Available online at www.sciencedirect.com
ScienceDirect
Computer methods in applied mechanics and engineering
Comput. Methods Appl. Mech. Engrg. 196 (2006) 551–565
www.elsevier.com/locate/cma

Adaptive dual boundary element method for solving oblique incident wave passing a submerged breakwater

K.H. Chen [a], J.T. Chen [b,*]

[a] *Department of Information Management, Toko University, Chia-Yi, Taiwan*
[b] *Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan*

Received 8 May 2003; received in revised form 10 August 2005; accepted 9 June 2006

Abstract

In this paper, an adaptive mesh scheme in boundary element computations for solving the propagation of oblique incident wave passing a breakwater is developed. The purpose is to demonstrate cost savings engendered through adaptivity. The computation is performed on meshes of constant boundary elements, and are adapted to the solution by locally changing element sizes (h-version). Two error indicators obtained from the dual integral equations are used as local error norm, which are essential ingredients for all adaptive mesh schemes in boundary element method (BEM). The evaluation of two local error norms is the norm between the obtained boundary condition and the given boundary condition. The two error tracking curves are in good agreement with their shapes. Two examples show that the adaptive mesh based on the error indicators converge to the solution more efficiently using the same number of elements than does uniform mesh discretization for each segment of the boundary. To check the validity of the present formulation, the transmission and reflection coefficients are determined by using the developed dual BEM program, and are compared well with those of experiment and analytical solution using the eigenfunction expansion method.

Keywords: Adaptive mesh; Dual boundary integral equation; Breakwater; Oblique incident wave; Hypersingular equation; Local error norm; Error indicator

1. Introduction

Nowadays, a submerged breakwater is often constructed to protect a harbor from waves of the open sea. The primary function is to reduce the wave energy transmitted through it and to have the advantages of allowing water circulation, fish passage, providing economical protection. A suitable arrangement of a barrier may act as a good model for a breakwater. Numerical solutions solving the water scattering problem of incident wave passing a submerged breakwater have been developed on the basis of the boundary element method [12,17,20,21], respectively. Following the theory of dual integral equations and BEPO2D program developed by Chen and Hong [3,4], the dual BEM program has been modified to solve the water wave problem of normal incident water wave past a submerged thin barrier by Chen et al. [7]. The reflection and transmission of oblique incident water wave past a submerged barrier with a finite width were studied using the conventional BEM under the linear wave theory [17]. Later, the extension to the problems of oblique incident wave passing a ‘‘thin’’ water barrier has been done by using the dual BEM [7].

Boundary element method is often utilized to solve several kinds of water wave problems as above mentioned. The discretization process, which transforms a continuous system into a discrete system using finite number of degrees of freedom, results in errors. The discretization error is defined as a measure of difference between the exact solution and the numerical approximation. A quantitative

* Corresponding author. Tel.: +886 2 2462 2192x6140/6177; fax: +886 2 2463 2375.
E-mail address: jtchen@mail.ntou.edu.tw (J.T. Chen).

