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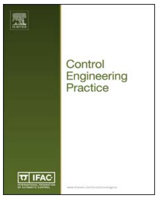

Robust pole location with experimental validation for three-phase grid-connected converters

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ABSTRACT

This paper provides design and experimental validation of robust current controllers for three-phase grid-connected converters. The main objectives here are: (i) to show that a simple polytopic model can be used for designing robust controllers for predominately inductive grids; (ii) to help in the choice of the control design parameter, based on a trade-off between an upper bound of the transient settling times and the control gain sizes. Linear matrix inequality based conditions are used to design the robust control gains with lower numerical complexity than similar conditions on literature. It is shown that small values the radius of pole location lead to better bounds for the transient responses, at the price of higher control gains. Good tracking of references for the grid currents is also illustrated in practice, allowing the closed-loop system to inject active and reactive power into the grid. Simulation and experimental results prove that the system connected to the grid can provide three-phase currents complying with requirements of an important international standard.

1. Introduction

Grid-connected converters are of great importance in renewable energy source systems, as wind and photovoltaic power plants, playing the role of controlling active and reactive power (Liserre, Teodorescu, & Blaabjerg, 2006; Teodorescu, Liserre, & Rodríguez, 2011; Willis & Scott, 2000). In this context, the control of the grid injected current is fundamental, and standards as the IEEE 1547 present limits of distortion, as total harmonic distortion (THD) no larger than 5%, and prescribed limits of each harmonic for the grid injected current (see IEEE, 2011). For this application, LCL output filters can provide a compact structure efficient for harmonic attenuation. Such filters exhibit a resonance peak that needs to be damped and, to keep high efficiency, active damping becomes an interesting alternative. Among the grid current controllers, one can cite PI-based and resonant controllers (Castilla, Miret, Matas, Garcia de Vicuna, & Guerrero, 2009; Dannehl, Fuchs, Hansen, & Thøgersen, 2010; Eren, Bakhshai, & Jain, 2012; Khajehoddin, Karimi-Ghartemani, Jain, & Bakhshai, 2011, 2014; Maccari et al., 2012; Massing, Stefanello, Gründling, & Pinheiro, 2012; Parker, McGrath, & Holmes, 2014; Peña-Alzola et al., 2014; Zmood & Holmes, 2003). PI-based and resonant controllers place poles at limit of stability, reducing the stability margins. For continuous-time design, PI-based controllers convert the tracking of a sinusoidal reference in a regulation problem, placing poles near the origin, and resonant controllers place imaginary poles at chosen frequencies. Both techniques must cope with rejection of disturbances, as grid harmonic voltages, and operation under uncertain parameters. Aiming at set-point tracking, PI is insufficient or even incapable of dealing with disturbance rejection. The results in Sun, Li, and Lee (2016) demonstrate that, in PI design, the objectives of set-point tracking and disturbance rejection are sometimes conflictive. To improve the disturbance rejection, a two-degrees-of-freedom PI is analyzed. Moreover, an experimental application in Sun, Li, Hu, Lee, and Pan (2016) further shows that PI is quite weak in handling strong disturbances even when it was well tuned. A robust control solution that can well accommodate both the disturbances and uncertainties are necessary. In this sense, robust control techniques become of interest.

Robustness against disturbances and parametric uncertainties is a fundamental topic in control theory and applications (Ackermann, 1993; Sun, Li, & Lee, 2015; Zhou, Doyle, & Glover, 1996). In practical applications, as in grid-connected converters, one has that physical plant parameters are not precisely known, but lie inside intervals for which only the lower and upper bounds are given. This leads to the design of controllers that ensure stability and performance for the

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entire parameter space (e.g. robust $\mathcal{H}_2$, $\mathcal{H}_\infty$, DLQR, pole placement controllers). In order to obtain robust controllers given by a set of fixed gains, it is important to use suitable models for the plant, as for instance, polytopic or linear fractional models (Gahinet, Nemirovskii, Laub, & Chilali, 1995). It is also important to obtain control design conditions that are able to handle the model with uncertainties. In this sense, linear matrix inequality (LMI) based conditions are attractive. LMIs can easily include performance criteria, as $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms and pole placement, and are efficiently solved by specialized algorithms (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994; Chilali, Gahinet, & Apkarian, 1999; Gahinet et al., 1995; Sturm, 1999). In case of plants affected by polytopic uncertainty, robust controllers can be designed in terms of a finite set of LMIs, assuring robust stability and performance for the whole domain of uncertainty. In this context, state feedback robust controllers are important, and the use of LMIs that provide less conservative results is of great interest, as given in de Oliveira, Bernussou, and Geromel (1999), Peaucelle, Arzelier, Bachelier, and Bernussou (2000), de Souza, Trofino, and de Oliveira (2000), Trofino and de Souza (2001), Shaked (2001), Apkarian, Tuan, and Bernussou (2001), de Oliveira, Geromel, and Bernussou (2002), Ebihara and Hagiwara (2004), Geromel and Korogui (2006), Oliveira, de Oliveira, and Peres (2011), and Morais, Braga, Oliveira, and Peres (2013). In general, the improvement of the accuracy of the synthesis conditions comes at the price of a larger computational effort. To establish a good tradeoff between complexity of the plant model and complexity of the control design conditions, aiming on robust stability and high performance, is still a matter that depends on the application under consideration.

