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Nonlinear state estimation for suspension control applications: a Takagi-Sugeno Kalman filtering approach

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A B S T R A C T

A new nonlinear state estimation approach, which combines classical Kalman filter theory and Takagi-Sugeno (TS) modeling, is proposed in this paper. To ensure convergence of the TS observer, conditions are derived that explicitly account for the TS model's confined region of validity. Thereby, the secured domain of attraction (DA) of the TS error dynamics is maximized within given bounds. The TS Kalman filtering concept is then applied to a hybrid vehicle suspension configuration, whose nonlinear dynamics are exactly represented by a continuous-time TS system. The benefit of the novel estimation technique is analyzed in comparison with the well-known EKF and UKF variants in simulations and experiments of a passive and an actively controlled suspension configuration in a quarter-car set-up. Employing a real road profile as disturbance input, the TS Kalman filter shows the highest estimation quality of the concepts studied. Moreover, as its computational complexity adds up to only one third of the one involved with the classical methods, the new approach operates remarkably efficient.

1. Introduction

Conventional passive suspensions are increasingly replaced by controlled suspension configurations already in medium class production vehicles. This is due to the fact that mechatronic components cannot only ease the primary conflict between ride comfort and ride safety, due to an uneven road surface, but at the same time address the objective to meet constraints on the maximum suspension deflection, see e.g. Hrovat (1997), Fischer and Isermann (2004). As fully active actuator configurations provide an entirely free force generation in the whole relevant frequency range of up to 25 Hz, these systems offer the largest potential to resolve the aforementioned conflict of goals, see Sharp and Crolla (1987), Jones (2005). However, this capability comes along with a substantial power demand, which together with the high manufacturing cost up to date prevents the integration of fully active actuators into the suspension of production vehicles. As only affecting the low-frequent motion of the body mass, the power demand can be reduced by using slow-active actuators with a bandwidth of up to 5 Hz. One example of this system class is the so-called spring mount adjustment, which features a low-bandwidth actuator in series to a passive spring, see Sharp and Hassan (1987). This hardware configuration has been integrated in series production in several upper class models since 1999, see Pyper et al. (2003). Yet, in current production vehicles mainly semi-active configurations are used to influence the movement of sprung and unsprung masses indirectly, by systematically changing the characteristics of individual, technically passive force components. For instance, in case of continuously variable dampers (CVD) the amount of damping can be adapted, see e.g. Karnopp et al. (1974), Savaresi et al. (2010). While both the chassis and the wheel dynamics can be addressed, the power consumption is comparatively small. However, the combination of a spring mount adjustment and a continuously variable damper, see Fig. 1 (left), is even able to combine the advantages of active and semi-active components: While the variable damper is able to control both the body and the wheel dynamics in an economic fashion, the slow-active actuator focuses on the motion of the body mass within the low-frequency range. As has been shown in Koch et al. (2010a), the performance potential of this so-called *hybrid* suspension comes close to that of a fully active suspension configuration at considerably lower power demand.

No matter which actuator set-up is considered, in order to exploit the performance potential of a mechatronic suspension configuration, an adequate control approach has to be applied. Therefore, relevant information about the current vehicle driving state must be at least partially known to control the dynamics in a beneficial manner. Depending on the control law that is employed,

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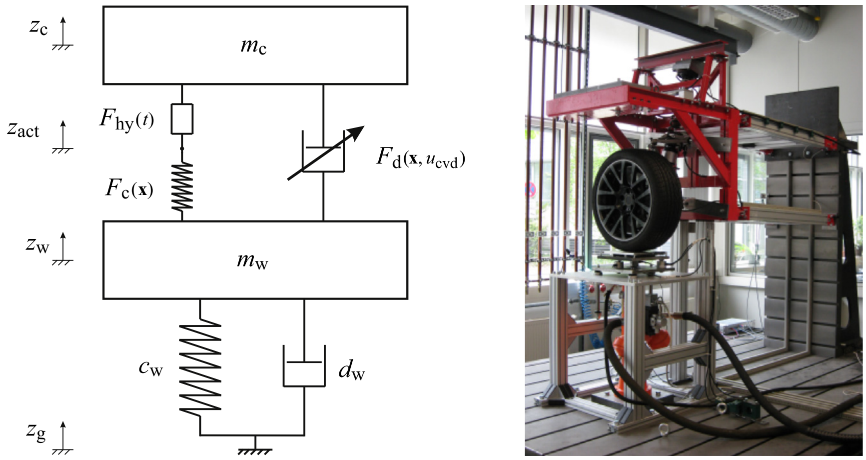

Fig. 1. Model (left) and test rig (right) of hybrid quarter-car configuration.

the quantities needed may range from solely the absolute body and the damper relative velocity, see e.g. Karnopp et al. (1974), the relative suspension deflection and the dynamic wheel load, see e.g. Koch et al. (2011), Pletschen et al. (2013), up to the full vertical dynamic state vector if state feedback control is applied, see e.g. Venhovens (1994), Hrovat (1997), Gao et al. (2006), Koch et al. (2008), Spirk and Lohmann (2012). Due to the complexity and cost of sensors, measuring all of these quantities is not possible. Frequently, in modern production vehicles only the suspension deflection as well as the body and wheel accelerations are available as measurement signals. Hence, estimation techniques have to be applied to calculate the required system states from the measured quantities at hand. Obviously, to a large extent the controller performance depends on the accuracy of the estimator scheme applied.