0045-7825/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.cma.2006.06.002

analysis for determining the quality of numerical simulation can be obtained from error estimation. The quantitative measure provides a criterion for adapting characteristics of discrete model for mesh generation so as to derive more reasonable result. Obtaining a reliable error estimation [2,9–11,13,14,22,24] is to guarantee a certain level of accuracy for the numerical results, and is a key factor of the adaptive mesh procedure [2,13,14,23]. Thus, estimation of the discretization error in the numerical method is the first step in adaptive mesh generation. A large number of studies on adaptive BEM have been done by Kamiya et al. [13], using sample point error estimation. However, the error stems not only from the discretization procedure, but also from the mismatch of the collocation points on the boundary. Zarikian et al. [24] and Paulino et al. [22] used pointwise error estimation to study the convergency of the interior problem by using the hypersingular residuals. Both the singular integral equation (UT) and hypersingular integral equation (LM) in the dual BEM can independently determine the unknown boundary data for the problems without a degenerate boundary [3,5]. The residuals obtained from these two equations can be used as indexes of error estimation. This provides a guide for remeshing without the problem of mismatch of the collocation points on the boundary in the sample point error method. By creating more divisions in the boundary mesh where the estimated error is large, the error curve will change where the error is redistributed. If the boundary density is the exact data, then both equations (UT and LM) are automatically satisfied to have zero residual. The actual error resulted from one equation (UT or LM) is smaller as the mesh becomes fine. The actual error can not be large to simultaneously satisfy the two constraints although rigorous proof is not offered. Since the posteriori error originates from the inconsistency by substituting the solved boundary data into the UT or LM equation instead of comparison with the exact solution, it is useful that the posteriori error on each element indicates a relative quantity in comparison with that of the other elements in the same level of mesh. Since the exponentially decaying Green function is accompanied with the boundary density in the boundary integrals, the strategy to refine the elements where error is relatively large is reasonable. This strategy has been applied well by solving for Laplace and Helmholtz problems [6,16].

Here, we will focus on the adaptive BEM for the problems of oblique incident wave passing a barrier in two dimensions. Mathematically speaking, we will extend the Laplace and Helmholtz equation to the modified Helmholtz equation. To the authors' best knowledge, no paper on the adaptive BEM for the water wave scattering problems has been found.

Following the successful experience in the Helmholtz equation [6], the dual boundary integral equations are extended to solve the modified Helmholtz equation which governs the problems of oblique incident wave passing a submerged breakwater. Two residuals, from the dual integral equations are used for local error norm as the error indicators which are essential ingredients for all adaptive mesh schemes in the dual boundary element method (DBEM). The solving process of two local error norms are obtained as follows: first of all, the boundary unknown data can be solved using the singular equation (UT method) (or hypersingular equation (LM method)) and let the solved boundary unknown data be known. Next, we reformulate the boundary condition, by using the hypersingular equation (LM method) (or singular equation (UT method)). Thus, the evaluation of two local error norms can be the distance between the reformulated boundary condition and the given boundary condition. According to the residuals by collocating the points along the boundary, error tracking curves can be constructed pointwisely. Based on the error indicators, adaptive scheme can be considered. Two examples will be performed to demonstrate the cost effectiveness of adaptive scheme. To check the validity of the present formulation, the transmission and reflection coefficients will be

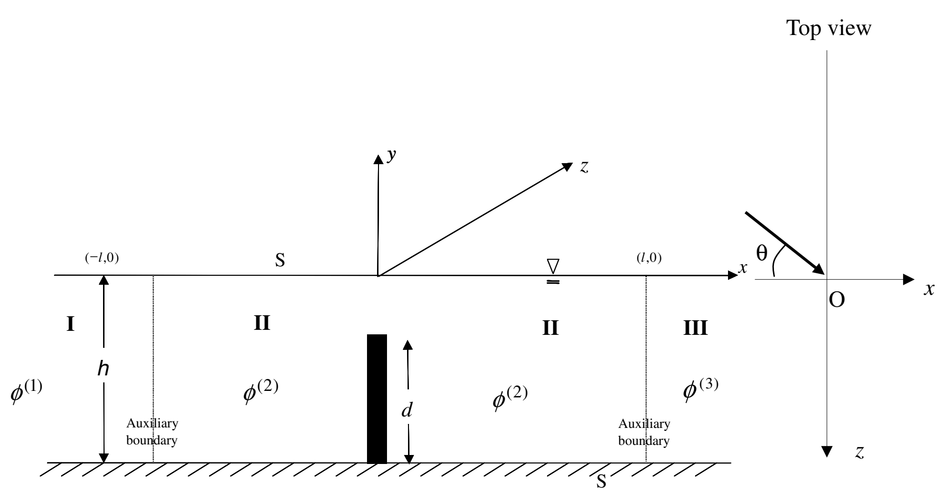

Fig. 1. Definition sketch of the water scattering problem of oblique incident wave past a rigid barrier.