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Optimal coupled and decoupled perimeter control in one-region cities

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A B S T R A C T

Perimeter controllers, located at a regional border, can manipulate the transfer flows across the border to optimize the regional operational performance. The macroscopic fundamental diagram (MFD), that relates average flow with accumulation, is used to model the traffic flow dynamics. In this paper, two cases of perimeter control inputs are considered: coupled and decoupled control. For both cases, the explicit formulations of the optimal feedback control policies and proofs of optimality are provided for three criteria. The proofs are based on the modified Krotov-Bellman sufficient conditions of optimality, where the upper and lower bounds of state variables are calculated.

1. Introduction

In the last decade, network traffic flow modeling with the Macroscopic Fundamental Diagram (MFD) representation has intensively attracted the traffic flow and control researchers. The MFD simplifies the modeling task of the traffic flow dynamics for large-scale urban networks, as it provides aggregate relationships between traffic variables at an urban region.

The MFD provides a unimodal, low-scatter relationship between network vehicle density (veh/km) or accumulation (veh) and network space-mean flow (outflow) (veh/h) for different network regions, if congestion is roughly homogeneous in the region. The physical model of the MFD was initially proposed by Godfrey (1969), but the theoretical elements for the existence of the MFD were provided later by Daganzo (2007). The MFD was first observed with dynamic features in congested urban network in Yokohama by Geroliminis and Daganzo (2008), and investigated using empirical or simulated data by Buisson and Ladier (2009), Ji, Daamen, Hoogendoorn, Hoogendoorn-Lanser, and Qian (2010), Mazloumian, Geroliminis, and Helbing (2010), Daganzo, Gayah, and Gonzales (2011), Gayah and Daganzo (2011), Zhang, Garoni, and de Gier (2013), Mahmassani, Williams, and Herman (1987), Olszewski, Fan, and Tan (1995), Lin, Kong, and Huang (2014), Leclercq, Chiabaut, and Trinquier (2014) and others.

Homogeneous networks with small variance of link densities have a well-defined form of MFD (as illustrated in Fig. 1(a)), i.e. low scatter of flows for the same densities (or accumulations), Geroliminis and Sun (2011b), Mazloumian et al. (2010), Daganzo et al. (2011), Knoop, Hoogendoorn, and van Lint (2013), Mahmassani, Saberi, and Zockaie (2013). Note that heterogeneous networks might not have well-defined forms of MFD, mainly in the decreasing part of the MFD, as the scatter becomes higher as accumulation increases and hysteresis phenomena has been found to exist (Buisson & Ladier, 2009; Daganzo et al., 2011; Geroliminis & Sun, 2011a; Ramezani, Haddad, & Geroliminis, 2015; Saberi & Mahmassani, 2012). As a solution, these networks might be partitioned into more homogeneous regions with small variances of link densities, Ji and Geroliminis (2012). Note that the network topology, the signal timing plans of the signalized intersections inside the region, and the infrastructure characteristics affect the shape of the MFD, see e.g. Geroliminis and Boyacı (2012).

The MFD concept has been utilized to introduce control policies that aim at improving mobility and decreasing delays in large urban networks, Daganzo (2007), Haddad and Geroliminis (2012), Geroliminis, Haddad, and Ramezani (2013), Hajiahmadi, Haddad, Schutter, and Geroliminis (2015), Haddad, Ramezani, and Geroliminis (2013), Aboudolas and Geroliminis (2013), Keyvan-Ekbatani, Kouvelas, Papamichail, and Papageorgiou (2012), Knoop, Hoogendoorn, and Van Lint (2012), Zhang et al. (2013), Gayah, Gao, and Nagle (2014). E.g. perimeter control strategies, i.e. manipulating the transfer flows at the perimeter border of the urban region, have been introduced for single-region cities in Daganzo (2007), Keyvan-Ekbatani et al. (2012), Haddad and Shraiber (2014), and for multi-region cities in Geroliminis et al. (2013), Haddad et al. (2013), Hajiahmadi et al. (2015), Aboudolas and Geroliminis (2013), Ramezani, Haddad, and Geroliminis (2015), Haddad (2015). In this paper, the perimeter control for a single urban region modelled by an MFD is treated.

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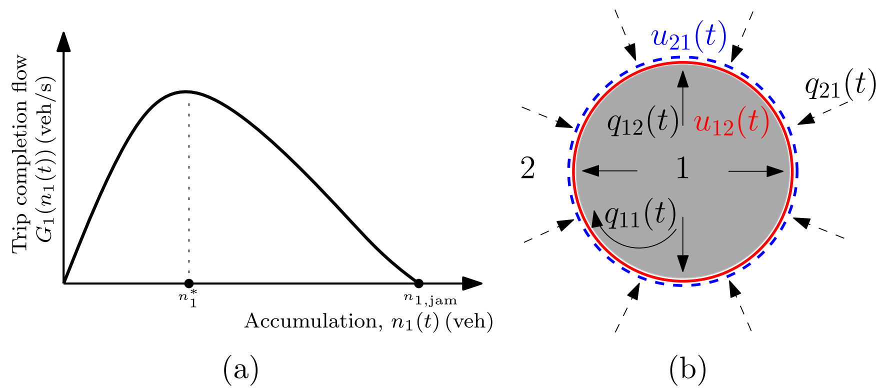

Fig. 1. (a) A schematic MFD which is Lipschitz, continuous, non-negative, and unimodal function, (b) An urban region with three traffic demand $q_{11}(t)$, $q_{12}(t)$, $q_{21}(t)$, and perimeter control with inputs $u_{12}(t)$ and $u_{21}(t)$.

