Flexible extended harmonic domain approach for transient state analysis of switched systems

Uriel Vargas [a,*], Abner Ramirez [a], George Cristian Lazaroiu [b]

[a] Electrical Engineering Department, CINVESTAV Guadalajara, Guadalajara, Mexico
[b] Department of Power Systems, University POLITEHNICA of Bucharest, Splaiul Independentei 313, 060042 Bucharest, Romania

ARTICLE INFO	ABSTRACT
	A mature mathematical technique to model power systems, aimed at computation of harmonics dynamics in a step-by-step fashion, is the extended harmonic domain (EHD). The EHD is capable of accounting for an arbitrary number of linearly-spaced sequentially-numbered harmonics; however, the dimensions of an EHD-based model are prone to explode if a large number of harmonics are considered. This fact makes computationally impossible the harmonics dynamics study when high-frequencies, inherent of electronic devices, are involved. This paper presents a flexible extended harmonic domain (FEHD) approach that permits to include an arbitrary number of (non-sequential) harmonics in any state variable of the system. The reasoning to use distinct harmonic content for each state variable is that in a general network (especially those involving alternative energy sources), state-variables exhibit distinct harmonic content. Such different contents mainly depend on the network topology, e.g., switched devices involved, filter arrangements utilized, etc. As shown in the paper, the proposed FEHD approach is able to provide reduced-order EHD models including high-frequency ripple information; thus, leading to computational savings while keeping accuracy compared to traditional EHD-based models and averaged-value models. A case study, involving a three-phase photovoltaic system in closed-loop operation, is presented to validate the proposed methodology. The results provided by the proposed methodology are verified with those given by the power system computer-aided design/electromagnetic transients including DC (PSCAD/EMTDC) simulation tool. © 2017 Elsevier B.V. All rights reserved.

1. Introduction

Power electronic converters (PECs) have become an integral part of electrical distribution networks, especially in the distributed generation area.

Inclusion of PECs in power electrical networks conveys both positive and negative impacts into quality of power [1–3]. Analysis of harmonic dynamics in switched networks plays a major role to understand power quality phenomena. Besides harmonic distortion, harmonics dynamics permit to study potential risks of harmonic resonance, mechanical vibration in transformers, flickering, among others [1–3].

There are multiple publications on the analysis of PECs at fundamental power frequency. In those publications, converters are simplified and represented via controllable voltage sources at fundamental frequency. This consideration does not allow a detailed harmonic study [4]. On the other hand, there are few studies focusing on power quality assessment in switched networks, having as basis electromagnetic transient (EMT) software tools, e.g., [5,6]; however, those studies require a post-processing routine based on the windowed fast Fourier transform (WFFT) to obtain the dynamics of harmonic frequencies. In Ref. [7], a power quality assessment study, based on steady-state, is carried out from experimental data.

Traditionally, switched networks are simulated via time-domain (TD) techniques; nevertheless, as consequence of switching phenomena, very small time-steps have to be employed leading to both large simulation times and excessive computational resources. This drawback has been alleviated by employing generalized averaged-value models (GAVMs) at cost of inability of showing ripple information [8]. Extended harmonic domain (EHD) models provide and alternative method for harmonic transient simulations, readily providing harmonics dynamics in a step-by-step fashion [9].

* Corresponding author.
E-mail addresses: uvargas@gdl.cinvestav.mx (U. Vargas), abner.ramirez@cinvestav.mx (A. Ramirez), cristian.lazaroiu@upb.ro (G.C. Lazaroiu).

EHD has been employed to model different electrical and electronic systems including photovoltaic (PV) and wind generation systems [10–19], and to obtain steady-state solutions in a single matrix/vector operation [20–32]. However, as consequence of high-frequencies, inherent of PWM schemes, EHD models present the issue of large dimensions. For example, a scalar instantaneous variable corresponds in the EHD to a $2h+1$ (being h the maximum harmonic under study) size EHD vector. This diminishes attractiveness of classical EHD for application to modern electrical network simulations. In Ref. [19], reduced-order EHD models of wind and photovoltaic generation systems are presented; nonetheless, all state variables in the system must contain the same predefined number of harmonics, based on the distinct PECs involved in the system. A hybrid method, which relies on interfacing generalized averaged model and EHD, has been proposed in Ref. [32]; however, some improvements such as controls are still pending.

In Ref. [33], three reduced-order EHD models have been proposed: i) representing a balanced three-phase system as a single phase system, taking into account appropriate EHD matrices, ii) considering harmonic pattern of PWM modulation techniques as known, i.e., for a frequency modulation index selected as an odd multiple of three, odd and triplen harmonic components vanish, and iii) eliminating differential equations based on the previous knowledge of steady-state solution given by a full harmonic domain (HD) model. However, reformulation of the accurate EHD model, in ii) and iii), involving the new set of selected harmonics, is not studied.

