTI output feedback controller may exist to robustly stabilize a delay plant family within the margin. In contrast, lower bounds on the delay margin were developed by the authors in Qi, Zhu, and Chen (2016), which provide instead, an interval of delay values ensuring that the delay plant can be robustly stabilized over the interval. It was shown therein as well that the lower bounds can be extended to MIMO delay systems with time-varying delays.

The purpose of this note is to extend the aforementioned results to MIMO LTI plants with constant, uncertain delays. Specifically, we seek to generalize the bounds in Middleton and Miller (2007) to MIMO systems. While following the spirit of Middleton and Miller (2007), MIMO systems do result in complications and our development sheds new insights. First, for MIMO systems, delay margin ceases to be an applicable measure. For this purpose, we derive upper bounds on the *delay radii*, and for more specialized instances, their exact expressions; delay radius has been previously introduced in Qi et al. (2016) and was shown to be useful in characterizing robust stabilizability of MIMO delay systems. Second and more importantly, the stabilization of a MIMO plant is particularly complicated by the directionality properties of plant unstable poles and proves significantly more challenging. This difficulty manifests itself through the interactions among the delays and the unstable poles, which exhibits a strong directional dependence. For example, our result shows that when the direction of a pole is orthogonal

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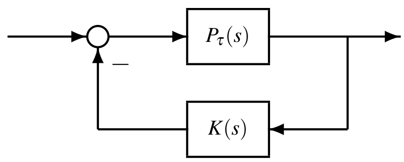

Fig. 1. Feedback control of a delay plant.

to the direction in which the delay takes place, the pole imposes no constraint on the allowable length of that delay. It is also shown that nonminimum phase zeros will confine further the range of delays for which robust stabilization is possible, when the directions of the nonminimum phase zeros are aligned with those of the delays.

Our results complement the lower bounds on the delay radii obtained previously in Qi et al. (2016) and contribute to a series of endeavors in tackling the longstanding delay margin problem (Devanathan, 1995; Foias, Özbay, & Tannenbaum, 1996; Ju & Zhang, 2015; Middleton & Miller, 2007; Qi et al., 2016) focused on the use of LTI controllers. It is worth noting however, that when more sophisticated classes of controllers are considered, then the delay margin can in fact be made infinite. Indeed, this has been shown to be possible by employing linear periodic controllers (Miller & Davison, 2005), nonlinear periodic controllers (Gaudette & Miller, 2014), and nonlinear adaptive controllers (Bekiaris-Liberis & Krstic, 2013; Bresch-Pietri, Chauvin, & Petit, 2012); in other words, with these controllers, an LTI delay plant can be stabilized for arbitrarily long uncertain delays. Nonetheless, LTI controllers are still desired for their ease of implementation.

The notation used in this paper is standard. We denote the open right-half plane by $\mathbb{C}_+ := \{s : \mathrm{Re}(s) > 0\}$. For any two unitary vectors $u, v \in \mathbb{C}^n$, we denote the principal angle $\angle(u, v)$ between the directions spanned by $u, v \in \mathbb{C}^n$ via

$$\cos \angle(u, v) = |u^H v|.$$

The two directions are said to be orthogonal if $\cos \angle(u, v) = 0$, and parallel if $\cos \angle(u, v) = 1$.

2. Delay radii of MIMO systems

The MIMO delay plants under consideration in this paper are in the form of

$$P_{\hat{\tau}}(s) = \Lambda_{\hat{\tau}}(s) P_0(s), \tag{3}$$

where $P_0(s) \in \mathbb{C}^{n\times m}$ is the transfer function matrix of the delay-free part, and $\Lambda_{\hat{\tau}}(s) \in \mathbb{C}^{n\times n}$ is a transfer function matrix consisting of l output delays:

$$\Lambda_{\hat{\tau}}(s) = V \mathrm{diag}\left(e^{-\tau_1 s}, \ldots, e^{-\tau_l s}\right) V^H, \quad \tau_1 \geq 0, \ldots, \tau_l \geq 0,$$

with $V = \begin{bmatrix} v_1 & v_2 & \cdots & v_l \end{bmatrix}$, $v_i \in \mathbb{C}^n$, being a unitary. Let the delays be represented in the delay parameter space by the vector $\hat{\tau} = [\tau_1, \ \ldots, \ \tau_l]^T \in \mathbb{R}^l$.

