Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 196 (2007) 4059–4074
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

# The control volume formulation to model the convective–diffusive unsteady heat transfer over the 1-D semi-infinite domain

N.J. Rodriguez [a,*], K. Davey [b], J.L.S. Gaytan [c]

[a] Instituto Tecnológico de Oaxaca, DGEST, Av. Ing. Víctor Bravo Ahuja # 125, Esq. Calz. Tecnologico, C.P. 68030, Oaxaca, Mexico
[b] The University of Manchester, P.O. Box 88, Sackville Street, Manchester M60 1QD, UK
[c] Centro de Ingeniería y Desarrollo Industrial, CIDESI, 76130 Querétaro, Mexico

Received 11 April 2006; received in revised form 5 March 2007; accepted 14 March 2007
Available online 21 April 2007

## Abstract

The aim of this work is to develop a formulation called the control volume capacitance method which can provide stabilized numerical results for convective–diffusive problems. The formulation employs the control volume approach to derive an artificial capacitance by equating heat and energy flow unsteady equations. The formulation is designed for materials that exhibit combined unsteady heat transfer and mass transport. Here, energy and in particular thermal capacitance are related to physical (real/material) and non-physical (non-real/mesh) quantities, respectively. The use of the control volume approach ensures that predicted temperature fields correspond exactly with the unsteady flow energy equation and so providing an extremely stable formulation. Unsteady convective–diffusive heat transfer is performed in the 1-D semi-infinite domain to demonstrate the applicability of the method. To validate the method, predictions are compared against exact and popular numerical schemes.


*Keywords:* Control volume; Capacitance; Heat; Convection; Oscillation

## 1. Introduction

The standard Galerkin FEM solution (called the Bubnov–Galerkin method) of the convective–diffusive equation is known to yield numerical oscillation for values of the Peclet number grater than one [1]. A number of numerical schemes have been proposed in order to prevent numerical instabilities, that is, that the solution has a physical meaning. In the first attempts to solve this problem the under-diffusive character of the Galerkin FEM (and the analogous central finite difference scheme) for convective–diffusive problems was corrected by adding "additional diffusion terms" to the governing equations [1,2]. The relationship of this approach with the upwind finite difference method [2] leads to the derivation of a variety of Petrov–Galerkin FEM. All these methods can be interpreted as extensions of the standard Galerkin Variational form of the FEM by adding residual-based integral terms computed over the element domains.

Among the many stabilization methods we name the Upwind FEM [3,4], the Streamline Upwind Petrov–Galerkin (SUPG) method and the pressure stabilizing/Petrov–Galerkin (PSPG) formulation for incompressible flows are some of the most prevalent stabilized methods [5–9], the Taylor–Galerkin method [10,11], the generalized Galerkin method [12,13], the Galerkin Least Square method and related approaches [14–16], the Characteristic Galerkin method [17,18], the Characteristic Based Split method [19], the Sub-grid Scale method [20–23], the Residual Free Bubbles method [24], the Discontinuous Enrichment Method [25], the Streamline Upwind with Boundary Terms method [26], the so called "shock-capturing" or "discontinuity-capturing" schemes [32–36] and the Finite Calculus (FIC) approach based on expressing the equation of balance of fluxes in a domain of finite size [37,38].

* Corresponding author. Tel./fax: +52 951 501 5016x231.
*E-mail address:* juarez1@hotmail.com (N.J. Rodriguez).

0045-7825/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.cma.2007.03.021

It is interesting that many of the stabilized FEM can be recovered using the FIC method. The FIC method has been successfully applied to problems of convection–diffusion [37–41], convection–diffusion–absorption [42,43], incompressible fluid flow [44–47] and incompressible solid mechanics [48,49].

Basically all the methods make use of a single stabilization parameter which suffices to stabilize the numerical solution along the velocity (streamline) direction. The computation of this stabilization parameter, known as "$\tau$", for multidimensional problems is usually based on extensions of the optimal value of the parameter for the simpler 1-D case. It involves a measure of the local length scale and other parameters such as the element Reynolds and Courant numbers. Various element lengths and $\tau$'s were proposed starting with those in [5,6,50,51], followed by the one introduced in [36], and those proposed in the subsequently reported SUPG-based methods. Specific attempts to design the stability parameter for multidimensional problems in the context of the Petrov–Galerkin formulation have been recently reported [27–31].

