Control Engineering Practice 63 (2017) 104–115

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Ensemble-based simultaneous input and state estimation for nonlinear dynamic systems with application to wildfire data assimilation

Huazhen Fang[a,*], Thayjes Srivas[b], Raymond A. de Callafon[b], Mulugeta A. Haile[c]

[a] Department of Mechanical Engineering, University of Kansas, Lawrence, KS 66045, USA
[b] Department of Mechanical & Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093, USA
[c] Vehicle Technology Directorate, US Army Research Laboratory, Aberdeen, MD 21005, USA

ARTICLE INFO

Keywords:
State estimation
Input estimation
Bayesian estimation
Nonlinear systems
Data assimilation
Ensemble Kalman filter

ABSTRACT

This paper presents a study of *simultaneous input and state estimation* for nonlinear dynamic systems, which is formulated as a filtering problem and defining the *simultaneous input and state filtering* (SISF) approach. The problem considers both unknown input and state variables, where the inputs offer a general description of unknown signals driving or existing in a system. To deal with the problem, a set of ensemble-based approaches for both filtering and smoothing are developed in a Bayesian statistical framework. The fundamental notion is to approximately represent the probability distributions of the unknown input and state variables conditioned on output measurements by ensembles of samples, propagate the ensembles to track the evolution of the probability distributions, and then extract the input and state estimates from the ensembles. The computational efficiency of the proposed method allows application characterized by high-dimensional nonlinear dynamic state representations. The results can be regarded as an extension of the celebrated ensemble Kalman filter (EnKF), which is intended for only state estimation by design, to the new inquiry on SISF. The proposed SISF approach is validated on the forty-dimension Lorenz'96 model. Furthermore, an elaborate demonstration of the effectiveness is illustrated on data-driven wildfire data assimilation, where uncertain wind conditions that act as an input driving the wildfire are estimated using SISF.

1. Introduction

This paper investigates *simultaneous input and state estimation* for nonlinear dynamic systems. Recent years have seen a growing research interest on this topic. While there are many results for linear systems, the study remains far from adequate for nonlinear systems, especially for high-dimensional ones. In this work, a filtering problem is formulated, motivating the development of a *simultaneous input and state filtering* (*SISF*) approach for general-form large-size nonlinear systems. The proposed methods can find wide application in the fields of control systems, signal processing and data assimilation to address a broad range of problems such as disturbance rejection (She, Kobayashi, Ohyama, and Xin, 2004), maneuvering target tracking (Li and Jilkov, 2002), fault detection (Patton, Frank, and Clarke, 1989; Schubert, Kruger, Wozny, and Arellano-Garcia, 2012), weather forecasting (Kitanidis, 1987) and oceanography (Fang and de Callafon, 2011).

Literature review: Since the 1960s, the control research community has shown considerable interest in the subject of SISF. The probably earliest work relevant to this topic is Friedland (1969), where state estimation in the presence of unknown inputs was studied. The inputs were assumed only partially unknown therein, modeled as stochastic processes with known statistics. This problem was investigated subsequently in a large body of work, e.g., Kitanidis (1987), Darouach and Zasadzinski (1997), Darouach, Zasadzinski, and Boutayeb (2003), and Cheng, Ye, Wang, and Zhou (2009), based on the minimum variance unbiased estimation (MVUE). Compared to these studies, joint estimation of both inputs and states is more appealing, because it can offer broader and deeper insights into a system in operation. The state of the art comprises many important results about SISF for linear dynamic systems. In this respect, a lead was taken in Mendel (1977) with an approach based on the Kalman filter (KF) to concurrently estimate the states and the white process noise (Mendel, 1977). From then on, many SISF methods have been proposed, and most of them are obtained through modifying the existing state estimation techniques, including the KF (Hsieh, 2000, 2010), moving horizon estimation (MHE) (Pina and Botto, 2006), H_∞-filtering (You, Wang, and Guan, 2008), sliding mode observers (Bejarano, Fridman, and Poznyak, 2007; Floquet, Edwards, and Spurgeon, 2007), and MVUE (Chavan, Fitch, and Palanthandalam-Madapusi, 2014; Fang and de Callafon, 2012; Fang, Shi, and Yi, 2008,