Kalman filter theory is often the method of choice in linear suspension state estimation applications, mainly for three reasons: firstly, its optimality properties in case of normally distributed uncertainties; secondly, its convenience in implementation, see Zarchan and Musoff (2005), and thirdly, the fact that the road height velocity of typical road profiles can approximately be represented by Gaussian white noise processes as discussed in Venhovens (1994). The design of Kalman filters with respect to active suspension control based on linear plant models is presented e.g. in Sharma et al. (1994) and Yu and Crolla (1998). Both apply Kalman filters to a fully active suspension setting, including an actuator in parallel to a linear spring.

Yet, when passive or semi-active dampers are part of the suspension configuration, their significant nonlinear force characteristics cannot be neglected without substantial deterioration in estimation quality. However, Krener and Respondek (1985) illustrated that for a class of nonlinear systems the state estimation error dynamics can be represented in such a way that errors evolve linearly. With regard to the suspension control problem, this concept has been adopted e.g. in Lindgärde (2002), Koch et al. (2010b) and Delvecchio et al. (2010). In these works, a linear suspension representation is extended by a nonlinear damper model, which generates a fictitious force input based on the damper relative velocity estimate. In Koch et al. (2010b), the estimation accuracy of state variables is further enhanced by running several Kalman filters of this kind in parallel.

Linear Kalman filter theory has been generally expanded to nonlinear systems by means of the *Extended* (*EKF*) and the *Unscented Kalman filter* (*UKF*) variants, see e.g. Zarchan and Musoff (2005) and Simon (2006). The fundamental idea of the widely used EKF is to linearize the original nonlinear system around the current Kalman filter state estimate at each instant of time, where the Kalman filter estimate itself is determined on the basis of the current linearized system dynamics. However, since the EKF involves the linear propagation of state mean and covariance, it may give unreliable state estimates when strong nonlinearities dominate the system dynamics. To overcome this issue Julier et al. (1995), presented a new sample-based approach for nonlinear Kalman filtering—namely the UKF—which reduces linearization errors by building on the so-called *unscented transformation*. Instead of linearizing the nonlinear dynamics, the system equations are evaluated at a number of so-called *sigma points*. Then, the ensemble mean and covariance of these nonlinearly transformed vectors will give an adequate estimate of the true mean and covariance of the nonlinear system state. The interested reader is referred to Simon (2006) and Grewal and Andrews (2008) for details. However, both the EKF and the UKF are computationally intensive, as they either involve the evaluation of Jacobians or the calculation of matrix square roots. Moreover, as a result of the approximations used by these nonlinear filters in one way or another, a rigorous proof of filter convergence is impeded, see e.g. Grewal and Andrews (2008). Applications of EKF and UKF concepts with respect to the vertical suspension control problem can be found in Koch et al. (2010b) and Graf et al. (2012).

By contrast, in the framework of *Takagi-Sugeno* (*TS*) *models* (Takagi and Sugeno, 1985), nonlinear system dynamics are represented by a convex interpolation of linear state space models evaluated at the vertices. By making use of the *sector nonlinearity transformation*, see Tanaka and Wang (2001), an *exact* description of the nonlinear dynamics can be obtained for a quite general class of nonlinear systems at least on a compact region of interest incorporating the origin. Hence, as the underlying subsystems of the TS model are linear, one can apply linear control and observer theory on each of the given linear models, before constructing a global controller or observer for the overall nonlinear system, respectively. Stability analysis as well as control and observer design for TS systems can then be posed as *linear matrix inequality (LMI)* problems, see e.g. Bergsten (2001) and Lendek et al. (2010).

Taking the benefits of linear Kalman filter theory and exact TS modeling together, a TS Kalman filtering approach has been presented in Pletschen and Badur (2014), which allows for state feedback in vertical suspension control. Therefore, the given nonlinear model of a hybrid suspension system in a quarter-car framework is represented as a family of linear state space models. Then, local Kalman filters are designed to estimate the states for each of these linear subsystems. Finally, the global state estimator for the nonlinear vehicle suspension is derived by appropriately combining the local observers. Simon (2003) describes a similar approach for fuzzy discrete-time systems, where optimality