Among the most popular stabilize classical finite element methods for convective–diffusive problems are the SUPG method, which have been successfully applied to many different situations [6,15,4]. The method corresponds to adding a consistent term providing an additional diffusion in the streamline direction. The amount of such additional diffusion is tuned by the parameter $\tau$ that must be chosen in a suitable way. Several recipes have been proposed for the choice of $\tau$ [52]. The method has been proved to have a solid mathematical basis in several cases of practical interest [5,6,27]. Nevertheless, the need for a suitable convincing argument to guide the choice of $\tau$ is still considered as a major drawback of the method by several users. Another major shortcoming of Petrov methods is that the setting for upwind parameters is not known *a priori* [53]. Moreover Petrov methods are known to aggravate the well known inaccuracies in the results when the Peclet number is lower than one.

A limited number of analytical solutions are available for convective–diffusive heat transfer problems. These are mostly for the semi-infinite solid which moves with velocity $v$ along the $x$-axis and has various surface conditions at the boundary $x = 0$. Positive velocities correspond to an accreting medium (such as a snowfield which is being supplemented by continuous falls). Negative values of velocity correspond to removal of material at $x = 0$ by erosion or similar processes.

Unfortunately problems can occur for the accreting medium when convection overpowers the diffusion process. Numerical instabilities are a common result and an example of what happens is illustrated in Figs. 1 and 2.

The aim of this work is to develop a formulation called the control volume capacitance method (CVCM) which can provide stabilized numerical results for convective–diffusive problems. The formulation which is based on the control volume approach eliminates the need of employing stabilization parameters $\tau$. Instead, additional diffusion is incorporated into the capacitance term. To demonstrate the applicability of the method, unsteady convective–diffusive heat transfer is performed in the classical 1-D semi-infinite domain.

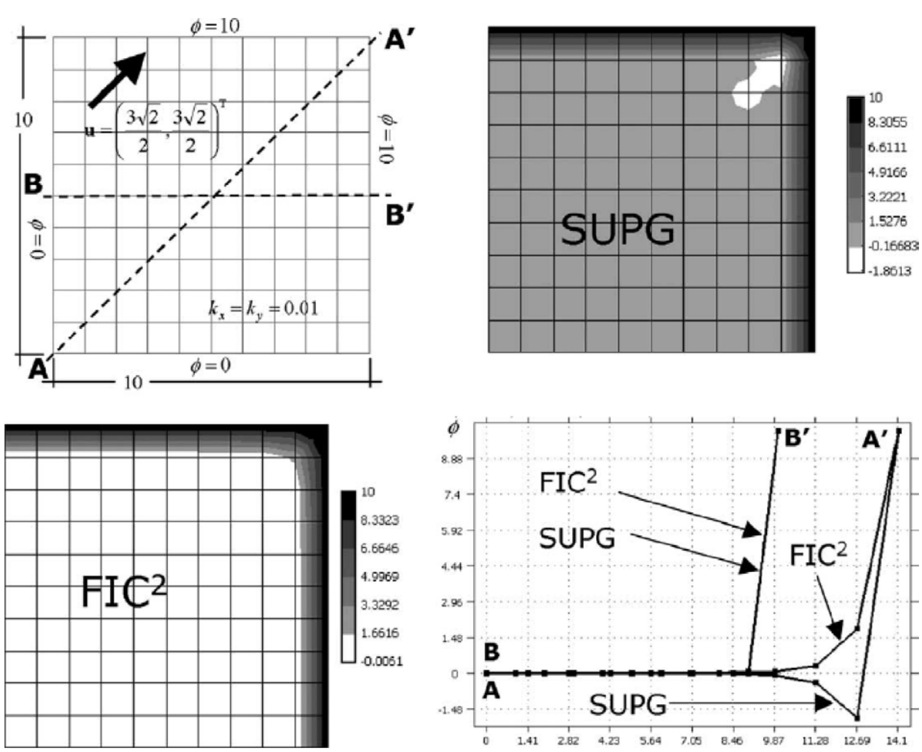


Fig. 1. Square domain with uniform Dirichlet conditions, upward diagonal velocity and zero sources. SUPG and FIC solutions obtained with a structured mesh of 10 × 10 linear four node square elements [64].