The synthesis of state feedback robust controllers based on LMIs was successfully applied to power converters, with experimental validation. For instance, in Olalla, Leyva, El Aroudi, and Queinnec (2009, 2010, 2011), and Maccari, Montagner, Pinheiro, and Oliveira (2012), one has application of robust DLQR for DC–DC converters, with design in continuous-time domain. In Li, Sun, and Dai (2013), an $\mathcal{H}_\infty$ controller based on LMIs is designed to control the load voltage of an inductively coupled power transfer system. In Pereira, Flores, Bonan, Coutinho, and da Silva (2014), one has application of robust state feedback based on LMIs for uninterruptible power supplies (UPS), designed in continuous-time domain. Controllers designed in discrete-time domain are important for implementation in DSP and microcontrollers, suitable for industry applications. Discrete-time state feedback based design with experimental validation was addressed, for instance, in Ribas, Maccari, Pinheiro, Oliveira, and Montagner (2014), for $\mathcal{H}_\infty$ control of UPS output stage. In Gabe, Montagner, and Pinheiro (2009), one investigates a robust discrete-time control for grid-connected converters with experimental validation, using partial state feedback based on LMIs, with control gains design based on an iterative process, including resonant controllers. This work indicates the viability of this control design for this application. In Maccari et al. (2014), robust discrete-time pole location for single-phase grid-connected converters was addressed, with the design of the resonant controller gains not depending on iterative process. Robust stability and performance were illustrated, and the experimental validation of the results according to pertinent norm was carried out. In Maccari, Santini, Oliveira, and Montagner, (2013), a robust DLQR based on LMIs was successfully applied for three-phase grid-connected converters.

This work presents the design and analysis of a controller by means of LMIs for three-phase inverters connected to a grid subject to an uncertain inductance L_g. The contribution with respect to the robust pole location in Maccari et al. (2014) is its extension, with experimental validation, for the three-phase case, and also the analyses of the relationships between the radius r and the size of the control gains and with the settling times of the transient responses, the evaluation of phase and gain margins and the interpretation of the $\mathcal{H}_\infty$ norm as an output admittance of the closed-loop system. With respect to Maccari, Do Amaral Santini, Pinheiro, de Oliveira, and Foletto Montagner (2015), the advances are to provide a control design alternative based solely on the choice of one parameter, and to show that the numerical complexity of the control computation in this paper is lower than the one in Maccari et al. (2015), allowing a design much simpler and faster, mainly when the number of resonant controllers increase. Time domain simulations and experimental results confirm the good tracking of sinusoidal references in coordinates α and β, and also confirm the good quality of the grid injected three-phase currents, which comply with requirements of harmonic content from the IEEE 1547 Standard.

2. Continuous-time uncertain model

Consider the circuit depicted in Fig. 1. The LCL filter represents the plant, whose control variables are given by the output voltages of the three-phase inverter, the controlled outputs are the currents injected in the grid, and the disturbance inputs are given by the grid voltages, which can also include harmonics. The control signals are generated by a digital signal processor (DSP), based on the measurements of the state variables of the LCL filter. The grid is supposed as predominantly inductive, being modeled, per phase, by a voltage source in series with an inductance L_{g2}, assumed as an uncertain parameter, belonging to a given interval. It is known that the DC bus voltage control and the synchronization with the grid voltage are important issues for grid connected applications (Bianchi, Egea-Alvarez, Junyent-Ferré, & Gomis-Bellmunt, 2012; Umbrí et al., 2014). However, in the current paper it is assumed that the DC input voltage is constant and that the voltage at the point of common coupling (PCC) is already synchronized with the grid voltage (Cardoso, de Camargo, Pinheiro, & Gründling, 2008).

In order to obtain a model for the system described in Fig. 1, it is also assumed that the converter uses ideal switches and the switching frequency is much higher than the grid fundamental frequency, allowing, for control design purposes, to neglect the effect of PWM harmonics in the voltages generated by the converter (Teodorescu et al., 2011).

Thus a set of equations in the state variables for the LCL filter can be obtained, generating a model with 9 variables, 3 control inputs and 3 disturbance inputs, represented by

$$\frac{d\mathbf{x}_{abc}}{dt} = \mathbf{A}_{abc}\mathbf{x}_{abc} + \mathbf{B}_{uabc}\mathbf{u}_{abc} + \mathbf{B}_{dabc}\mathbf{v}_{dabc} \tag{1}$$

with

$$\mathbf{A}_{abc} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{A}_{p1} & \mathbf{0}_{3\times3} \\ \mathbf{A}_{p2} & \mathbf{0}_{3\times3} & -\mathbf{A}_{p2} \\ \mathbf{0}_{3\times3} & \mathbf{A}_{p3} & \mathbf{0}_{3\times3} \end{bmatrix},$$

$$\mathbf{B}_{uabc} = \begin{bmatrix} -\mathbf{A}_{p1} \\ \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} \end{bmatrix}, \quad \mathbf{B}_{dabc} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} \\ -\mathbf{A}_{p3} \end{bmatrix} \tag{2}$$

and with

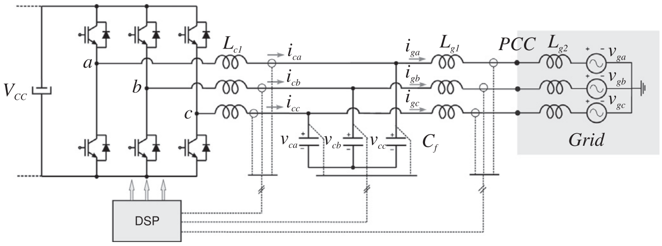

Fig. 1. Three-phase inverter connected to the grid by means of LCL filter.