Different control approaches have been proposed to solve perimeter control problems for *single-region cities*. The pioneer work in this field is related to Daganzo (2007), where a bang-bang control has been presented as an optimal control policy for maximizing the rate at which trips are served at an urban region. A Proportional-Integrator (PI) gating controller has been designed for an urban region in Keyvan-Ekbatani et al. (2012). The formulated nonlinear system is linearized around a priori known set-point chosen carefully within a value range in the uncongested regime of the MFD having positive slope and close to the critical state (total time spent) of the MFD function. The work in Keyvan-Ekbatani et al. (2012) aims at regulating the dynamic system around the desired chosen set-point, at which the system's total time spent is minimized. Note that the work in Keyvan-Ekbatani et al. (2012) do not allow direct consideration of the control constraints, but impose them after the design process, e.g. adjusting or fine-tuning the controller gains.

In Haddad and Shraiber (2014), a robust perimeter controller has been designed for an urban region with the MFD representation. The designed controller is a fixed PI-controller with proportional and integrator gains, which stabilizes the linearized system against MFD and parameter uncertainties. The robust control in Haddad and Shraiber (2014) was designed based on the principles of Quantitative Feedback Theory, Houpis, Rasmussen, and Garcia-Sanz (2006). The robust controller is also designed to handle control constraints within the design level in a systematic way, i.e. the control constraint is integrated in the closed-loop control with the help of the so-called describing function. Note that the describing function should be carefully chosen to guarantee satisfying the control constraint. The results showed that the controller has performed well for the whole state set, and not necessary for a value range nearby a set-point.

Following Daganzo (2007), Keyvan-Ekbatani et al. (2012), Haddad and Shraiber (2014), the current paper also deals with perimeter control problems for *single-region cities*. In Daganzo (2007), an optimal solution was presented analytically for maximizing the rate at which trips are served. However, the optimal control solution for one objective function and the optimality proof were presented for a basic dynamic model, i.e. one vehicle conservation equation without decomposing accumulations based on their destinations. The current paper focuses on a dynamic model that decomposes the accumulation into two vehicle conservation equations. The presented model in Haddad and Shraiber (2014) is utilized. Note that the model in Keyvan-Ekbatani et al. (2012) has a different form with different state variables. However, both works (Haddad & Shraiber, 2014; Keyvan-Ekbatani et al., 2012) formulated similar regulating control problems to which PI controllers were designed to regulate around a reference point. In this paper, *optimal* control problems are formulated with control constraints for a perimeter traffic flow at an urban region, the optimal feedback control policies are derived, and proofs of optimality are provided for three criteria: (i) maximum *total travelled distance* (TTD), (ii) minimum *total time spent* (TTS), and (iii) minimum integrated errors from accumulation *reference*, with the help of the modified Krotov-Bellman sufficient conditions of optimality. Both coupled and decoupled controllers are treated. The region is assumed to be a homogeneous region having a well-defined form of MFD with two traffic flow demands generated inside the region with internal and external destinations, and a generated traffic flow outside the region with a destination to the region.

2. Optimal perimeter control: problem definition

This paper deals with a perimeter control problem for a homogeneous urban region having a well-defined form of MFD, as schematically shown in Fig. 1. The flow dynamic equations for a homogeneous urban region have been already formulated in Haddad and Shraiber (2014), and they are briefly presented in this paper as follows. There are two state variables denoted by $n_{11}(t)$ and $n_{12}(t)$(veh), which respectively represent the number of vehicles traveling in the region with destination inside and outside the region at time t. The total accumulated number of the vehicles in the region is $n_1(t) = n_{11}(t) + n_{12}(t)$. The MFD links the *accumulation*, $n_1(t)$, and *trip completion flow*, defined as the output flow of the region. The MFD provides a low-scatter relationship, if congestion is roughly homogeneous in the region. The MFD is denoted by $G_1(n_1(t))$(veh/s), and it is assumed to be *concave*, *twice differentiable*, *non-negative*, and *strictly unimodal*. This assumption is based on many simulation and empirical results, e.g. in Geroliminis and Daganzo (2008). The MFD is defined as the trip completion flow for the region at $n_1(t)$: (i) the sum of a transfer flow, i.e. trips from the region with external destination (outside the region), plus (ii) an internal flow, i.e. trips from the region with internal destination (inside the region). The transfer flow is calculated corresponding to the ratio between accumulations, i.e. $n_{12}(t)/n_1(t)\cdot G_1(n_1(t))$, while the internal flow is calculated by $n_{11}(t)/n_1(t)\cdot G_1(n_1(t))$.

The traffic flow demands generated in the region with internal and external destinations are respectively denoted by $q_{11}(t)$ and $q_{12}(t)$ (veh/s), while $q_{21}(t)$ (veh/s) denotes a generated traffic flow outside the region with destination to the region, as schematically shown in Fig. 1(b). Following Haddad and Shraiber (2014), a perimeter control is introduced on the border of the urban region, where its inputs $u_{12}(t)$ and $u_{21}(t)(-)$ control the ratios of flows, $0 \le u_{12}(t), u_{21}(t) \le 1$, that cross the border from inside to outside and from outside to inside the region at time t, respectively, see Fig. 1(b). The control input constraints $0 \le u_{12}(t), u_{21}(t) \le 1$ describe the physical limits of controlling the ratios of flows that cross the border at time t. However, these values of upper and lower limits, i.e. 1 and 0, might raise practical implementation issues, e.g. if $u_{21}(t) = 0$ this means that the perimeter controller must prevent all vehicles traveling towards the region to enter probably for some time interval, which might result unreasonable control policy for the travellers. Moreover, with these values of limits