Electric Power Systems Research 151 (2017) 329–337

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Earth wires currents calculation by tableau analysis

Jacek Klucznik

Gdansk University of Technology, Faculty of Electrical and Control Engineering, Narutowicza 11/12, 80-233, Poland

ARTICLE INFO

Article history:
Received 3 October 2016
Received in revised form 5 May 2017
Accepted 5 June 2017

Keywords:
Fault current distribution
Earth faults
Earth wires
Overhead transmission lines

ABSTRACT

The paper presents a new method to compute a phase-to-earth fault current distribution in overhead transmission line. The method is based on the tableau analysis of an unbalanced, multi-wire model of a transmission line. It allows for the calculation of current distribution in overhead earth wires along the transmission line. Additionally, it may be used to calculate currents in a tower's earthing system as well as voltages over every tower of the transmission line. The method can be used for analysis of any type of transmission line (single circuit, double circuit and multi-circuit lines) with any number of earth wires. The calculation results may be used for complex designs of transmission line earthing systems, earth wire thermal rating selection, and the evaluation of shock hazards.

1. Introduction

1.1. Background

Earth faults are one of the most common disturbances in a power system. It is estimated that more than 80% of the malfunctions in transmission lines are associated with single phase-to-earth faults. During an earth fault, electric current flows through the earth, through a tower's construction, and through overhead earth wires (shield wires). Therefore, during earth faults, significant currents flow in areas where they do not usually flow in the normal operating state, thus causing many risks. Earth fault currents cause the earth potential to rise near the line, resulting in increased step and touch voltages, which can be a threat to human life. Currents in overhead earth wires can result in heat damage because the cross-section of earth wires is always less than that of phase wires. These threats necessitate a precise determination of currents and voltages during an earth fault, not only for the faulty tower but also for the whole transmission line.

For the safe operation of transmission lines and for the safety of humans, it is necessary to calculate the following values:

- Current distribution in earth wires to enable the selection of sufficient thermal ratings for their conductors.
- Tower earthing current (i.e. currents flowing through the tower and its footing resistance to the earth), to calculate earth potential rise, a factor for evaluating electric shock hazards near towers.

1.2. Overview of existing computational methods

The problem of calculating current distribution during a phase-to-earth fault is not new. Over the last fifty years, many researchers have attempted to solve this problem. Selected ideas, found in recently developed computational methods, are briefly reviewed below.

Some existing methods are based on finite-difference equations [1–3]. This approach is based on the assumption that the transmission line is sufficiently long to be treated as an infinite line. Moreover, all spans of the transmission line are assumed to have equal length, self-impedances, mutual impedances, and tower footing resistance. The above assumptions allow for reducing the transmission line to an equivalent lumped impedance to estimate a value of fault current initially. Then, the earth wire current and tower current are recursively determined for each section of the transmission line.

One of the most popular methods, used in many practical projects, is the double-sided elimination method [4]. In contrast to methods based on finite-difference equations it has no limitation with regard to line length. The analysed transmission line (or lines) is reduced to an equivalent simple circuit by conversion span by span from both sides of the line simultaneously. The simplified, equivalent circuit allows for fault current and earth current calculation. The rest of the currents in the earthing system can be obtained afterwards. The method is limited to transmission lines with a single earth wire.

In paper [5], the authors used the so-called driving-point impedance technique for earth wire current computations. The main improvement in the method is the use of the multi wire system, which allows for complicated wire layout analysis. However, the method is limited to a single line connecting two substations.

E-mail address: jacek.klucznik@pg.gda.pl

http://dx.doi.org/10.1016/j.epsr.2017.06.001

A different way to solve the problem is presented in papers [6–8], where the authors use a ladder circuit to calculate components of initial fault current. When a calculation of the tower earthing current or tower voltage is needed for a specified tower, the current and the voltage can be estimated from the single phase-to-earth initial fault current and a so-called division factor (split factor) [9,10]. Another method used to calculate the distribution of earth fault current, is based on the two-port theory, this is presented in paper [11]. All those methods are incapable of analysing complex, branched transmission lines.

In recent years the problem of earth fault current distribution calculation has become commonplace again. Nowadays, researchers focus on improving computational techniques to analyse complex and more realistic study cases i.e. multi circuit transmission lines, transmission lines running in parallel, different types of earth wires (in the case of double earth wires systems). Two trends are commonly seen in recently published papers. The first tendency is the use of universal power system analysis software to analyse problems of earthing systems. Many researchers e.g. [12–15] use EMTP software for that purpose, which allows for the analysis of complex phase and earth wires layouts, by using a multi wire LCC model. The second direction of research projects being conducted at present is leading to the development of computational algorithms focused directly on the earthing system of transmission lines. Two new methods are particularly interesting. The first [13] is an improved concept of the Dawalibi method, where the authors extended the original method of double side elimination [4] to analyse the influence of a second circuit and a second earth wire. The second method [16] is based on the multi-wire model of a transmission line, where currents are calculated by iterative nodal analysis to include magnetic couplings.