Remark 1. We use the unitary vector v_i to represent the direction of the ith delay $e^{-\tau_i s}$. This formulation allows us to describe in a more general manner the spatial effect of each delay element, thus representing more fully the directional dependence displayed by delays arising in different channels whose lengths may differ from channel to channel. In the special case where v_i is the ith Euclidean coordinate, the delay part becomes

$$\Lambda_{\hat{\tau}}(s) = \mathrm{diag}\left(e^{-\tau_1 s}, \ \ldots, \ e^{-\tau_l s}\right),$$

which is the standard description modeling fully decoupled delays in each channel. Note that the present formulation is also more consistent with the interpretation that a delay can be considered an extreme nonminimum phase zero at the infinity; in this case, v_i can be used to characterize the direction of that nonminimum phase zero. ■

Assume that $P_0(s)$ can be stabilized by some controller $K(s)$ and define with the delay-free plant the system's output complimentary sensitivity function

$$T_0(s) = P_0(s)K(s)[I + P_0(s)K(s)]^{-1}.$$

It can be easily seen (Qi et al., 2016) that $P_{\hat{\tau}}(s)$ can be stabilized by $K(s)$ if and only if

$$\det[I + (\Lambda_{\hat{\tau}}(s) - I)T_0(s)] \neq 0, \quad \forall s \in \bar{\mathbb{C}}_+. \tag{4}$$

The condition (4) thus characterizes the region of the delays for which $P_{\hat{\tau}}(s)$ can be robustly stabilized by $K(s)$. In Qi et al. (2016), the authors introduced the notion of *delay radius* of the delay parameter vector $\hat{\tau} = [\tau_1, \ldots, \tau_l]^T$ to quantify this region, defined as

$$r_q = \inf\Big\{r : \text{ There exists no } K(s) \text{ to stabilize } P_{\hat{\tau}}(s), \ \forall \hat{\tau}, \ \|\hat{\tau}\|_q \leq r\Big\},$$

where

$$\|\hat{\tau}\|_q = \begin{cases} \left(\sum_i |\tau_i|^q\right)^{1/q} & q \in [1, \infty), \\ \max_i |\tau_i| & q = \infty. \end{cases}$$

Evidently, to determine the exact delay radius requires synthesizing a controller $K(s)$ that can stabilize the family of plants $P_{\hat{\tau}}(s)$ for all $\|\hat{\tau}\|_q < r_q$, which is a rather difficult problem. With this recognition, we seek to derive bounds on the delay radius.

3. Main results

In Middleton and Miller (2007), upper bounds are derived for the delay margin of SISO systems. In this section we extend the results of Middleton and Miller (2007) to MIMO systems, by deriving upper bounds on the delay radii. Of particular interest is the delay radius r_∞; it should be evident that any upper bound on r_∞ serves as an upper bound on r_q, for any $q \in [1, \ \infty)$. The results give estimates of regions in the delay parameter space for which no controller may exist to stabilize the plant.

We first quote two preliminary lemmas (see, e.g., Chen, 2000; Middleton & Miller, 2007).

Lemma 1. *Let $\alpha, \ \beta \geq 0$. Then,*
(i) $\tan^{-1}\alpha \leq \alpha$.
(ii) $\left|\tan^{-1}\alpha - \tan^{-1}\beta\right| \leq |\alpha - \beta|$.

Lemma 2. *Let $p \in \mathbb{C}_+$ be an unstable pole and $z \in \mathbb{C}_+$ a nonminimum phase zero of $P_0(s)$. Suppose that $K(s)$ stabilizes $P_0(s)$. Then, there exist some unitary vectors $\eta \in \mathbb{C}^n$ and $\zeta \in \mathbb{C}^n$ such that*

$$T_0(p)\eta = \eta.$$

$$\zeta^H T_0(z) = 0,$$

where η is called the input pole direction vector associated with the pole $p \in \mathbb{C}_+$, and ζ the output direction vector associated with the zero $z \in \mathbb{C}_+$.

In what follows we shall denote, for any vector η,

$$\mathscr{I} = \left\{i : \ v_i^H \eta \neq 0\right\}.$$

3.1. Real poles

Theorem 1. *Let $p \in \mathbb{C}_+$ be a real unstable pole of $P_0(s)$ with input direction vector $\eta \in \mathbb{C}^n$. Suppose that for all $i \in \mathscr{I}$,*

$$\tau_i^* \geq \frac{2}{p}. \tag{5}$$