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Nonlinear analysis and control of a variable-geometry suspension system

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ABSTRACT

The paper proposes methods for both the analysis and the synthesis of variable-geometry suspension systems. The nonlinear polynomial Sum-of-Squares (SOS) programming method is applied in the analysis and it gives the optimal utilization of the maximum control forces on the tires. Moreover, the construction of the system can be based on the nonlinear analysis. The variable-geometry suspension system affects the wheel camber angle and generates an additional steering angle, thus the coordination of steering and wheel tilting can be handled. An LPV (Linear Parameter-Varying) based control-oriented modeling and control design for lateral vehicle dynamics are also proposed. The novelty of the method is the combination of the LPV-based control design and the SOS-based invariant set analysis. The simulation example presents the efficiency of the variable-geometry suspension system and it shows that the system is suitable to be used as a driver assistance system. In the SIL (software-in-the-loop) simulation both the dSPACE-AutoBox hardware and the CarSim simulator are used as standard industrial tools.

1. Introduction and motivation

The variable-geometry suspension system is a novel mechanism with which road holding can be improved. The control input of the system is the camber angle of the front and rear wheels, with which the driver is supported to perform the various vehicle maneuvers, such as a sharp cornering, overtaking or double lane changing. By changing the camber angles the yaw rate of the vehicle is modified, which can be used to reduce the tracking error relative to the reference yaw rate. The suspension determines such components as the height of the roll center and the half-track change. The roll center can be modified by setting the camber angle of the wheels. Thus, during maneuvers the control system must guarantee various crucial vehicle performances such as trajectory tracking, roll stability and geometry limits. The advantages of the mechanism are the simple structure, low energy consumption and low cost compared to other mechatronic solutions, see Evers, van der Knaap, and Nijmeijer (2008).

Several papers for various kinematic models of suspension systems have been published. A review of the first variable geometry systems was presented by Sharp (1998). A nonlinear model of the McPherson suspension system was published by Fallah, Bhat, and Xie (2009) and Németh and Gáspár (2012), while Hong, Jeon, and Sohn (1999) proposed a linearized model of the suspension, which can be used for active suspension design. In Habibi, Shirazi, and Shishesaz (2008) the effect of unnecessary steering due to the chassis roll angle was considered. An optimization method of the McPherson suspension system was introduced in Lee, Won, and Kim (2009). By using this model the kinematic parameters, such as camber, caster and king-pin angles, were examined. The kinematic design of a double-wishbone suspension system was examined by Sancibrian, Garcia, Viadero, and Fernandez (2010). The performance requirements often lead to conflicts and require a compromise considering the kinematic and dynamic properties, see Vukobratovic and Potkonjak (1999). The vehicle-handling characteristics based on a variable roll center suspension were proposed by Lee, Lee, Han, Hedrick, and Catala (2008). A rear-suspension active toe control for the enhancement of driving stability was proposed by Goodarzia, Oloomia, and Esmailzadehb (2010). A series active variable-geometry suspension which is able to improve the pitch attitude control of the chassis was found in Arana, Evangelou, and Dini (2015). This solution offers several advantages compared to the semi-active suspension, e.g. fail-safe behavior and negligible unsprung mass increment. Another field of variable-geometry suspension is the steering of narrow vehicles. These vehicles require the design of an innovative active wheel tilt and steer control strategies in order to perform steering similarly to a car on straight roads but in bends they tilt as motorcycles, see Suarez (2012). The active tilt control system, which assists the driver in balancing the vehicle and performs tilting in the bend, is an essential part of a narrow vehicle system, see Piyabongkarn, Keviczky, and Rajamani (2004).

The intervention of variable-geometry suspension systems requires the lateral motion of the suspension arm. In a real implementation it is realized using an electro-hydraulic actuator (Iman, Esfahani, &

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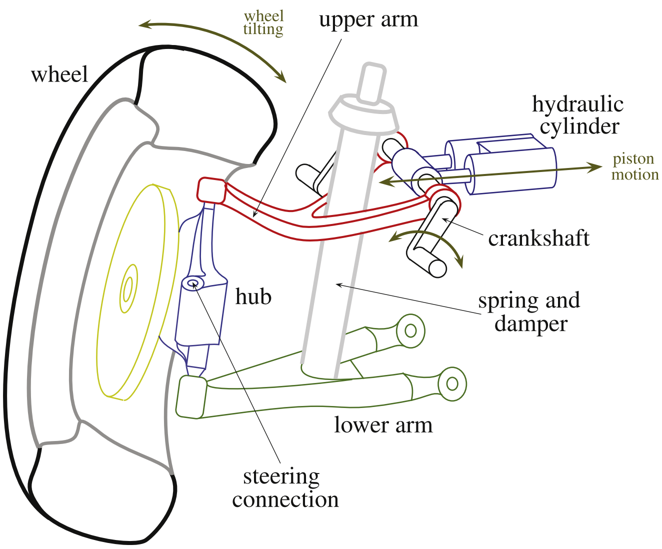

Fig. 1. Scheme of the suspension construction.

Mosayebi, 2010; Lee, Sung, Kim, & Lee, 2006; Schiehlen & Schirle, 2006) or an electric motor (Arana et al., 2015; Evers et al., 2008). In the paper the electro-hydraulic construction is considered as the real actuator of the system, see Fig. 1. In the illustrated example, the tilting and steering positioning of the wheel are yielded by the lateral motion of the upper arm. It is realized by the position control of the piston in the hydraulic cylinder (Németh, Varga, & Gáspar, 2014). The cylinder and the upper arm are connected through a crankshaft, whose rotation results in the lateral motion of the arm. The role of the crankshaft is to define the profile of the arm motion in lateral and vertical directions. Moreover, it has safety reasons in case of a hydraulic fault.