This paper is part of the second trend, where a new dedicated calculation method is proposed for solving the problem. The computing capabilities of modern computers are superior to devices used 15 or more years ago. In modern computer systems, the ability to store large amounts of data and apply sparse matrix computation techniques allows for the use of a much simpler and versatile method. The proposed method is based on the analysis of an unbalanced multi-wire transmission line model using Kirchhoff's laws and linear equation solving techniques.

2. Proposed calculation method

2.1. Tableau analysis of electrical circuits

Tableau analysis can be used to solve any linear electrical circuit [17], however, other methods such as nodal analysis, modified nodal analysis, or mesh analysis are more frequently used. The tableau analysis method is based on solving a set of linear equations that include Kirchhoff's current law (1), Kirchhoff's voltage law (2), and branch equations (3):

$$\mathbf{I}\mathbf{A}_I=\mathbf{0} \tag{1}$$

$$\mathbf{V}-\mathbf{A}_I^T\mathbf{E}=0 \tag{2}$$

$$\mathbf{K}_I\mathbf{I}+\mathbf{K}_V\mathbf{V}=\mathbf{V}_s \tag{3}$$

where

- **I**—vector of branch currents,
- **V**—vector of branch voltages,
- **E**— vector of nodal voltages,
- $\mathbf{V}_s$—vector of source voltages,
- $\mathbf{A}_I$—incidence matrix,
- $\mathbf{K}_I$—current coefficients matrix,
- $\mathbf{K}_V$—voltage coefficients matrix.

Eqs. (1)–(3) are rewritten as a single matrix equation (4) to solve for unknown vectors **I**, **E**, and **V**.

$$\begin{bmatrix}\mathbf{A}_i & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{1} & -\mathbf{A}_I^T\\ \mathbf{K}_I & \mathbf{K}_V & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{I}\\ \mathbf{V}\\ \mathbf{E}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{V}_s\end{bmatrix} \tag{4}$$

In this paper, a further reduction is proposed to decrease the number of equations. The vector of branch voltages **V** in (3) is replaced by a combination of nodal voltages (2). The final equation set can then be presented as:

$$\begin{bmatrix}\mathbf{A}_i & \mathbf{0}\\ \mathbf{K}_I & \mathbf{K}_V\mathbf{A}_I^T\end{bmatrix}\begin{bmatrix}\mathbf{I}\\ \mathbf{E}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{V}_s\end{bmatrix} \tag{5}$$

Solving the above equation, which can be done with linear algebra procedures, gives branch currents and nodal voltages for specified source voltages $\mathbf{V}_s$, allowing for the calculation of earth wire currents, tower earth currents, and tower voltages.

2.2. Defining the problem

The analysis of overhead transmission lines during a short-circuit requires a proper mathematical model. The proposed model of a single circuit transmission line with a single overhead earth wire is shown in Fig. 1. The model is based on the following assumptions:

- The transmission line model is divided into (n + 1) sections corresponding to spans.
- Each span (i) between towers t_{i-1} and t_i, is represented by four self-impedances (Z_{Ai}, Z_{Bi}, Z_{Ci}, Z_{Ei}) of phase wires (A, B, C) and

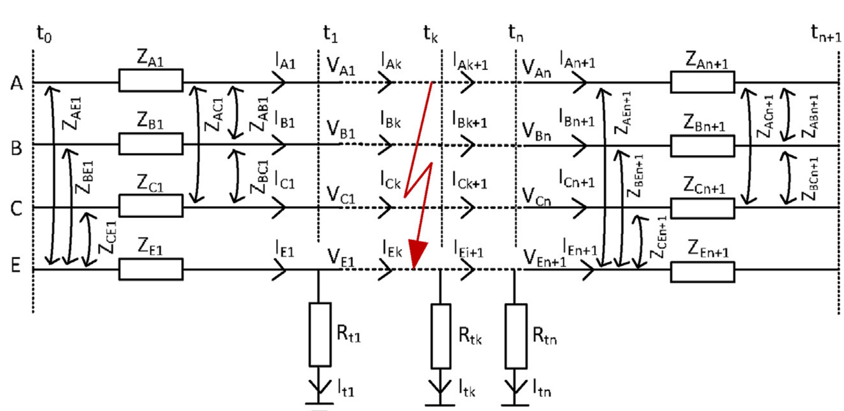

Fig. 1. Single circuit overhead transmission line model with single earth wire.