This paper proposes a *flexible* EHD (FEHD) approach. The FEHD permits to obtain reduced-order EHD switched network models by selecting distinct harmonics (not necessarily sequentially-numbered) in each part/state-variable of the system. The two basic steps of the FEHD approach are: a) to identify dominant frequencies for each part/state-variable of the switched network and b) to reformulate the frequency-domain convolution between switching functions and voltage/current variables. It is also shown in the paper that the FEHD approach is a generalization of interfacing GAVMs and classical EHD. Compared to existent research in the specialized literature on harmonic domain modeling, two major features of the proposed FEHD technique are: i) it achieves computational savings without losing accuracy, compared to classical (full-order) EHD models, and ii) it is able to provide ripple information, unlike GAVM technique. Furthermore, unlike TD-based models, FEHD does not require a post-processing routine as harmonics dynamics are provided intrinsically by the model. A case study of a switched network involving control actions is presented to support the above statements. The waveforms by the proposed approach are verified with the PSCAD/EMTDC software tool [34].

2. Switched system modeling

Electrical waveforms in power systems are typically assumed to be purely sinusoidal with constant magnitude and frequency; however, the increasing use of PECs and nonlinear loads has become a major issue as they introduce distinct frequencies superimposed into fundamental frequency voltage/current waveforms, impacting neighboring network elements, such as mains supply [3].

Conventionally, a switched system can be represented in TD as the nth order linear time-periodic (LTP) system:

$$\dot{x}(t) = a(t)x(t) + b(t)u(t),$$
$$y(t) = c(t)x(t) + d(t)u(t), \tag{1}$$

where a, b, c, and d represent the state, input, output, and feedthrough matrices, respectively; x, y, and u denote state-variable, output and input column vectors, respectively.

The EHD modeling technique has become a mature power system simulation approach where harmonics dynamics are readily available in a step-by-step fashion [9–19]. Basically, the EHD technique transforms the LTP system Eq. (1) into a linear time-invariant (LTI) system as shown in Eq. (2).

$$\begin{aligned}\dot{x}(t) &= a(t)x(t) + b(t)u(t) \\ y(t) &= c(t)x(t) + d(t)u(t)\end{aligned} \underset{TD}{\overset{EH}{\rightleftarrows}} \begin{aligned}\dot{X}(t) + NX(t) &= AX(t) + BU, \\ Y(t) &= CX(t) + DU,\end{aligned} \tag{2}$$

where the differentiation matrix, given by $diag\{-jh\omega_o, \ldots, -j\omega_o, 0, j\omega_o, \ldots, jh\omega_o\}$, is repeated n times (with ω_o being the fundamental power frequency) and arranged as a block-diagonal matrix N. Matrix A (similarly for B, C and D) in Eq. (2) is an expanded version of the original state matrix of order n, i.e., $a(t)$; any element of $a(t)$ involving switching functions becomes a Toeplitz-type sub-matrix in A containing its corresponding harmonic coefficients [2], as illustrated by Eq. (3). Such harmonic coefficients are provided by a PWM scheme, for example, for a given operating point.

$$A_k = \begin{bmatrix} a_o & a_{-1} & \cdots & a_{-h} & 0 & \cdots & 0 \\ a_1 & a_o & a_{-1} & \cdots & \ddots & \ddots & \vdots \\ \vdots & a_1 & a_o & \ddots & \cdots & \ddots & 0 \\ a_h & \vdots & \ddots & \ddots & \ddots & \cdots & a_{-h} \\ 0 & \ddots & \vdots & \ddots & a_o & a_{-1} & \vdots \\ \vdots & \ddots & \ddots & \vdots & a_1 & a_o & a_{-1} \\ 0 & \cdots & 0 & a_h & \cdots & a_1 & a_o \end{bmatrix}. \tag{3}$$

Finally, $X(t)$ in Eq. (2) corresponds to a complex-valued EHD column vector with the harmonic content of all state-variables [9]. For example, for the generic ith state we have:

$$x^i(t) = \sum_{k=-h}^{h} x_k(t)e^{jk\omega_o t} \underset{TD}{\overset{EH}{\rightleftarrows}} X^i(t), \tag{4a}$$

where:

$$X^i(t) = [x_{-h}(t) \quad \cdots \quad x_o(t) \quad \cdots \quad x_h(t)]^T. \tag{4b}$$

In Eq. (4b), T represents transpose and h denotes the highest harmonic under study. The dimensions of the EHD system Eq. (2) are $n \times (2h+1)$. It should be noted that traditional EHD models consider sequentially numbered harmonics, as in Eq. (4b).

Furthermore, the steady-state of the EHD system Eq. (2) can be readily obtained in the HD by setting to zero its derivative term, i.e.,

$$\begin{aligned}NX &= AX + BU, \\ Y &= CX + DU.\end{aligned} \tag{5}$$

EHD provides an elegant and natural framework for resolving LTP systems; however, the dimensions of EHD systems explode when considering a large number of harmonics, as seen in Eq. (4b). This implies a major drawback, especially when a finite difference method, involving inversion of matrices, is utilized for the solution of Eq. (2); nevertheless, a single matrix/vector operation in the HD, as dictated by Eq. (5), permits to provide the steady-state solution in a straightforward manner.

On the other hand, GAVMs provide a modeling framework in which specific harmonics can be accounted for. Ordinarily, low-order harmonics, e.g., DC and fundamental, are involved in GAVMs as the main objective is to represent *averaged dynamics*, consequently reducing computational resources [8]. A major drawback of GAVMs is their inability to provide ripple information.