The paper proposes methods for both the analysis and the synthesis of the variable-geometry suspension system. The efficiency of variable-geometry suspension has been presented in preliminary works (Németh, 2013; Németh & Gáspár, 2014). In several automotive applications the linearized model of lateral dynamics is sufficient to design controllers for driver assistance systems. However, the comprehensive analysis of the suspension system requires nonlinear techniques to understand its operation in detail. In the analysis the nonlinear polynomial Sum-of-Squares (SOS) programming method is applied to calculate the shape of the Controlled Invariant Sets of control systems. It is an efficient tool to find feasible solutions. Important theorems in SOS programming, such as the application of Positivstellensatz, were proposed in Parrilo (2003). The analysis provides information on the optimal utilization of the maximum control forces on the tires.

The vertical position of the actuator can be calculated using the result of the nonlinear analysis. In an earlier paper a simultaneous design of robust control and the construction of a variable-geometry suspension was proposed in order to enhance vehicle stability, see Németh and Gáspár (2013). It was shown that there was a trade-off between the control design and the construction design, thus an optimization criterion which contained both the performances of the suspension construction and the performances of control design was formulated. In the current paper the construction parameter is fixed using the result of the nonlinear SOS-based analysis. It provides an optimal balance between the camber angle and the control system.

The orientation of wheels is modified by the variable-geometry suspension system, which affects both the steering angle and the camber angle. Thus, the integration of steering and wheel tilting can be handled by the variable-geometry suspension system. A further contribution of the paper is the LPV (Linear Parameter-Varying) based control-oriented modeling and control design for lateral vehicle dynamics. Based on the LPV modeling approach the nonlinear effects can be considered in the state space description. Furthermore this state space representation of the LPV model is valid in the entire operating region of interest. The advantage of LPV methods is that the controller meets robust stability and nominal performance demands in the entire operational interval, since the controller is able to adapt to the current operational conditions (Bokor & Balas, 2005; Packard, Seiler, Hjartarson, & Balas, 2014). In the control design both the steering angle and wheel tilting are handled. The novelty of the method is the combination of the LPV-based control design and the results of the analysis based on the SOS-based invariant set.

The paper is organized as follows. In Section 2 the nonlinear vehicle model with nonlinear characteristics of tire forces is formed and the mechanism of variable-geometry suspension system is presented. In Section 3 the SOS programming method is applied to analyze the nonlinear polynomial system. An iterative method for Maximum Controlled Invariant Set is proposed and the actuator efficiency of the variable-geometry suspension system is presented as a demonstration example. In Section 4 the LPV-based control design of the variable-geometry suspension system is developed. In Section 5 the efficiency of the variable-geometry suspension system is presented using a simulation scenario. Section 6 contains some concluding remarks.

2. Modeling of lateral vehicle dynamics for purposes of analysis

The variable-geometry suspension system has different effects on the wheels: the modification of the steering and the camber angles. The relationship between the two angles is determined by the construction of the suspension system. In this section the nonlinear vehicle model is created and the mechanism of variable-geometry suspension system is presented.

2.1. Nonlinear vehicle model

The modeling of tire forces is a crucial point of vehicle dynamics. Several tire models have already been published, see e.g. Pacejka (2004), Kiencke and Nielsen (2000), and de Wit, Olsson, Astrom, and Lischinsky (1995).

In this paper the lateral tire force characteristics using polynomial functions are approximated. The advantage of this model is its formulation, which can be efficiently used in the analysis methods, e.g. Lyapunov-based stability and controllability examinations. Furthermore, the polynomial functions fit the lateral tire force characteristics appropriately. Some further papers also motivate the use of polynomial descriptions, such as Hirano, Harada, Ono, and Takanami (1993), Sadri and Wu (2013), and López, Olazagoitia, Moriano, and Ortiz (2014).

The lateral dynamics of the vehicle is formulated by the following dynamic model, see Fig. 2:

$$J\ddot{\psi} = F_{lat,1}(\alpha_1)l_1 - F_{lat,2}(\alpha_2)l_2 = F_1(\alpha_1)l_1 - F_2(\alpha_2)l_2 + G(\alpha_1)l_1\gamma \tag{1a}$$

$$mv(\dot{\psi} + \dot{\beta}) = F_{lat,1}(\alpha_1) + F_{lat,2}(\alpha_2) = F_1(\alpha_1) + F_2(\alpha_2) + G(\alpha_1)\gamma \tag{1b}$$

where m is the mass of the vehicle, J is the yaw-inertia, l_1 and l_2 are geometric parameters. β is the side-slip angle of the chassis, $\dot{\psi}$ is the yaw-rate, δ is the steering angle and γ is the camber angle of the wheel. $F_{lat,1}(\alpha_1)$ and $F_{lat,2}(\alpha_2)$ represent lateral tire forces, which depend on tire side-slip angles α_1 and α_2.

The relationships between the tire side-slip angles for the front and rear axles, the steering angle of the vehicle and the side-slip angle of the chassis are $\tan(\delta - \alpha_1) = (l_1\dot{\psi} + v\sin\beta)/(v\cos\beta)$ and $\tan(\alpha_2) = (l_2\dot{\psi} - v\sin\beta)/(v\cos\beta)$. At stable driving conditions the tire side-slip angle α_i is normally not greater than 10° and the equations can be simplified by substituting $\sin\beta \approx \beta$ and $\cos\beta \approx 1$. Moreover, the relative error of these simplifications is less than 1%. Thus, the following side-slip angles of the front and rear axles can be approxi-