* Corresponding author.
E-mail addresses: fang@ku.edu (H. Fang), thayjes.srivas@gmail.com (T. Srivas), callafon@ucsd.edu (R.A. de Callafon), mulugeta.a.haile.civ@mail.mil (M.A. Haile).

http://dx.doi.org/10.1016/j.conengprac.2017.03.005
Received 14 August 2016; Received in revised form 12 March 2017; Accepted 13 March 2017
Available online 20 April 2017
0967-0661/

2011; Gillijns and De Moor, 2007a, 2007b; Yong, Zhu, and Frazzoli, 2016). Though reaching a certain level of maturity for linear systems, SISF can become rather complicated when it comes to nonlinear systems, with only limited results reported to date. In Corless and Tu (1998) and Ha and Trinh (2004), SISF methods are developed for a special class of nonlinear systems, which consist of a nominally linear part and a nonlinear part. The work (Hsieh, 2013) decouples the unknown inputs from the nonlinear system and then extends linear SISF methods to handle the estimation. In Fang, de Callafon, and Cortés (2013) and Fang and de Callafon (2011), a Bayesian statistical perspective is undertaken to investigate nonlinear SISF. The design is then extended in Fang, de Callafon, and Franks (2015) and Fang and de Callafon (2013) to solve the simultaneous input and state smoothing problem. As is observed, although there is prolific research for linear systems, more effort is needed to substantiate the development of nonlinear SISF methods. This task will be nontrivial especially when large-scale nonlinear systems are considered, because a mix of the high dimensions and nonlinearities will pose serious challenges for the design of computationally efficient and easy-to-execute methods.

Built on the authors' previous work (Fang and de Callafon, 2015), this paper perceives that ensemble-based estimation underpinning the ensemble Kalman filter (EnKF) can provide a promising means to address the large-size nonlinear SISF problem. The EnKF is among the most popular methods for data assimilation. As a Bayesian statistical filtering method, it considers the unknown state as a random vector. It then represents its probability density function (pdf) by a group of point samples and performs state estimation via operation on the samples. Such a treatment gives rise to two advantages. First, simply based on the Bayesian Gaussian update, its conceptual formulation is straightforward and easy to understand; second, it can be readily implemented owing to its derivative-free and efficient computation (Evensen, 2003). Since its advent in Evensen (1994), the EnKF has been studied extensively, with many progresses reported in the literature, e.g., Li and Xiu (2009), Harlim, Mahdi, and Majda (2014), Grooms, Lee, and Majda (2014), Li and Reynolds (2009), Hunta, Kostelich, and Szunyogh (2007), and Kalnay (2010). Along with its popularity, it has provided significant impetus to the development of solutions to various high-dimensional issues, e.g., ocean, atmospheric, hydrological data assimilation (Clark et al., 2008; Evensen, 1994, 2009; Houtekamer et al., 2005; Salman, Kuznetsov, Jones, and Ide, 2006). The EnKF can be modified to estimate unknown inputs along with state estimation through the augmentation approach (Evensen, 2003; Yang and Delsole, 2009). Specifically, the original system is augmented to include a new dynamic model assumed for the inputs, and then the EnKF applied to the combined model. This augmented EnKF (AEnKF) understandably has performance depending on the integrity of the input model, which, however, is practically difficult to guarantee or verify. This suggests a necessity for further quests for improved nonlinear SISF methods.

Contribution statement: Considering high-dimensional nonlinear systems, this paper offers a systematic development of a set of ensemble-based SISF algorithms as its main contribution. First, an examination of the SISF problem from the perspective of Bayesian estimation shows that it can be translated into tracking the probability distributions of the input and state variables given the output measurements. Under some Gaussian distribution assumptions, this can be further simplified as tracking the first- and second-order statistics (i.e., mean and covariance) of the input and state variables. Then using the Monte Carlo integration, the tracking can be accomplished through operation on ensembles associated with the input and state variables, thus leading to the new algorithms. The obtained algorithms build on an important statistical understanding of the SISF problem and with the ensemble-based operation are suitable for large nonlinear systems as expected. The proposed algorithms are evaluated through extensive simulations on the Lorenz'96 model in a comparison with the AEnKF. They are consistently shown to be capable of providing better estimation performance.

