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An optimal vector fitting method for estimating frequency-dependent network equivalents in power systems

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ABSTRACT

Vector fitting (VF) algorithms have become popular and powerful tools for estimating models formed by rational basis function (RBF) expansions. In this paper, we first translate the well-known continuous time-domain VF method (cTD-VF) to a discrete time-domain framework. We denote this new domain VF method by dTD-VF. Differently from the cTD-VF, the dTD-VF formulation does not rely on a numerical approximation of convolution integrals and, as a result, it can be easily implemented with a variety of RBF sets. The second part of this paper shows that the proposed dTD-VF can also be transformed into a novel instrumental variable (IV)-dTD-VF technique, which is shown to have a guaranteed optimal solution at convergence. Moreover, this important optimality property does not depend on the nature of the noise that corrupts the data (for instance, if it is white or colored). Two case studies highlight the advantages of using the proposed methods. One of these examples consists of modeling the admittance characteristics of a power system implemented as a frequency-dependent network equivalent (FDNE).

1. Introduction

Linear system identification based on models formed by rational basis function (RBF) expansions involves, in the most general cases, the minimization of a multiple-parameter nonlinear least-squares objective function [1–4]. In order to address this nonlinear problem, several identification methods make use of the so-called *vector fitting* (VF) iterative algorithms [5].

Also known as robust variants of the original Sanathanan–Koerner [6] and Steiglitz–McBride [7] iterations, VF algorithms have become very popular within the power systems area, with reports of successful applications in transient analysis of frequency-dependent network equivalents (FDNE) [8,9], wideband modeling of transmission lines and transformers [10–12], and advanced packaging [13,14]. In a more recent perspective, VF formulations have also shown to be powerful tools for passive macromodeling implementations [5,15,16] as well as for estimating oscillatory (electromechanical) modes during post-disturbance moments in power systems [17].

For system identification based on time-domain data, the continuous time-domain VF technique introduced by [14] (here denoted by cTD-VF) remains as one of the most adopted techniques. As in its classical frequency-domain counterpart method [12], the cTD-VF is also specifically designed for estimating models formed by *continuous-time* partial fractions. However, since time-domain tabulated data are always in the form of *discrete-time* samples, the cTD-VF's practical implementation relies on a numerical approximation of convolution integrals [5,8,14]. Moreover, if the data used for estimation are corrupted by colored noise, recent results on VF algorithms show that they never converge to any local minimum of their corresponding nonlinear least-squares objective functions (NLSOFs) [18]. When it comes to power systems signals such as active power and operation frequency, the so-called ambient and measurement noise sources are usually regarded to be colored and white, respectively [19,20].

In this paper, we first translate the cTD-VF to a discrete time-domain framework. We denote this new domain VF method by discrete time-domain VF (dTD-VF). Since dTD-VF operates directly in the discrete time-domain, its practical implementation does not rely on a numerical approximation of convolution integrals. This also allows dTD-VF to be easily used not only with partial fractions, but also with more general RBF sets, such as the discrete-time Takenaka–Malmquist orthonormal basis functions [1,4].

As will be shown in Sections 3 and 5 of this paper, the dTD-VF formulation can be easily implemented and usually provide a good solution after convergence. As other standard VF algorithms [18],

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however, dTD-VF does not guarantee that, if it converges, a local optimum of its NLSOF is obtained. This fact motivates us to propose an instrumental variable (IV) version of the dTD-VF, which we denote by IV-dTD-VF. This IV formulation guarantees that the gradient local optimality condition of the NLSOF is necessarily satisfied at convergence. Moreover, this important result does not depend on the nature of the noise that corrupts the data. As a result, more accurate RBF models may be obtained even if this noise is colored.

The proposed IV-dTD-VF is shown to be naturally more computationally expensive than the dTD-VF and, therefore, we also propose in this paper an approximation procedure which minimizes this problem without losing the local optimality property of the method. Some of the ideas of this IV formulation can be related with the 'mode 2 iteration' introduced by Steiglitz and McBride in [7] and revisited in [21]. These two papers describe a polynomial-based system identification technique, whereas here we develop a VF (RBF) method. Since IV-dTD-VF uses a Steiglitz–McBride iteration-based approach for selecting poles of possibly general RBF sets, it can also be considered as a generalization of recently proposed (frequency-domain) IV techniques such as those in [22,23].

The paper is organized as follows. In Section 2, we briefly summarize the problem of linear RBF model identification in discrete time-domain. In Section 3, we introduce the dTD-VF method. In Section 4, we propose the IV version of the method, that is, the IV-dTD-VF formulation. In Section 5, two case studies are used to compare the proposed methods with the cTD-VF technique. The first case study consists of modeling the admittance characteristics of a power system implemented as a FDNE, whereas the second case study aims at identifying a third order system corrupted by colored noise. Finally, Section 6 addresses the conclusions of this work.

2. Problem statement

In the discrete time-domain, a stable single-input single-output (SISO) linear time-invariant system can be described in terms of its scalar input sequence $u_0(k)$ and its scalar output sequence $y_0(k)$ as [24]

$$y_0(k) = G_0(q)u_0(k) + v(k), \tag{1}$$

where q denotes the forward shift operator and $v(k)$ represents a sequence of stochastic additive disturbance at the system output, which can be but is not restricted to white noise. In power systems analysis, $u_0(k)$ and $y_0(k)$ may represent a variety of signals such as voltage, active or reactive power, operation frequency, etc. In the specific context of FDNE modeling, $G_0(q)$ commonly represents the frequency-dependent admittance or impedance characteristics of a single component (for instance, a cable or a transmission line) or, in a more realistic scenario, a combination of components [5,8]. In these cases, $u_0(k)$ and $y_0(k)$ naturally assume the form of voltage and current signals.