The applied contribution of the work lies in a case study for wildfire data assimilation. Wildfires are dangerous to the environment, ecology and human life, causing considerable damages worldwide every year. This has motivated an ongoing research of data-driven estimation of wildfire spread for effective monitoring and suppression. In the past, the EnKF has been used to address this challenge (Mandel, Beezley, Coen, and Kim, 2009). Here, the proposed SISF approach will be exploited to deal with not only the fire spread but also the unknown wind driving the fire growth, in the anticipation of achieving wildfire prediction at a higher accuracy. The simulation based on a practical wildfire case verifies the effectiveness of the proposed approach.

Organization: The remainder of this paper is organized as follows. Section 2 investigates the ensemble-based SISF problem for nonlinear systems with direct feedthrough. An extension of the results to systems without direct feedthrough is then presented in Section 3. A simulation-based study using the Lorenz'96 model is described in Section 4 to evaluate the effectiveness of the proposed algorithms and makes a comparison with the AEnKF technique. Section 5 then demonstrates the application of the ensemble-based SISF algorithms to the monitoring of wildfire spread. Finally, some concluding remarks are gathered in Section 6.

2. SISF for nonlinear systems with direct feedthrough

This section studies SISF for nonlinear systems with direct feedthrough. A Bayesian solution framework is presented first, followed by the development of an ensemble-based SISF algorithm.

2.1. The Bayesian paradigm

Consider the following general-form nonlinear dynamic system with direct input-to-output feed-through:

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k, \\ \mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k, \end{cases} \tag{1}$$

where $\mathbf{u} \in \mathbb{R}^{n_\mathbf{u}}$ is the input vector, $\mathbf{x} \in \mathbb{R}^{n_\mathbf{x}}$ the state vector, $\mathbf{y} \in \mathbb{R}^{n_\mathbf{y}}$ the measurement vector, and $\mathbf{w} \in \mathbb{R}^{n_\mathbf{x}}$ and $\mathbf{v} \in \mathbb{R}^{n_\mathbf{y}}$ mutually independent zero-mean white Gaussian noise sequences with covariances $\mathbf{Q}_k$ and $\mathbf{R}_k$, respectively. The mappings $\mathbf{f}: \mathbb{R}^{n_\mathbf{x}} \times \mathbb{R}^{n_\mathbf{u}} \to \mathbb{R}^{n_\mathbf{x}}$ and $\mathbf{h}: \mathbb{R}^{n_\mathbf{x}} \times \mathbb{R}^{n_\mathbf{u}} \to \mathbb{R}^{n_\mathbf{y}}$ define the state transition and measurement functions. For the above system, both $\mathbf{u}_k$ and $\mathbf{x}_k$ are unknown, and our objective is to estimate them simultaneously at any time k from the measurements $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k$, which is known as a filtering problem and referred to as SISF in this paper. The need for SISF can arise in many circumstances. An example is the monitoring of an industrial system subject to unknown disturbance, which requires the estimation of both the operational status (state) and the disturbance (input) (She et al., 2004). In maneuvering target tracking, the tracker often wants to concurrently estimate the state of the target, e.g., position and velocity, and the input, e.g., the acceleration (Li and Jilkov, 2002). Another example is the wildfire data assimilation to be discussed in Section 5. The spread of a wildfire often depends on local meteorological conditions such as the wind. On this account, a joint estimation of both the fire perimeters (state) and the wind speed (input) will help extract the maximum information from the measurement data toward improved fire growth monitoring.

The SISF problem can be dealt with from a Bayesian perspective, which has been a cornerstone historically for a variety of estimation techniques ranging from the classical KF to particle filters (Candy, 2009; Haug, 2012; Law, Stuart, and Zygalakis, 2015; Ristic, Gordon, and Arulampalam, 2004). Since $\mathbf{x}_k$ and $\mathbf{u}_k$ are unknown, they can be viewed as random vectors taking values subject to variations due to chance. Denote $\mathbb{Y}_k$ as the set of $\{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k\}$, which contains all measurements collected up to time k. Then for the sake of estimation, what is of interest is the joint conditional pdf $p(\mathbf{u}_k, \mathbf{x}_k | \mathbb{Y}_k)$, which