In this paper, the major identification goal consists of finding a RBF model whose dynamic behavior is sufficiently close to the dynamic behavior of $G_0(q)$. Such a RBF model must have a mathematical structure in the form:

$$G(q, \mathbf{c}, \mathbf{a}) = \frac{B(q, \mathbf{c}, \mathbf{a})}{\hat{F}(q, \mathbf{a})} = c_0 + \sum_{i=1}^{n} c_i \Phi_i(q, \mathbf{a}). \tag{2}$$

In (2), the ratio between polynomials $B(q, \mathbf{c}, \mathbf{a})$ and $\hat{F}(q, \mathbf{a})$ is expanded into a series of n rational basis functions $\{\Phi_i(q, \mathbf{a})\}_{i=1}^{n}$ [1], where $\mathbf{c}$ is the vector of unknown coefficients $\mathbf{c} = [\, c_0 \ \cdots \ c_n \,]^{\mathrm{T}}$ and $\mathbf{a}$ is the vector of unknown transfer function poles $\mathbf{a} = [\, a_1 \ \cdots \ a_n \,]^{\mathrm{T}}$. Note that $\mathbf{a}$ is also assumed to parametrize the RBF set $\{\Phi_i(q, \mathbf{a})\}$, whereas n also stands for the model order. Selecting this model order has already been discussed by several authors in the literature [1], mainly for one-parameter RBF sets such as Laguerre and Kautz functions.

In this paper, we address the general multiple-parameter case where the nth order RBF model is parametrized by a set of (possibly) different poles $a_1, \ldots, a_n$. In this context, partial fractions [5,25]

$$\Phi_i(q, \mathbf{a}) = \frac{1}{q - a_i}, \ \ i = 1, \ldots, n \tag{3}$$

or the well-known discrete-time Takenaka–Malmquist orthonormal basis functions defined by Eq. (4) [26,1] are commonly chosen to serve as RBFs.

$$\Phi_i(q, \mathbf{a}) = \frac{\sqrt{1 - |a_i|^2}}{q - a_i} \prod_{j=1}^{i-1} \left(\frac{1 - a_j^* q}{q - a_j} \right), \ \ i = 1, \ldots, n. \tag{4}$$

Based on a sequence of N time-domain input-output samples extracted from system (1), estimating the unknown RBF model parameters $\mathbf{c}$ and $\mathbf{a}$ in (2) leads to the following nonlinear least-squares problem

$$\begin{aligned} &\mathrm{argmin}_{\mathbf{c},\mathbf{a}} \sum_{k=1}^{N} (y_0(k) - G(q, \mathbf{c}, \mathbf{a}) u_0(k))^2, \\ &= \mathrm{argmin}_{\mathbf{c},\mathbf{a}} \sum_{k=1}^{N} \left(y_0(k) - \left(c_0 + \sum_{i=1}^{n} c_i \Phi_i(q, \mathbf{a}) \right) u_0(k) \right)^2 . \end{aligned} \tag{5}$$

In some cases, the model poles $\mathbf{a}$ may be chosen beforehand, based on a prior knowledge about the dominant dynamics (poles) of system $G_0(q)$. In such cases, (5) reduces itself to a linear least-squares problem, since only coefficients $c_0, \ldots, c_n$ remain unknown. Unfortunately, a prior knowledge about the dominant dynamics of the system is usually not available.

In the following sections, we propose two different methods for iteratively estimating $\mathbf{a}$ and $\mathbf{c}$. Both methods are based on transforming (5) into a sequence of linear problems, where coefficient sets are then estimated by making use of pre-specified update-dependent poles.

3. The dTD-VF method

Considering many successful frequency-domain VF techniques in the literature (see, e.g., [5,13]), we shall here address the complete nonlinear estimation problem in (5) by using an alternative model structure in the form

$$\bar{G}(q, \theta, \bar{\mathbf{a}}) = \frac{B(q, \theta, \bar{\mathbf{a}})}{F(q, \theta, \bar{\mathbf{a}})} = \frac{B(q, \theta, \bar{\mathbf{a}})/\hat{F}(q, \bar{\mathbf{a}})}{F(q, \theta, \bar{\mathbf{a}})/\hat{F}(q, \bar{\mathbf{a}})}, \tag{6}$$

with

$$\frac{B(q, \theta, \bar{\mathbf{a}})}{\hat{F}(q, \bar{\mathbf{a}})} = r_0 + \sum_{i=1}^{n} r_i \Phi_i(q, \bar{\mathbf{a}}), \tag{7}$$

$$\frac{F(q, \theta, \bar{\mathbf{a}})}{\hat{F}(q, \bar{\mathbf{a}})} = 1 + \sum_{i=1}^{n} d_i \Phi_i(q, \bar{\mathbf{a}}). \tag{8}$$

In these equations, $\bar{\mathbf{a}}$ is assumed to be a set of n specified poles and θ is the (alternative) coefficient vector:

$$\theta = [\, r_0 \ \ \cdots \ \ r_n \ \ d_1 \ \ \cdots \ \ d_n \,]^{\mathrm{T}}. \tag{9}$$

The main reason for defining the model structure (6) is to provide an alternative way for estimating the poles of a RBF model. Since $\bar{\mathbf{a}}$ is known, then the poles of $\bar{G}(q, \theta, \bar{\mathbf{a}})$ (roots of polynomial $F(q, \theta, \bar{\mathbf{a}})$) depends solely on the unknown coefficients $\{d_i\}$. Once $\{d_i\}$ are estimated, the corresponding poles of $\bar{G}(q, \theta, \bar{\mathbf{a}})$ can be